Math 201-003 Fall 2019 (Darij Grinberg): homework set 3
due date: Wednesday 2019-11-11 at the beginning of class, or earlier by Blackboard

Exercise 1. Prove that

$$\det(AB) = \det A \cdot \det B$$

holds for any 2×2-matrices $A = \begin{pmatrix} a & b \\ a' & b' \end{pmatrix}$ and $B = \begin{pmatrix} c & d \\ c' & d' \end{pmatrix}$.

[In class, I said that this holds for square matrices of any size, but I didn't give a proof. You cannot use this general fact without proof here.]

Exercise 2. Let $a, b, c, d, e, f, g, h, i, j, k, \ell, m, n, o, p$ be any numbers.

(a) Find a simple formula for the determinant

$$\det \begin{pmatrix} a & b & c & d \\ \ell & 0 & 0 & e \\ k & 0 & 0 & f \\ j & i & h & g \end{pmatrix}.$$

(b) Find a simple formula for the determinant

$$\det \begin{pmatrix} a & b & c & d & e \\ f & 0 & 0 & 0 & g \\ h & 0 & 0 & 0 & i \\ j & 0 & 0 & 0 & k \\ \ell & m & n & o & p \end{pmatrix}.$$

(Do not mistake the "o" for a "0".)

[**Hint:** Part **(b)** is simpler than part **(a)**.]

Exercise 3. Find

$$\det \begin{pmatrix} 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 0 & 3 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 2 & 2 & 4 & 3 & 0 \\ -3 & 0 & 1 & 2 & 0 \end{pmatrix}.$$

Exercise 4. Here is a 5×5-matrix: $\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 1 & 0 & 2 & 2 & 1 \end{pmatrix}$. Its determinant is 0.

Find a 0 entry which can be replaced by a 1 to give a nonzero determinant. (You can box this entry in the matrix. Note that you cannot replace more than one entry simultaneously.)