Lecture Notes Combinatorics
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version of 30 May 2017, downloadable from `http://www.math.kit.edu/iag6/lehre/combinatorics2017s/media/script.pdf`
Errata and addenda by Darij Grinberg

I will refer to the results appearing in the notes "Lecture Notes Combinatorics" by the numbers under which they appear in these notes (specifically, in its version of 30 May 2017).

8. Errata

- **general:** Let me clarify the meaning of some notation that is not standardized across the literature:
  - The symbol $\mathbb{N}$ stands for the set $\{0, 1, 2, \ldots\}$.
  - The symbol $\subset$ means "proper subset", while the symbol $\subseteq$ stands for "subset".
- **page 6:** In the proof of the "Claim" at the top (about the tileability of the $m \times n$ board), replace the equations

  $$\begin{aligned} \#\text{ squares colored with } 1 &= ks_1s_2 + s_1r_2 + s_2r_1 + r_2 \\ \#\text{ squares colored with } 2 &= ks_1s_2 + s_1r_2 + s_2r_1 + r_2 - 1 \end{aligned}$$

  by

  $$\begin{aligned} \#\text{ squares colored with } 1 &= ks_1s_2 + s_1r_2 + s_2r_1 + r_1 \\ \#\text{ squares colored with } 2 &= ks_1s_2 + s_1r_2 + s_2r_1 + r_1 - 1. \end{aligned}$$

  (After all, the main diagonal in the $r_1 \times r_2$ rectangle crosses as many squares as the shorter side, and the latter is r_1, not r_2.)
- **page 7:** "Temperly" $\to$ "Temperley".
- **page 9, §1.1:** "where" $\to$ "were".
- **page 10, §1.1.1:** I would replace "the parts are disjoint" by "the parts $S_1, \ldots, S_k$ are disjoint sets" in order to clarify what is meant by "parts".
- **page 10, §1.1.2:** Pedantic remark: The S_i need to be required to be finite. Otherwise, it could happen that one of the S_i is infinite but their product is nevertheless $\varnothing$ (since any set times $\varnothing$ is $\varnothing$).
- **page 10, §1.1.2:** In the formula "$|S| = |S_1| \times |S_2| \times \cdots \times |S_m|$", replace "$m$" by "$k$".

- **page 11, §1.1.3:** In the first sentence of the Example, replace "If T is" by "Let T be".
- **page 11, §1.1.3:** "*substraction*" $\to$ "*subtraction*"
- **page 11, §1.1.4:** "is a a" $\to$ "is a".
- **page 11, §1.1.5:** The sets $S_1, \ldots, S_m$ (and their disjointness) are irrelevant here, since the statement only uses cardinalities, which can just as well be arbitrary integers. But is this really worth calling the "pigeonhole principle"? To me, the "pigeonhole principle" is one of the following two facts:
  - An injective map $f : A \to B$ between two finite sets A and B is automatically bijective if $|A| \geq |B|$.
  - A surjective map $f : A \to B$ between two finite sets A and B is automatically bijective if $|A| \leq |B|$.

  Note that the first of these two facts is tacitly used on page 13 (in "we have $S_n = P(n, n)$"), so it is worth stating these two facts here.

  In contrast, I know your "pigeonhole principle" as the "Lake Wobegon principle" ("where all the children are above average").
- **page 12, §1.1.6:** "how two count" $\to$ "how to count".
- **page 12, §1.2:** "the first n natural number" $\to$ "the first n natural numbers".
- **page 12, §1.2:** I would not require X to be finite in the first sentence here, but only in Definition 1.1. I am sure you end up using tuples of elements from infinite sets at some point...
- **page 13, §1.2:** In the "string" section, I would replace "is denoted by" by "is defined by". After all, we are not defining $s(i)$ here, but rather the notions of "position" and "character". That said, I think the word "position" is out of place here. When I hear "position", I rather think of a preimage (i.e., a "position of x in the string s" is an i such that $s(i) = x$), not an image under s. (And that's how you use this word several times further down.) It is probably best to replace "The i-th position (or *character*)" by "The i-th *character* (or the *character in position* i)".
- **page 13, §1.2:** The same problem with the meaning of "position" appears in the "sequence" section.
- **page 13, §1.2:** After "but could have, for instance,", add "if".
- **page 13, §1.2.1:** I think it is worth explaining that the definition of a permutation you state here is only one of two (competing) standard definitions of "permutation". (The other definition – used, e.g., in Bourbaki – defines a permutation of a set X to be a bijection from X to X. The two definitions coincide when $X = [n]$, but otherwise are not equivalent.)

- **page 13, §1.2.1:** In the definition of "k-permutation", I would allow arbitrary k; in particular, k can be 0. This is very likely to come up useful in the base of some induction proof or otherwise...
- **page 14, Theorem 1.2:** In part (i), I would allow the case $k = 0$. (Of course, in part (ii) I would not.)
- **page 14, proof of Theorem 1.2:** The word "permutation" is being abused for "k-permutation" here. I would suggest correcting this imprecision here and elsewhere where it appears (e.g., on the next page: "k permutations" $\to$ "k distinct k-permutations"), since it conflicts with your own definition of "permutation" a page ago.
- **page 15, §1.3:** Again, the finiteness of X is unnecessary here (except in Definition 1.3, but you already require it there).
- **page 15, Definition 1.3:** In the definition of "k-Combination of a Multiset", the indexing of the r_i and s_i is somewhat unclear (it presumes that the elements of X are numbered, and this numbering is inherited by M, but this is not given a priori). Wouldn't it be clearer to instead define "A *k-combination* of M is a multiset of size k with types in X in which each element occurs at most as often as in X" ?
- **page 16, Definition 1.3:** The definition of "k-Permutation of a Multiset" confuses me: What is an "ordered multiset"? What are "different orderings of elements of the same type"? For me, a *k-permutation* of a multiset M is simply defined as a k-tuple in X^k (where X is the set of types of M) in which each element of X appears at most as often as in M. This definition seems to match the examples you give below.
- **page 17:** "There $n - k_1 - \cdots - k_{j-1}$ indices left to choose from" $\to$ "There are $n - k_1 - \cdots - k_{j-1}$ indices left to choose from".
- **page 19:** The comma in front of the "however" should be a period or a semicolon; otherwise, it is unclear whether it refers to the part of the sentence before or after the "however".
- **page 19, Example:** The list after "Trying to list them all" has some misplaced commas and parentheses.
- **page 19, Theorem 1.8:** In "numbers $r_1, r_2, \ldots r_k$", add a comma before "r_k".
- **page 19, last line:** "(i.e. number of subsets of $[n]$)" $\to$ "(i.e. number of k-subsets of $[n]$)".
- **page 21:** "number of 2 permutations" $\to$ "number of 2-permutations".
- **page 21:** "only of one" $\to$ "only one".

- **page 21:** "of n element multisets" $\to$ "of n-element multisets".
- **page 24:** In the "arbitrary number of labels per box" section, specifically in its item 1, you should assume $k > 0$ (otherwise, it won't be $(k-1) \cdot \mid$ but rather $0 \cdot \mid$ in the multiset).
- **page 24:** In the "arbitrary number of labels per box" section, specifically in its item 3, replace "to the remaining boxes" by "to the remaining", and replace "non of" by "none of".
- **page 25:** In the "≥ 1 ball per box" section, the comma after "This amounts to $2^n - 2$ possibilities" should be a period; otherwise, it is ambiguous whether the "however" affects the part of the sentence before or after it.
- **page 26:** "to an arrangements" $\to$ "to an arrangement".
- **page 27, §1.5.2:** "non of" $\to$ "none of".
- **page 28:** "flee" $\to$ "flea".
- **page 29:** "(for $0 \leq i \leq k$)" $\to$ "(for $1 \leq i \leq k$)" (in the last case of the proof of $p_k(n) = p_k(n-k) + p_{k-1}(n-1)$).
- **page 29:** In the "≥ 1 ball per box" section, replace add a plus sign before the last "1" in "$n = \underbrace{1 + 1 + \cdots 1}_{n\text{times}}$", and add a whitespace between "n" and "times".
- **page 29:** In the "arbitrary number of balls per box" section, replace "how many boxes i should be non-empty" by "how many boxes should be non-empty (call this number i)".
- **page 29:** The summation sign in the "arbitrary number of balls per box" section should start at $i = 0$ instead of at $i = 1$ (unless you want to require $n \geq 1$, but why would you?). The same applies to the southeasternmost cell in Table 1 on page 30.
- **page 30, §1.5.5:** In the table, replace "non-negati ve" by "non-negative".
- **page 30, Example 1.14:** "A monotone lattice paths" $\to$ "A *monotone lattice path*".
- **page 30, Example 1.14:** Add a semicolon before "see Figure 12".
- **page 31, footnote [2]:** "week" $\to$ "weak". (This is not the first time I am seeing this typo; I remember spotting a "weekly increasing sequence" in the literature.)

- **page 32, proof of Theorem 1.15:** "tiles of with" $\rightarrow$ "tiles with". Better yet, I would replace "tiles of with sizes of the first n odd numbers" by "tiles with sizes $1, 3, 5, \ldots, 2n-1$ (the first n odd positive integers)".
- **page 33, proof of Theorem 1.16 (i):** "n permutations" $\rightarrow$ "n-permutations" (what a difference a little hyphen can make).
- **page 35, proof of Theorem 1.18:** Remove the colon after "For general".
- **page 35, proof of Theorem 1.18:** "placed all number" $\rightarrow$ "placed all numbers".
- **page 37, §1.7.2:** "to connect i with $\pi(i)$" $\rightarrow$ "to connect i with $\pi(i)$" (the "i" should be in mathmode). Also, I would suggest reformulating this sentence in a less telegraphic way, such as: "to draw a diagram in which the numbers $1, 2, \ldots, n$ are arranged (in order) along a "top" horizontal line and the same numbers are also arranged (in order) along a "bottom" horizontal line (with each "top" number i vertically facing the corresponding "bottom" i), and each top number i is connected with the bottom $\pi(i)$".
- **page 38:** "are group by" $\rightarrow$ "are grouped by".
- **page 38:** "this composition" $\rightarrow$ "this decomposition".
- **page 39, between Theorem 1.20 and Theorem 1.21:** "smallest k" $\rightarrow$ "smallest positive integer k".
- **page 39, proof of Theorem 1.21:** In "parsing i through i", remove the second "i".
- **page 39, §1.7.3:** After "*discriminant* of π", add "$\in S_n$" in order to clarify what the symbol n later refers to.
- **page 40, Theorem 1.22:** "the product" $\rightarrow$ "a product". (This appears twice in the theorem and twice in the proof.)
- **page 43, proof of Recursion (i):** In "(case 2)", replace every "ϕ" by "π" starting with "If $i < x$" and ending at "For example".
- **page 43, proof of Recursion (i):** In "(case 2)", replace "$x < i$" by "$x \leq i$" (in the sentence that begins with "Otherwise").
- **page 46, Corollary 2.2, and various other places:** Somehow $N(S)$ has become a number here, despite previously being a set. (You probably mean $|N(S)|$.) Would suggest getting the notations back in sync.
- **page 49, Lemma:** You can replace all appearances of $2n$ by m here, where m is any integer satisfying $m > r \geq 0$. This makes the proof somewhat easier to grasp, in my opinion. Of course, when you use the Lemma later on (on page 51), you apply it to $m = 2n$, but you can just say that.

- **page 50, proof of Lemma:** When you say "the remaining positions" in Case 1, you mean "the positions $2, 3, \ldots, 2n-1$" (not "the positions $2, 3, \ldots, 2n$" as one would normally expect in this context).
- **page 52, §2.1.1:** For the clarity, I'd add a comma before "the total number".
- **page 52:** "we proof" $\rightarrow$ "we prove".
- **page 53:** "We now proof" $\rightarrow$ "We now prove".
- **page 53, proof of Theorem 2.8:** I would replace "for appropriate c_T (to be determined!)" by "for $c_T = \sum\limits_{T \subseteq S \subseteq A} (-1)^{|A|-|S|}$"; this would be clearer and more concrete.
- **page 54:** Remove the comma before "is the number $k = \gcd(m, n)$".
- **page 54:** In the definition of the ϕ-function, replace "$n \geq 2$" by "$n \geq 1$". Otherwise, one of the addends in the sum in Theorem 2.10 is undefined.
- **page 54, Theorem 2.9:** Here, you want to require that $a_i > 0$ for all i, and that the p_i are distinct primes.
- **page 54(?):** It would be good to explain that sums of the form $\sum\limits_{d|n}$ are understood to be sums over all **positive** divisors d of n (rather than all divisors).
- **page 55:** "they do not have a square as a factor" $\rightarrow$ "they do not have a square > 1 as a factor".
- **page 55, proof of Corollary 2.12:** "$-1^{|S|}$" $\rightarrow$ "$(-1)^{|S|}$".
- **page 56, proof of Theorem 2.13:** I would replace "Where $c_{d'}$ are numbers (to be determined!) that count how often $f(d')$ occurs as a summand" by the shorter and clearer "where $c_{d'} = \sum\limits_{d'|d|n} \mu\left(\frac{n}{d}\right)$".
- **page 59, Example 3.1 (i):** "is one" $\rightarrow$ "is 1". Otherwise, it is too easily misunderstood as "is an infinite geometric series".
- **page 63, proof of Lemma 3.9:** The last line of this proof has a "$(n-1)!\,(n-1) \cdot n \cdot n$" in the denominator; this should be a "$(n-1)!\,(n-1)! \cdot n \cdot n$".
- **page 64:** You want to replace "$\mathbb{R}[x]$" by "$\mathbb{R}[[x]]$"; after all, you are getting a power series, not a polynomial. For the same reason, "linear combination of the basis" should be replaced by "infinite linear combination of the

topological basis" (though it is probably more elementary to avoid talking about bases altogether, and just say instead that each $f \in \mathbb{R}[[x]]$ has a unique representation of the form

$$\sum_{n=0}^{\infty} a_n x^n \qquad \text{with } a_n \in \mathbb{R},$$

a unique representation of the form

$$\sum_{n=0}^{\infty} a_n p(x, n) \qquad \text{with } a_n \in \mathbb{R},$$

a unique representation of the form

$$\sum_{n=0}^{\infty} a_n \frac{x^n}{n!} \qquad \text{with } a_n \in \mathbb{R},$$

and a unique representation of the form

$$\sum_{n=0}^{\infty} a_n e^{-x} \frac{x^n}{n!} \qquad \text{with } a_n \in \mathbb{R}.$$

As to the "basis" $\left\{\frac{1}{n^x}\right\}_{n \in \mathbb{N}}$, I would leave it out completely; it is not a basis of $\mathbb{R}[[x]]$ in any meaning of this word, but rather a topological basis of a much different ring, which looks a bit like formal power series but uses multiplication instead of addition for exponents. (It is the ring of formal Dirichlet series.) Since you seem to only be mentioning Dirichlet series as an aside, I am not sure if they are worth mentioning at all.

- **page 64, Theorem 3.11 and before:** I am pretty sure all the three sums "$\sum_{n=0}^{\infty} \frac{a_n}{n^s}$", "$\sum_{n=0}^{\infty} \frac{\mu(n)}{n^s}$" and "$\sum_{n=0}^{\infty} \frac{1}{n^s}$" should start at $n = 1$ rather than at $n = 0$.
- **page 85:** "A partition of $[n]$ into" $\to$ "A partition of $[n]$".
- **page 85, proof of Theorem 4.2:** It is worth pointing out that this proof relies on the concept of power series from complex analysis rather than the formal kind of power series ("power series = sequence of coefficients"). If you just used the formal kind of power series, then e^{e^x} would be undefined.
- **page 86, §4.1.1:** The definition of "crossing" here is not symmetric in A and B. You probably want to add "or $\{i, k\} \subseteq B, \{j, l\} \subseteq A$" at the end of the sentence.

- **page 87, proof of Theorem 4.3:** "every part contains either only numbers that are bigger than k or only numbers that are at most k" is wrong (strictly speaking), since the part S satisfies both. Apparently, you are talking not about the partition $\mathcal{P}$ but about the partition obtained from $\mathcal{P}$ upon removing the part S.
- **page 87, §4.2:** "as the sum" $\to$ "as a sum".
- **page 87, §4.2:** I suggest including a rigorous definition of a partition. (Namely: A *partition* of a nonnegative integer n is defined as a weakly decreasing finite tuple of positive integers whose sum is n.) This notion is a bit too important for a "definition by example".

  Also, the notation "$n = \lambda$" for "λ is a partition of n" is really non-standard and, in my opinion, bad (it abuses the equality sign, tempting to jump to nonsensical conclusions like "if $n = \lambda$ and $n = \mu$ then $\lambda = \mu$").
- **pages 87–88:** The notion of the "size" of a partition should be defined. (Namely: The *size* of a partition λ is defined to be the nonnegative integer n such that λ is a partition of n.)
- **page 88:** In the recursive formula for $p_k(n)$, the equality sign is missing.
- **page 88 and further on:** "Ferrer" $\to$ "Ferrers".
- **page 89, proof of Theorem 4.5:** "$\sum\limits_{i\geq 0}$" $\to$ "$\sum\limits_{i\geq 1}$".
- **page 89:** In the long formula (after "This is no coincidence"), the product $\prod\limits_{k=0}^{\infty}\left(1+x^k\right)$ should start at $k = 1$, not at $k = 0$.
- **page 91, proof of Lemma 4.7:** The notion of a "staircase" should be defined – for example, as a sequence of squares of the form

  $$((i,j), (i+1, j-1), (i+2, j-2), \ldots, (i+g, j-g)) \qquad \text{for some } g \geq 0.$$

  Since the parts of the partition are distinct, it is clear that if a staircase starts in the top-right square of the diagram, then each square of this staircase must be the rightmost square of its row. But this is worth mentioning.
- **page 91, proof of Lemma 4.7:** "define the bottom" $\to$ "define the *bottom*" (to italicize the word "bottom").
- **page 91, proof of Lemma 4.7:** "three types partitions" $\to$ "three types of partitions".
- **page 92, proof of (ii):** "We can iterate through all of them by alternatingly adding a column and a row" is not quite correct; I suggest replacing it by "We can iterate through all of them by alternatingly adding a column (if $S = B$) or a row and a column (if $S = B - 1$)".

- **page 92, Claim (iii):** "partitions" $\to$ "partitions of n" (both times).
- **page 93, §4.2:** In the formula at the end of §4.2, replace "$3i+1$" by "$(3i+1)$", since the sum otherwise does not encompass the 1.
- **page 94, Theorem 4.10:** After "of the same shape", add "of size n".
- **page 94, proof of Theorem 4.10:** In the definition of $\min(X)$, replace "$X \setminus \{x,y\}$" by "$X \setminus \{(x,y)\}$".
- **page 94, proof of Theorem 4.10:** It would be great to have a more precise definition of $S(X)$. Here is my suggestion: "Number the points $(x,y) \in \min(X)$ as $(x_1,y_1), (x_2,y_2), \ldots, (x_p,y_p)$ in the order of increasing x-coordinates. Then, we easily see that $x_1 < x_2 < \cdots < x_p$ and $y_1 > y_2 > \cdots > y_p$. The shadowline $S(X)$ is then defined as the broken line

  $$(x_1,+\infty) \to (x_1,y_1) \to (x_2,y_1) \to (x_2,y_2) \to (x_3,y_2) \to (x_3,y_3) \\ \to \cdots \to (x_p,y_{p-1}) \to (x_p,y_p) \to (+\infty,y_p),$$

  which enters each point in $\min(X)$ going down and leaves it going right."
- **page 95, Algorithm 1:** "convex" $\to$ "concave". More precisely, by "convex bends", you mean the set of all points at which concave bends occur.
- **page 96, Example:** "There are still convex bends" $\to$ "There are still concave bends".
- **page 97:** After defining "increasing", it is worth defining "decreasing" as well: Namely, a subset Y of a point set X is said to be *decreasing* if it can be written as $Y = \{(x_1,y_1), (x_2,y_2), \ldots, (x_k,y_k)\}$ with $y_1 \geq y_2 \geq \cdots \geq y_k$ and $x_1 \leq x_2 \leq \cdots \leq x_k$.
- **page 97:** Before the first Claim here, I would add a "zero-th" claim: If i is arbitrary, then no two points in X_i have the same x-coordinate, and no two points in X_i have the same y-coordinate.

  (This is proven by induction on i, since the x-coordinate of each point in X_{i+1} equals the x-coordinate of precisely one point in X_i (because the concave bends of a shadowline steal their coordinates from the original convex bends), and the same holds for y-coordinates. But this needs to be said, since you keep using this claim tacitly; in particular, it is the reason why you can restrict yourself to considering increasing rather than "weakly increasing" chains.)
- **page 97, first Claim:** "shadows lines" $\to$ "shadowlines".

- **page 97, proof of the first Claim:** The "$X_{1+|Y|}$" at the end is false. You probably mean the X'_i for $j = 1 + |Y|$ or something like that.

  (I would find an inductive proof clearer here anyway. You could even factor such a proof out of the long proof of Theorem 4.10. Its main tool is the following general statement: If U is a nonempty finite set of points in the plane, no two of which have the same x-coordinate and no two of which have the same y-coordinate, then each longest chain of U has exactly one point in common with $\min(U)$, and furthermore, if we remove this common point from the longest chain, then we are left with a longest chain of $U \setminus \min(U)$. This is easy to prove, and once it is proven, your first Claim follows by an easy induction.)

- **page 97, proof of the fourth Claim:** "where constructed" $\to$ "were constructed".

- **page 97, proof of the fourth Claim:** It is worth explaining in a footnote why "the shadow lines of the same phase are non-crossing". The proof is not hard (in fact, each point on S_i^{p+1} lies northeast of at least one point of S_i^p, because each convex bend of S_i^{p+1} lies northeast of at least one convex bend of S_i^p), but I don't consider it trivial.

- **page 97, proof of the fourth Claim:** "is therefore at least the x-coordinate of the leftmost concave bend of S_i^j" $\to$ "is therefore at least the x-coordinate of the leftmost concave bend of $S_i^j, \ldots, S_i^{n_i}$". Indeed, I believe it may happen that one of the paths S_i^k with $k > j$ has its leftmost concave bend further left than S_i^j (or at least I am not convinced that this can be ruled out). The argument, however, still works because the shadowlines are non-crossing.

- **page 98:** "a corresponding shadowlines" $\to$ "a corresponding shadowline".

- **page 98:** "do not have convex bends" $\to$ "do not have concave bends".

- **page 98:** Add a semicolon before "if it didn't".

- **page 98:** "that where not visited" $\to$ "that were not visited".

- **page 98:** I find it far from obvious that the inverse map is well-defined. For example, why are there no unused points left after the "reconstruction" of the shadowlines?

- **page 100, proof of Lemma 4.11 (iii):** Add a "$<$" sign before "i_k" and a "$<$" sign before "$\pi(i_k)$".

- **page 102:** In "which is replaced by $(x_{i-1} - x_i) = -(x_i - x_{i-1})$", replace both "$i - 1$"s by "$i + 1$"s.

- **page 102:** "must be fall behind" $\rightarrow$ "must fall behind".
- **page 102, Lemma 4.13:** FYI: There is an elementary proof of Lemma 4.13 without any polynomial tricks. I have outlined it in `https://math.stackexchange.com/a/1969211/` (and exposed it in detail in the reference given therein, which does not however appear very useful).
- **pages 102–103:** "then g is a polynomial of degree n over x_1" $\rightarrow$ "then g is a polynomial of degree at most m in x_1". (The word "over" is very ambiguous here – after all, one usually says "over the ring of constants".)
- **page 103:** I would not use the notation p' here, as it (falsely) suggests the derivative of p.
- **page 103:** Remove the period at the end of $(\star)$.
- **page 104, proof of Theorem 4.14:** In the definition of t, you should add that $t(n_1, \ldots, n_m)$ is defined to be 0 when when $x_1 \geq \cdots \geq x_m \geq 0$ is not satisfied. Thus, t is really defined on all m-tuples of integers. You use this for the uniqueness argument.
- **page 105, proof of Theorem 4.14:** In the first computation on page 105 (in the proof of (2)), you have an "$(x_m - 1)!$" in the denominator. This should be an "$(x_{m-1} - 1)!$".
- **page 105:** In the definition of a "hook", I would point out that the square (i, j) itself is counted as part of the hook. Also, I would replace "union" by "set".
- **page 106, proof of Theorem 4.15:** You should WLOG assume $n_m > 0$ here; otherwise, the hook $h_{i,1}$ does not start in $(m, 1)$.
- **page 108, Table 3:** Strictly speaking, the "completely left of" relation should be defined only on nonempty intervals, not on all intervals (because defining it on all intervals would lead to $I < \varnothing < J$ for each I and J, which would fail to be a poset).
- **page 108:** The word "y-monotone" can be somewhat misleading (I first thought "graph of a monotonically increasing function", but this is not what you mean); I think it is worth defining.
- **page 123, proof of Theorem 5.17:** "a antichain" $\rightarrow$ "an antichain".
- **page 124, proof of Observation 5.18:** "$|\mathcal{F}| \leq 2^n - \mathcal{F}$" $\rightarrow$ "$|\mathcal{F}| \leq 2^n - |\mathcal{F}|$".
- **page 125, Second Way:** "$\sum\limits_{A \in \mathcal{F}} \{\sigma \mid \sigma \text{ meets } A\}$" $\rightarrow$ "$\sum\limits_{A \in \mathcal{F}} |\{\sigma \mid \sigma \text{ meets } A\}|$".