Separation of Two Inseparable Logics

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Separation of Test-Free Propositional Dynamic Logics over Context-Free Languages

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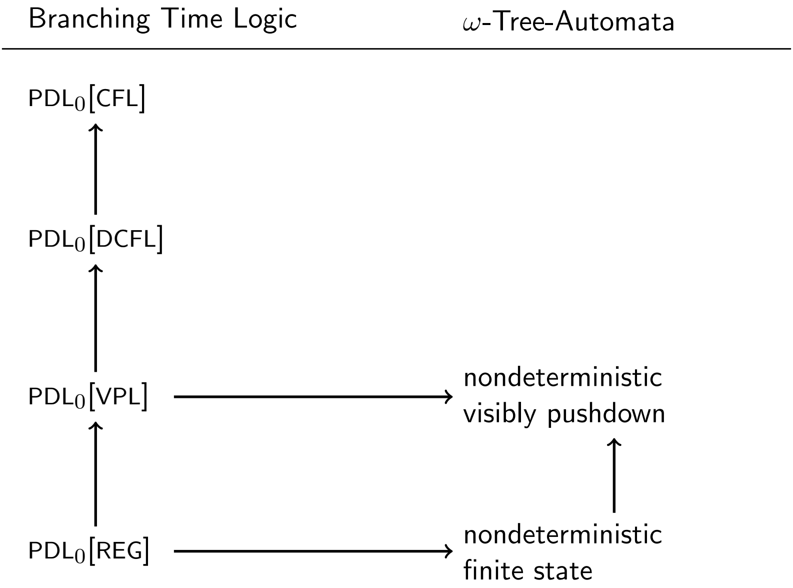
Motivation—Where are we?



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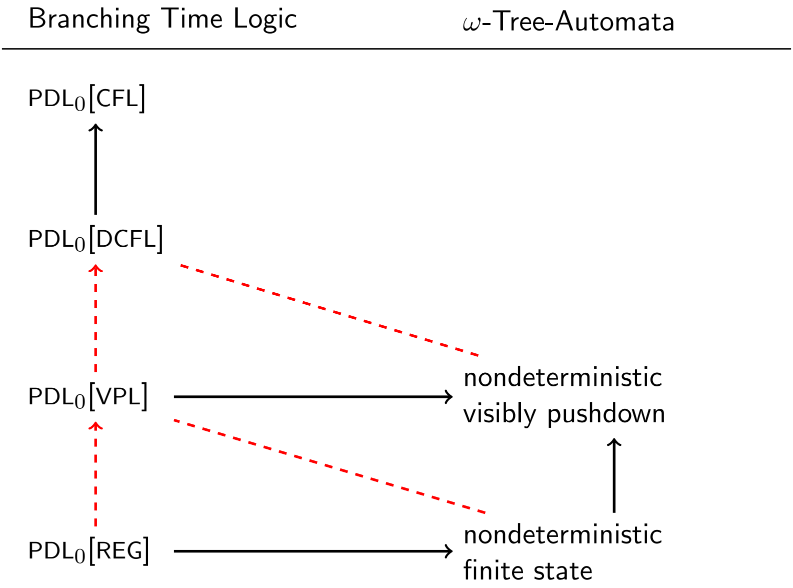

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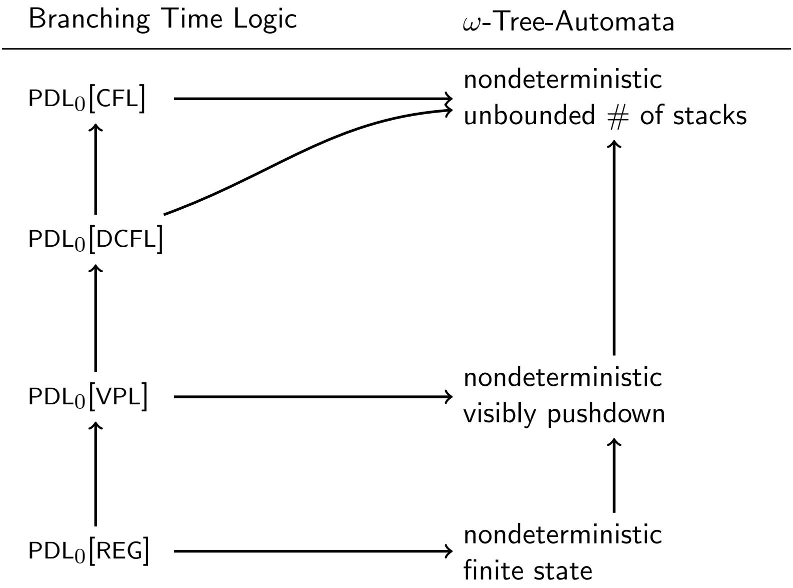
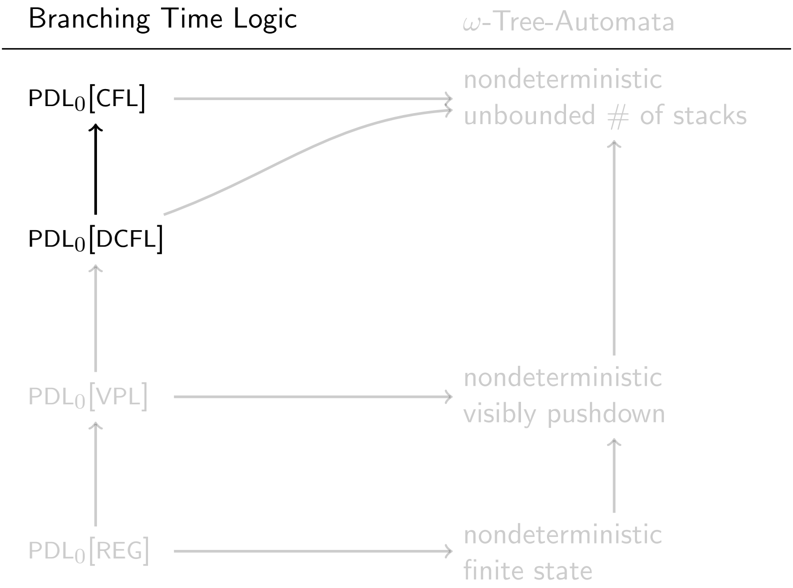


Motivation—What is our Target?



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Syntax of $\mathrm{PDL}_0[\cdot]$

[Fischer/Ladner '79], [Berman/Paterson '81] [Harel/(Pnueli/Stavi '83 | Raz '90)]
[Löding/Lutz/Serre '04], [Axelsson/Hague/Kreutzer/Lange/L. '10]

Let $\mathfrak{A}$ be a class of languages $L \subseteq \Sigma^+$.

Definiton (Syntax of $\mathrm{PDL}_0[\mathfrak{A}]$)

$$\varphi ::= \mathtt{ff} \mid \mathtt{tt} \mid p \mid \neg p \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathtt{EF}^L \varphi \mid \mathtt{AG}^L \varphi$$

where $L \in \mathfrak{A}$, and p denotes a proposition.

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Let $\mathfrak{A}$ be a class of languages $L \subseteq \Sigma^+$.

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$$\begin{aligned}\varphi \;::=\; & \mathtt{ff} \mid \mathtt{tt} \mid p \mid \neg p \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \\ & \mathtt{EF}^L \varphi \mid \mathtt{AG}^L \varphi\end{aligned}$$

where $L \in \mathfrak{A}$, and p denotes a proposition.

Test-free means

- no tests such as $\langle(\varphi?\ a)^*\rangle\psi$,
- no $\mathtt{EU}/\mathtt{ER}/\mathtt{AU}/\mathtt{AR}$-formulas.

Semantic of $\text{PDL}_0[\cdot]$

- Interpreted over a labeled transition system.

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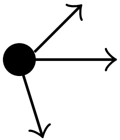

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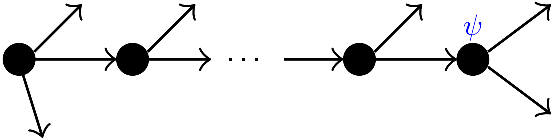

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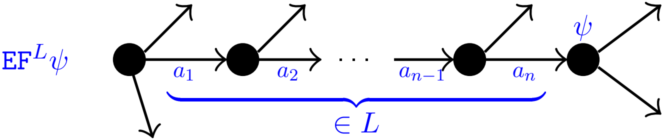

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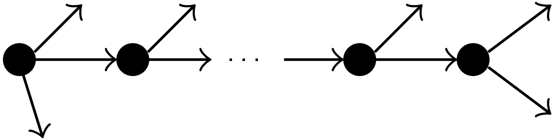

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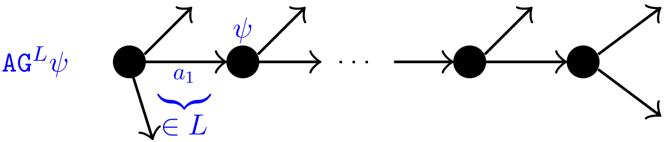

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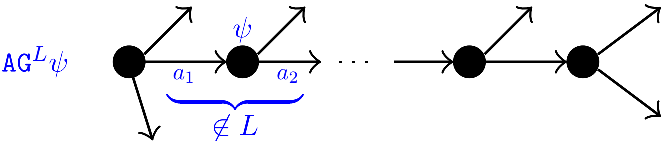

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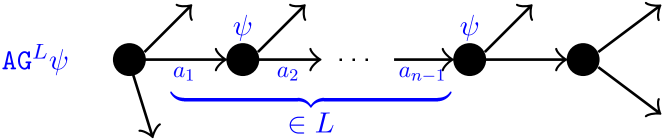

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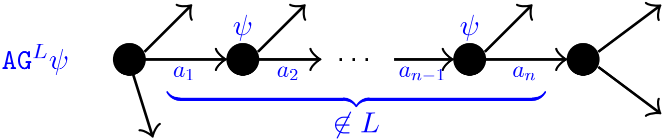

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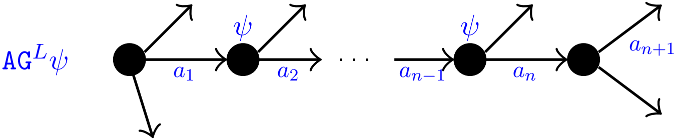

Example (Producer-Consumer Scenario)

- Producer can put items on a stack.
- Consumer can remove them and request further items.
- $\Sigma := \{p, c, r\}$.
- $L := \{w \in \Sigma^* \mid |w|_c = |w|_p$ and $|u|_c \leq |u|_p$ for any prefix u of $w\}$.
- Specification:

$\mathtt{AG}\,\mathtt{EX}^p\mathtt{tt}$	At any time it is possible to produce an object.
$\mathtt{AG}^L(\mathtt{AX}^c\mathtt{ff} \wedge \mathtt{EX}^r\mathtt{tt})$	Whenever the buffer is empty, it is impossible to consume and possible to request.
$\mathtt{AG}^{\overline{L}}(\mathtt{EX}^c\mathtt{tt} \wedge \mathtt{AX}^r\mathtt{ff})$	Whenever the buffer is non-empty, it is possible to consume and impossible to request.

where $\mathtt{AX}^c\varphi := \mathtt{AG}^{\{c\}}\varphi$, $\mathtt{AG}\varphi := \mathtt{AG}^{\Sigma^*}\varphi$, and $\mathtt{EX}^c\varphi := \mathtt{EF}^{\{c\}}\varphi$.

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$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Setting up the Scenery

- Σ alphabet, $|\Sigma| \geq 2$.
- $\mathsf{Palindromes} := \{w \in \Sigma^* \mid w = w^R\} \in \mathsf{CFL} \setminus \mathsf{DCFL}$.
- Extended alphabet $\Sigma_\$:= \Sigma \,\dot{\cup}\, \{\$\}$.
- Goal: none $\mathsf{PDL}_0[\mathsf{DCFL}]$-formula is equivalent to the $\mathsf{PDL}_0[\mathsf{CFL}]$-formula $\mathtt{EF}^{\mathsf{Palindromes}\cdot\$}\mathtt{tt}$.
- Assumption for contradiction: $\mathsf{PDL}_0[\mathsf{DCFL}] \ni \vartheta \equiv \mathtt{EF}^{\mathsf{Palindromes}\cdot\$}\mathtt{tt}$.
- Wlog. no propositions in ϑ.

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—A Simple World

A simple world: ϑ uses neither $\wedge$ nor $\mathtt{AG}$.

- $\mathtt{EF}^L \bigvee_i \psi_i \equiv \bigvee_i \mathtt{EF}^L \psi_i$,
- $\mathtt{EF}^{L_1} \mathtt{EF}^{L_2} \psi \equiv \mathtt{EF}^{L_1 \cdot L_2} \psi$, and
- $\mathtt{EF}^L \mathtt{ff} \equiv \mathtt{ff}$.

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- $\mathtt{EF}^{L_1} \mathtt{EF}^{L_2} \psi \;\equiv\; \mathtt{EF}^{L_1 \cdot L_2} \psi$, and
- $\mathtt{EF}^L \mathtt{ff} \;\equiv\; \mathtt{ff}$.

Therefore,

- $\vartheta \;\equiv\; \mathtt{EF}^L \mathtt{tt}$ where $L := \bigcup_i \Pi_j L_{i,j}$ and $L_{i,j}$ are DCFLs over $\Sigma_\$$.
- $$\text{Palindromes} = \{w \mid w\$ \text{ is a prefix of a word in } L\} = L/(\$ \cdot \Sigma_\$^*) \;\cap\; \Sigma^*$$
  as $\mathtt{EF}^{\text{Palindromes}\cdot\$} \mathtt{tt} \;\equiv\; \vartheta \;\equiv\; \mathtt{EF}^L \mathtt{tt}$.

In this world, Palindromes is a finite union of DCFL-concatenations.

$\mathrm{PDL}_0[\mathrm{DCFL}] \lneq \mathrm{PDL}_0[\mathrm{CFL}]$—The Real World

Theorem [Bojańczyk 2008]

For $L \subseteq \Sigma^\omega$ the following are equivalent:

- The tree language "a path belongs to L" is definable in ECTL.
- L is a finite union of languages of the form

$$A_0^* a_1 A_1^* a_2 \cdots A_{n-1}^* a_n A_n^\omega$$

where $a_1, \ldots, a_n \in \Sigma$ and $A_0, \ldots, A_n \subseteq \Sigma$.

Intractability

$\wedge$- and $\mathtt{AG}$-subformulas cannot be eliminated
if the elimination method also applies to CTL.

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Strategy

Planned Definition

A language L is good iff ...

Planned Theorem

If $\mathtt{EF}^{L\cdot\$}\mathtt{tt} \equiv \vartheta \in \mathsf{PDL}_0[\mathsf{DCFL}]$ then L is good.

Planned Theorem

The language Palindromes is not good.

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$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Eliminations I

Theorem

If

$$\vartheta \equiv \mathtt{EF}^L\mathtt{tt}$$

then

$$\vartheta \equiv \bigvee_{\substack{\psi:\ \mathtt{EF}\text{-formula in } \vartheta,\\ \models \psi \to \mathtt{EF}^L\mathtt{tt}}} \psi$$

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Eliminations II

Proofsketch.

Transformations:

- Turn ϑ into a disjunctive normal form.
- Complete formula to

$$\vartheta \vee \bigvee \left\{ \bigwedge \Phi \mid \Phi \text{ set of } \mathtt{EF}\text{-formulas in } \vartheta, \ \models \bigwedge \Phi \rightarrow \vartheta \right\}$$

Proofs:

- It is admissible to remove terms containing $\mathtt{AG}$-formulas.
- In each term it suffices to pick one $\mathtt{EF}$-formula. □

$\mathrm{PDL}_0[\mathrm{DCFL}] \lneq \mathrm{PDL}_0[\mathrm{CFL}]$—Extraction I

Therefore, have for some DCFLs L_i over Σ.

$$\mathtt{EF}^{\mathsf{Palindromes}\cdot\$}\mathtt{tt} \quad\equiv\quad \vartheta \;\equiv\; \bigvee_i \mathtt{EF}^{L_i}\psi_i$$

Definiton

$$[\![\psi]\!] := \Big\{ a_1 \cdots a_n \in \Sigma^* \;\Big|$$  $$\models \psi \Big\}$$

Thus, $\mathsf{Palindromes} = \bigcup_i (L_i \cap \Sigma^*) \cdot [\![\psi_i]\!] \;\cup\; (L_i/(\$\Sigma_\$^*) \cap \Sigma^*)$

$$= \bigcup_i L_i' \cdot R_i'$$

for DCFLs L_i' over Σ, and for arbitrary languages R_i' over Σ.

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$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Palindromes vs. Concat.

Lemma

Suppose $LR \subseteq$ Palindromes and $|L| \geq 2$. Then

$$R \subseteq u^* V$$

for some word u and a finite language V.

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Palindromes vs. Concat.

Proofsketch.

Let $\ell, \ell\, u \in L$ and $r \in R$.

$\mathrm{PDL}_0[\mathrm{DCFL}] \lneq \mathrm{PDL}_0[\mathrm{CFL}]$—Palindromes vs. Concat.

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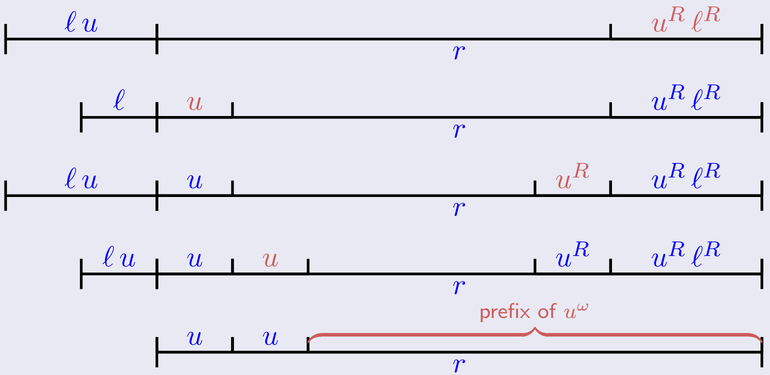

□

$\mathsf{PDL}_0[\mathsf{DCFL}] \not\leq \mathsf{PDL}_0[\mathsf{CFL}]$—Good vs. Bad

What can we say about R if $LR \subseteq$ Palindromes for $|L| = 1$?

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Good vs. Bad

What can we say about R if $LR \subseteq$ Palindromes for $|L| = 1$?
Almost nothing: R could be Palindromes $\cdot\, L^R$.

Definiton

A language L is good iff $L = \bigcup_{i=1}^{n} L_i R_i$ for L_i DCFL and $|L_i| \geq 2$.

Planned Theorem

If $\mathtt{EF}^{L\cdot\$}\mathtt{tt} \equiv \vartheta \in \mathsf{PDL}_0[\mathsf{DCFL}]$ then L is good.

Planned Theorem

The language Palindromes is not good.

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Palindromes are Bad

Theorem

Palindromes is not good.

Proof.

Otherwise,

$$\text{Palindromes} = \bigcup_i L_i R_i = \bigcup_{\substack{i \\ R_i \text{ finite}}} \underbrace{L_i R_i}_{\text{DCFL}} \cup \bigcup_i u_{i,0} u_{i,1}^* \; u_{i,2}^* u_{i,3}$$

Choose u s.t. it is not a prefix of the right union.

$$u \backslash \text{Palindromes} = \bigcup_i \underbrace{u \backslash L_i R_i}_{\text{DCFL}}$$

But Palindromes is not a finite union of DCFLs.

□

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$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Extraction II

Reminder

$\mathtt{EF}^{\mathsf{Palindromes\$}}\mathtt{tt} \equiv \vartheta \equiv \bigvee_i \mathtt{EF}^{L_i}\psi_i$ implies that

$$\mathsf{Palindromes} = \bigcup_i L_i \cdot R_i$$

for DCFLs L_i over Σ, and for arbitrary languages R_i over Σ.

That's almost good except for "$|L_i| > 1$".

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Extraction III

Solution if $|L_i| = 1$ for some i

- Let $a \in \Sigma$ s.th. a is a prefix of the sole word of L_i.
- We have that $\mathtt{EF}^{a\backslash \mathsf{Palindromes}\cdot\$}\mathtt{tt} \equiv \bigvee_i \mathtt{EF}^{a\backslash L_i}\psi_i =: a\backslash\vartheta$.
- $\mathsf{Palindromes} = \bigcup_i L_i R_i = \bigcup_{\substack{i \\ |L_i|\geq 2}} L_i R_i \cup a \bigcup_i \underbrace{a\backslash L_i R_i}_{=[\![a\backslash\vartheta]\!]}$

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- $\mathsf{Palindromes} = \bigcup_i L_i R_i = \bigcup_{\substack{i \\ |L_i|\geq 2}} L_i R_i \cup a \bigcup_i \underbrace{a\backslash L_i R_i}_{= [\![a\backslash\vartheta]\!]}$
- If $[\![a\backslash\vartheta]\!]$ is good then so Palindromes.

$\rightsquigarrow$ Ind. on a suitable measure yields that Palindromes is good.

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Extraction IV

Definiton (Measure)

A measure is finite subset of $(\omega+1)^*$, ordered by multi-set order based on lexicographical order on $(\omega+1)^*$.

$$\mu(\ell) := \{[]\} \qquad \ell \text{ literal or constant}$$

$$\mu(\varphi_1 \circ \varphi_2) := \mu(\varphi_1) \cup \mu(\varphi_2) \qquad \circ \in \{\wedge, \vee\}$$

$$\mu(Q^L\varphi) := \begin{cases} \{|w| :: m \mid m \in \mu(\varphi)\} & \text{if } L = \{w\} \\ \{\omega :: m \mid m \in \mu(\varphi)\} & \text{otherwise} \end{cases} \qquad Q \in \{\mathtt{AG}, \mathtt{EF}\}$$

Example: $\mu\left(\,\mathtt{EF}^{\{abba\}}(\mathtt{EF}^{\{c^n | n \leq 7\}}p \,\wedge\, \mathtt{EF}^{\{ba\}}\neg p)\,\right) = \{[4,\omega],[4,2]\}$

Properties

- All considered transformations weakly decrease the measure.
- $\mu(\mathtt{EF}^L\psi) < \mu(\mathtt{EF}^{a\backslash L}\psi)$ if $L = \{w\}$ and a prefix of w.

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$—Extraction V

Theorem

If $\mathtt{EF}^{L\cdot\$}\mathtt{tt} \equiv \vartheta \in \mathsf{PDL}_0[\mathsf{DCFL}]$ then L is good.

Corollary

$\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$.

Conclusion

- A separation of $\mathsf{PDL}_0[\mathsf{DCFL}]$ and $\mathsf{PDL}_0[\mathsf{CFL}]$ seems to be impossible by means of automata theory.
- Elimination of outermost $\wedge$- and $\mathtt{AG}$-formulas is possible.
- Iterated elimination is non-uniform
—compared to Bojańczyk's impossibility result.
- $\mathsf{PDL}_0[\mathsf{DCFL}] \lneq \mathsf{PDL}_0[\mathsf{CFL}]$.

Work in Progress and Future Work

- Extension to the $\mathtt{EU}/\mathtt{AR}$-fragment of $\mathsf{XCTL}[\cdot] \geq \mathsf{PDL}_0[\cdot]$.
  Interpretation of $\mathtt{E}(\psi_1 \mathtt{U}^L \psi_2)$ compared to $\mathtt{EF}^L \psi$ as $L \cdot [\![\Psi]\!]$?
- Extension to the $\mathsf{PDL}[\cdot]$.
- Extension to the $\mathtt{EG}/\mathtt{AF}$-fragment and so to the whole $\mathsf{XCTL}[\cdot]$.
- Extension to the $\Delta\mathsf{PDL}^?[\cdot]$.
- Generalize to separations like:
  $\mathsf{PDL}[\mathfrak{A}] \lneq \mathsf{PDL}[\mathfrak{B}]$ if $\mathfrak{A}$ is a "reasonable" subset of $\mathfrak{B}$.

$$\mathsf{now} \models \mathtt{EF}^{(\mathsf{question}\ \ \mathsf{answer})^*}\mathtt{AX}\ \mathtt{ff}$$