Mathematics via Problems, part 2: Geometry
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Errata and addenda by Darij Grinberg (version of January 18, 2024)

1. Errata

Some of the items below are not corrections of literal mistakes but rather suggestions written according to my taste and ideology. I hope they are nevertheless helpful.

1.1. Chapter 1

- **notation:** The notion of an "excircle" should be defined as well, and its different spellings ("excircle", "exscribed circle", "escribed circle") should be unified.
- **1.3.7 (a), suggestion:** Probably you mean Problem 1.3.4, not 1.3.3 here, but anyway the problem is easier without that suggestion.
- **solution to 1.4.6 (b):** "follows from part (a) and from" → "follows from $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$ and from".
- **solution to 1.4.11 (a):** That "the condition of the problem defines a line" is not obvious (unless you know about Cartesian or trilinear coordinates), and probably worth its own problem, as it is a technique that has many other uses (e.g., the Gauss line of a complete quadrilateral).
- **1.8.13 (b):** "of an inscribed circle" → "of the inscribed circle".
- **solution to 1.8.3:** After "$(90^\circ + \measuredangle C/2)$", add "$-90^\circ$".
- **solution to 1.8.4:** "triangles TLK" → "triangles TKL".
- **solution to 1.8.8:** "$KT|^2$" should be "$|KT|^2$".
- **note after 1.9.9:** An even better reformulation of the generalized Napoleon's theorem is as follows:

  Let ABC and MNP be two triangles, and T any point in the plane. Let A_1, B_1, C_1 be three points in the plane such that $\triangle ABC_1 \sim \triangle NMT$ and $\triangle BCA_1 \sim \triangle PNT$ and $\triangle CAB_1 \sim \triangle MPT$. Here, the symbol "$\sim$" means "directly similar" (i.e., similar and having the same orientation), and we understand a triangle to be an ordered triple of its vertices (so $\triangle ABC$ is not similar to $\triangle BAC$).

  Then, $\triangle A_1B_1C_1 \sim \triangle MNP$.

- *Some additional comments:* The generalized Napoleon's theorem goes back to the paper

  J. F. Rigby, *Napoleon Revisited*, Journal of Geometry **33** (1988), pp. 129–146 (`https://dynamicmathematicslearning.com/napoleon-revisited-rigby.pdf`),

  where it appears as Theorem 3.1. It was recently reproved elementarily in

  Khakimboy Egamberganov, *A Generalization of the Napoleon's Theorem*, Mathematical Reflections **3** (2017). (`https://www.awesomemath.org/wp-pdf-files/math-reflections/mr-2017-03/generalization_of_the_napoleons_theorem.pdf`).

  It is also easy to prove using complex numbers.

1.2. Chapter 2

- **2.1.4:** I think this should be in §2.2, due to its use of the fact that $\measuredangle ABC + \measuredangle CDA = 180^\circ$ for a cyclic quadrilateral $ABCD$.

- **2.3.5:** This is false. Maybe a misstated copy of 1.2.2?

- **hint to 2.6.12:** The sufficiency part needs a lot more work. Why do the circles ω and δ' exist? This seems plausible in the complex plane $\mathbb{C}^2$ from dimensional analysis, but the goal is probably to prove existence in the real plane $\mathbb{R}^2$, and anyway dimensional analysis does not always give correct results. (Most if not all texts leave the same gap when proving Casey's theorem, so I suspect it is too hard for a problem.)

1.3. Chapter 3

- **3.4.11 and 3.4.16:** Shouldn't these problems go into §3.5, as they use rotational homothety?

- **solution to 3.5.12:** I don't quite understand this; are we really supposed to decrease the area? Also, the "no longer" in the last sentence should probably not be there.

- **3.7.5:** Remove the word "intersects".

1.4. Chapter 4

- **solution to 4.1.1:** In part (b), replace "$\overrightarrow{XA_n} + \overrightarrow{XO}$" by "$\overrightarrow{OA_n} + \overrightarrow{XO}$".

- **solution to 4.1.2:** "subtract" should be "add".

- **beginning of §4.2:** "it is undestood that the line on which the point lie has a fixed direction" is confusing. The ratios $\frac{\overrightarrow{AC}}{\overrightarrow{AD}}$ and $\frac{\overrightarrow{BC}}{\overrightarrow{BD}}$ do not depend on the direction. It would be much clearer to define ratios $\frac{\overrightarrow{UV}}{\overrightarrow{XY}}$ of segments lying on parallel lines first (either by picking a direction and showing that the result does not depend on it, or just defining it as the scalar λ for which the vector $\overrightarrow{UV}$ equals $\lambda \cdot \overrightarrow{XY}$).
- **4.2.4:** "intersect O" should be "contain O".
- **4.3.10:** The words "circumscribes" and "inscribes" are being used in an uncommon way here. (Usually one says that a polygon is inscribed into a circle, not that it inscribes a circle.) It is worth either explaining them or using the more common formulation.
- **solution to 4.3.14:** "By Problem 4.3.9" → "By Problem 4.3.8".

1.5. Chapter 5

- **beginning of §5.2:** I am not convinced that "Any Möbius transformation other than a similarity can be represented as the composition of an inversion and a motion". Is this obvious? How can it be proved?
- **5.2.2:** "$(a\overline{z} + b/\ (c\overline{z} + d)$" should be "$(a\overline{z} + b)\ /\ (c\overline{z} + d)$".

1.6. Chapter 7

- **7.4.11:** "Semi-regular bodies" are not defined.
- **7.4.31:** "Surface area" is not defined.

1.7. Chapter 8

- **solution to 8.1.9:** In the long computation, "$++$" should be a single "$+$".
- **solution to 8.1.10:** I don't understand this proof. (But there is a simple solution using Bretschneider's formula.)
- **solution to 8.2.8:** Varignon's theorem has never been stated or mentioned before. It is not exactly high-school material, so I think it should be a problem somewhere earlier in the book.
- **solution to 8.3.2:** This tacitly assumes that the ray F_1X does intersect the ellipse. True, but is it obvious at this point?

- **solution to 8.3.3:** This tacitly assumes that l meets the ellipse only at P. True, but is it obvious at this point?