An involutive introduction to symmetric functions
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`http://www.ma.rhul.ac.uk/~uvah099/Maths/Sym/SymFuncs2017.pdf`
version of 20 March 2019
Errata and addenda by Darij Grinberg

1. Errata

- **pages 1–2, Preface:** Something similar to your solution to Question 21 appears in the proof of Theorem 6.3 of:

  Anthony Mendes, Jeffrey Remmel,

  Counting with Symmetric Functions,

  Springer 2015.

  Might be worth a brief comparison.

  Also, the book

  Nicholas A. Loehr,

  Bijective Combinatorics,

  CRC Press 2011

  is worth mention: its Chapters 11 and 12 have a similar goal as your notes (but differ in their coverage and proofs).

- **page 4, §1.3:** "acts as a group of linear transformations of $\widehat{\mathbf{C}}[x_1, x_2, \ldots]$ by linear extension of $x_i\sigma = x_{i\sigma}$" → "acts as a group of formally continuous $\mathbf{C}$-algebra endomorphisms of $\widehat{\mathbf{C}}[x_1, x_2, \ldots]$ by requiring $x_i\sigma = x_{i\sigma}$ (where a $\mathbf{C}$-linear map from $\widehat{\mathbf{C}}[x_1, x_2, \ldots]$ is said to be *formally continuous* if it respects not just finite $\mathbf{C}$-linear combinations, but also infinite ones as long as they are well-defined)".

  I admit that invoking some kind of continuity appears a bit incongruous in a combinatorics text, but I don't see an easy way to avoid it.

- **page 5, Lemma 1.2:** It would be helpful to explain what size the 0-1 matrices are supposed to have. Namely, they either are $\infty \times \infty$-matrices, or they are finite matrices, but in the latter case you should say that two such matrices count as equal if they only differ in zero rows at the bottom or zero columns on the right[1] (otherwise you'll overcount them).

[1]i.e., the two 0-1 matrices $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ are considered to be identical

- **page 6, proof of Proposition 1.3:** Please explain how exactly you regard X as a matrix. (Per se, X is just a family of complex numbers indexed by pairs of partitions of n. To make it a matrix, you need to totally order these partitions in a way that extends the dominance order.)
- **page 6:** Under the example, it would be good to clarify that expressions such as $X_{\nu\mu}^{-1}$ mean $\left(X^{-1}\right)_{\nu\mu}$ (and not $\left(X_{\nu\mu}\right)^{-1}$).
- **page 6:** Under the example, after "we get $\sum\limits_{\mu \trianglerighteq \kappa} X_{\mu\kappa}^{-1} e_\mu = \operatorname{mon}_{\kappa'}$", add a period.
- **page 7, §1.5:** In "$H(t) = \prod\limits_{i=1}^{\infty} \dfrac{1}{1 - x_i t} \in \widehat{\mathbf{C}}[[t]]$", replace "$\widehat{\mathbf{C}}$" by "$\widehat{\mathbf{C}}[x_1, x_2, \ldots]$".
- **page 8, §1.5:** Various bugs in the last sentence of §1.5: First of all, it's Question 5, not "Question 6". Second, there is no "Problem Sheet 1" in the notes, so I'd say "in Section 6" instead. Finally, "the matrix R" is only defined in the question, so it makes no sense to refer to this matrix here by its name.
- **page 8, §1.6:** When defining $\widehat{\operatorname{ev}}_N$, you should again require that it be a formally continuous $\mathbf{C}$-linear map.
- **page 8, §1.6:** I'd replace "As for Λ, this ring is graded" by "Just as Λ, this ring is graded". (Indeed, "As for Λ, this ring is graded" might be misread as "On the other hand, Λ is graded".)
- **page 9:** In the definition of q_N, it is a bit inappropriate to say that q_N is "defined by $x_N \mapsto 0$ and $x_k \mapsto x_k$ for $k < N$", since q_N is a map from the **symmetric** polynomials and thus does not act on single variables like x_k alone. Instead, it is better to say that q_N is the restriction of the $\mathbf{C}$-algebra homomorphism $\mathbf{C}[x_1, \ldots, x_N] \to \mathbf{C}[x_1, \ldots, x_{N-1}]$ that sends $x_N \mapsto 0$ and $x_k \mapsto x_k$ for $k < N$. (Once again, it needs to be required that it is a $\mathbf{C}$-algebra homomorphism; otherwise, the definition is incomplete.)
- **page 10, Example 1.7 (1):** "has constant degree" $\to$ "has homogeneous degree".
- **page 10, Example 1.7 (1):** When you say "taking fixed points does not commute with inverse limits", I think you really mean "taking the completion does not commute with inverse limits".
- **page 10, Example 1.7 (2):** I suspect "$\widehat{\mathbf{C}}[[t]]$" should be "$\mathbf{C}[[t]]$" or "$\widehat{\mathbf{C}}[x_1, x_2, \ldots][[t]]$".
- **page 10, Example 1.7 (3):** Add comma before "have many properties".

- **page 11, Lemma 1.8:** Maybe a short reminder on what $\operatorname{Sym}^n(Bt)$ means would be nice here. Also, you could rewrite $\operatorname{Tr}\operatorname{Sym}^n(Bt)$ as $(\operatorname{Tr}\operatorname{Sym}^n B)\,t^n$ in order to avoid talking of linear algebra over the base ring $\mathbf{C}[t]$.
- **page 11, Example 1.9:** At the end of $(\star)$, the period should be a comma.
- **page 12, Example 1.9:** I'd replace "and then $(xz)^m$" by "and then $(zx)^m$".
- **page 13, §1.10:** After "Hence $p_n = nh_n - \sum\limits_{k=1}^{n-1} p_k h_{n-k}$", add "for each $n \in \mathbf{N}$" (not for $n = 0$).
- **page 13, §1.11:** In "the coefficient of $p_\alpha t^n$ is $\prod\limits_{k=1}^{n} 1/\left(k^{a_i} a_i!\right)$", replace both "$i$"s by "$k$"s.
- **page 15, §1.12:** "let β be a composition of $\beta_1 + 2\beta_2 + \cdots = b$" $\rightarrow$ "let β be a composition, set $b = \beta_1 + 2\beta_2 + \cdots$,". (Note that $\beta_1 + 2\beta_2 + \cdots$ is not the size of β.)
- **page 16, Definition 1.14:** "be the set of λ-tableaux with content α" $\rightarrow$ "be the set of semistandard λ-tableaux with content α".
- **page 18, Example 2.2:** Obvious as it may be to us combinatorialists, it's probably necessary to mention that "paths" are supposed to only consist of steps 1 unit to the east and steps 1 unit to the north (rather than arbitrary steps back and forth).
- **page 21:** It is worth pointing out that Theorem 2.1 immediately yields a new proof of Proposition 1.16.
- **page 21, proof of Corollary 2.3:** In "$(\lambda_1 - 1 + c_1, \ldots, \lambda_k - k + c_k)$", replace each "$k$" by "$M$".
- **page 21, proof of Corollary 2.3:** In "$\{c_1, \ldots, c_k\} = \{1, \ldots, k\}$", replace each "$k$" by "$M$".
- **page 22, §3.1:** "Given a N-strict" $\rightarrow$ "Given an N-strict".
- **page 22, §3.1:** In "and Γ_n has as a basis $\{a_{\lambda+\delta} : \lambda \vdash n,\ \ell(\lambda) \leq N\}$", replace "$\lambda \vdash n$" by "$\lambda \vdash n - N(N-1)/2$".
- **page 23, §3.2:** "starting in the SW corner with coordinate $(0, \lambda_1')$ and ending at the NE corner at coordinate $(\lambda_1, 0)$" $\rightarrow$ "starting in the SW corner with coordinate $(\lambda_1', 0)$ and ending at the NE corner at coordinate $(0, \lambda_1)$".

  More importantly, these two "corners" do not actually belong to the rim. Instead, they are being used as a starting line and a finish line. This should probably be explained somehow lest readers get confused about whether row/column indexing starts at 0 (it does not, but your definitions of the corners suggests it does).

Maybe it would also be neat to see the actual walk drawn into the picture, as a sequence of arrows?

- **page 23, §3.2:** "and for each step right" $\to$ "and for each step upwards".

- **page 23, §3.2:** The word "represents" (as in "an abacus represents a partition") probably needs to be defined. It's a bit unexpected, since you have never explicitly said that you are planning to use abaci to represent partitions.

- **page 23, Definition 3.3:** After "Fix an abacus for λ with exactly N beads and no final gaps", I'd point out that this abacus is actually uniquely determined by λ and N.

- **page 23, §3.2:** "acts transitively" $\to$ "acts transitively and freely". (You use the freedom later.)

- **page 24, §3.2:** In the computation of $a_{(3,1)+(2,1,0)}$, the "$+x_3^2x_1^5 - x_1^2x_3^5$" part should be "$-x_3^2x_1^5 + x_1^2x_3^5$".

- **page 24, Lemma 3.6:** This lemma should require $\ell(\lambda) \leq N$.

- **page 24, §3.2:** Every $A \in \operatorname{Abc}(\lambda)$ and $\sigma \in \operatorname{Sym}_N$ satisfy $x^{A\sigma} = x^A\sigma$. This simple fact is used tacitly in the proof of Lemma 3.6, so I'd suggest stating it somewhere before that proof.

- **page 24, Theorem 3.7:** Here you probably want to require $\ell(\lambda) \leq N$.

- **page 24, §3.3:** In the example, replace "e_r" by "e_2".

- **page 25:** In the first case ("If there are no collisions") of the definition of $J(A, S)$, I briefly stumbled over the question of what to do if the first bead we want to move right is already in the rightmost position. Thinking about the purpose of the construction, I soon realized that in this case, the abacus is simply extended by one gap to the right before moving the bead. This is probably worth writing out.

- **page 25:** In th proof of Claim (2), the sentence that begins with "Let $g = \prod_k x_k^{\alpha_k}$" has incorrect notations. First, the "k" should be renamed into another letter, since k already stands for the label of the bead into which the bead labelled j bumps. Second, "$\neq i, j$" should be "$\neq j, k$".

- **page 26, Theorem 3.8:** Here you probably want to require $\ell(\lambda) \leq N$.

- **page 26, proof of Corollary 3.9:** It might be worth explaining what a "Young's Rule addition" is (i.e., adding boxes in such a way that no two boxes are added in the same column).

- **page 26, proof of Corollary 3.9:** Replace "the added boxes" by "the boxes added in the i-th step".

- **page 27, §3.5, and many places below:** What you call "hook" is commonly called "rim hook" or "ribbon" or "border strip" in the theory of symmetric functions (e.g., in the books by Stanley and by Loehr). The notion of a "hook" means something different (a "hook" is a partition of the form $(a, 1, 1, \ldots, 1)$). While these notions sometimes lead to the same result (e.g., removing a hook is tantamount to removing a rim hook), often enough they don't (e.g., when you say "μ/λ is a hook", you always mean "μ/λ is a rim hook", not "μ/λ is a parallel translate of a partition of the form $(a, 1, 1, \ldots, 1)$"), so I would advise you to fix the notation by find/replace to avoid teaching an unusual terminology that conflicts with the standard.

- **page 27, §3.5:** Add comma after "denoted $\operatorname{ht}(\mu/\lambda)$".

- **page 27, §3.5:** "that it meets" is somewhat inappropriate here, because "it" is a formal symbol μ/λ (you have not defined skew diagrams yet) and has no "physical body" that could meet a row. Instead you might want to say "that have a nonempty intersection with the set $[\mu] \setminus [\lambda]$".

- **page 27, §3.5:** I'd add the remark that (for any partition λ) we say that λ/λ is a 0-strip, and that its sign $\operatorname{sgn}(\lambda/\lambda)$ is defined to be 1 (contrary to what the definition of sign would suggest). This convention is important in making Corollary 3.13 work (keep in mind that α_i can be 0 in a composition α).

- **page 28, caption to Figure 1:** In "$\circ \bullet^b \circ \circ \bullet \circ \bullet \bullet \circ \circ \bullet$", either a "$\bullet$" is missing before the "^{b}", or the "^{b}" itself signifies a bead (which, if true, should be pointed out).

- **page 28, caption to Figure 1:** "$(6, 5, 5, 5, 4)$" $\to$ "$(6, 5, 5, 5, 4, 1)$".

- **page 28:** I'd replace "where B represents μ" by "where $B \in \operatorname{Abc}(\mu)$", since "represent" (besides being a bit of a weasel word) has so far been used for unlabelled abaci only.

- **page 29, proof of Theorem 3.11:** In "$\tau = \sigma(j, i_1, \ldots i_h)$", add a comma before "$i_h$".

- **page 29, Definition 3.12:** Replace "and let $(\alpha_1, \ldots, \alpha_k) \models n$" by "and let $\alpha = (\alpha_1, \ldots, \alpha_t) \models n$". (Notice that "$k$" has become a "$t$" since you use the letter t further down; but of course, you can just as well replace all "t"s by "k"s instead.)

- **page 30, proof of Corollary 3.13:** I'd mention here that you are using Theorem 3.11 for all $r \in \mathbf{N}_0$, not just for $r \in \mathbf{N}$. (Of course, Theorem 3.11 for $r = 0$ is obvious.)

- **page 30, §3.6:** After "just observe that $P(1,2,2,1) = (2,2,1,1)$", add "$= P(1,1,2,2)$", in order to clarify what this has to do with 2-hooks.

- **page 30, §3.6:** You say that "$(3,2,1)$ has no 2-hooks", but this makes no sense until you define what it means for a partition to have an r-hook. (You only defined when μ/λ is a r-hook.) I suggest you say "there is no 2-hook of the form $(3,2,1)/\kappa$" instead, or define this concept.

- **page 30, §3.6:** "see Question 17 and 19": Is Question 19 really about applying Murnaghan-Nakayama?

- **page 30, §4.1:** Add a comma after "denoted $\mathrm{w}(t)$".

- **page 30, Definition 4.1:** "when k is read from left to right" $\to$ "when w is read from left to right".

- **page 30, Definition 4.1:** "when k is read right to left" $\to$ "when w is read right to left".

- **page 30, Definition 4.1:** "subword of unpaired entries" $\to$ "subword of k-unpaired entries".

- **page 30, Definition 4.1:** I think an example illustrating the concepts of "excess" and "record" used in this definition would be helpful. For example, in order to find the 1-unpaired 1s in 121321132, we make the following table:
$$\begin{pmatrix} 1 & 2 & 1 & 3 & 2 & 1 & 1 & 3 & 2 \\ 1 & 0 & 1 & 1 & 0 & 1 & 2 & 2 & 1 \\ * & & & & & & * & & \end{pmatrix}.$$
The top row is the word $w = 121321132$. The middle row shows, for each entry of this word, the excess of 1s over 2s in the part of the word reaching up to this entry (when the word is read from left to right). The bottom row has an asterisk $*$ in each column where the excess achieves a new record; thus, the 1-unpaired 1s in w are exactly the entries which have a $*$ under them. A similar table can be made for finding 1-unpaired 2s.

- **page 30, §4.1:** "of the form $k+1vk$" $\to$ "of the form $(k+1)\,vk$".

- **page 31, §4.1:** "the subword $k+1vk$" $\to$ "the subword $(k+1)\,vk$".

- **page 31, §4.1:** "If $k = 2$ the parenthesised word is 45)((411)((. The unpaired subword is 233, in positions 3, 10 and 11." is wrong. The opening parenthesis in position 4 is unpaired, too, and the unpaired subword is 2333.

- **page 31, proof of Lemma 4.2:** In "all $(k+1)\,s$", the "s" should be a textmode "s".

- **page 31, proof of Lemma 4.2:** You write: "since every $k+1$ to the left of position i is paired, this new k is unpaired". I believe this isn't so simple. Couldn't this new k grab a $k+1$ to its left that was previously paired with some other k in w, and thus mess up the pairing of parentheses?

  Let me suggest two valid proofs of this claim (though I cannot say any of them is particularly readable).

  I shall refer to the third sentence of Lemma 4.2 ("Changing the letters [...] entries of w") as Lemma 4.2 (b).

  First proof of Lemma 4.2 (b): Let w' be the word obtained from w by the change indicated in Lemma 4.2 (b).

  Regard the ks and $(k+1)$s in w as closing and opening parentheses, respectively. The paired ks and the paired $(k+1)$s then correspond to parentheses that are paired according to the usual rules of bracketing. This pairing has the following property: Between any paired parenthesis and its partner[2], there are no unpaired parentheses[3]. Therefore, any change to the unpaired parentheses in w does not interfere with the paired parentheses; in particular, it does not render their pairing invalid[4]. In general, such a change might introduce some new paired parentheses; however, the change indicated in Lemma 4.2 (b) cannot do this, because it replaces the unpaired subword $k^c\,(k+1)^d$ by a subword of the form $k^{c'}\,(k+1)^{d'}$, which clearly creates no opportunity for further pairing. Therefore, the paired parentheses in w' are exactly the paired parentheses in w (in particular, they occupy the same positions in w' as in w); consequently, the k-unpaired entries of w' are in the same positions as the k-unpaired entries of w. This proves Lemma 4.2.

  Second proof of Lemma 4.2 (b): We proceed by strong induction on the length of the word. Thus, we fix our w, k, c, d, c' and d', but we assume that Lemma 4.2 (b) is already proven for all words shorter than w in the place of w.

  A word is said to be *simple* if it has the form $(k+1)\,vk$, where v is a word (possibly empty) containing neither of the letters k and $k+1$. (Of course, the letter k is fixed here.) Let w' be the word obtained from w by the change indicated in Lemma 4.2 (b).

[2]The *partner* of a paired parenthesis is the other parenthesis that it is paired with.

[3]In fact, any unpaired parenthesis between them would have prevented them from getting paired with each other.

[4]"Invalid" would mean that two parentheses that were paired to each other before the change could end up not paired to each other after the change. This cannot happen, because there were no unpaired entries between them (as we have just seen), and so none of the letters between them have changed.

If the word w contains no simple factor, then Lemma 4.2 (b) is obvious (indeed, in this case, all ks and all $(k+1)$s are unpaired in w, and the same holds for w'). We thus assume that the word w contains a simple factor. In this case, we choose some simple factor of w; we denote this factor by u, and we let p and q be the positions (in w) of its first and last letter. For any word z having at least q letters, we let $\overline{z}$ be the word obtained from z by removing the letters at positions $p, p+1, \ldots, q$.

Now, the pairing of the ks and $(k+1)$s in w (regarded as closing and opening parentheses) has the following property: The $k+1$ in position p is paired with the k in position q (since there are no ks and no $(k+1)$s between them), and the pairing of the remaining ks and $(k+1)$s in w is precisely the same as if the simple factor u (starting at position p and ending at position q) was absent (i.e., it is the same as for the word $\overline{w}$). Exactly the same holds for the word w', because the simple factor u is unaffected by the change that transforms w into w' (indeed, the change only modifies unpaired letters, but there are no unpaired letters in u). Hence, in order to prove Lemma 4.2 (b) for our word w, it suffices to prove Lemma 4.2 (b) for the word $\overline{w}$ (since the word $\overline{w'}$ is obtained from $\overline{w}$ by the same change that transforms w into w'). But this follows from the induction hypothesis, since the word $\overline{w}$ is shorter than w. This concludes the proof of Lemma 4.2 (b).

- **page 32:** In the figure on top of the page, in the southeasternmost diagram, the bottom row should be 2444, not 2334.

- **page 32, Lemma 4.4:** Replace "$\alpha' = (\alpha_1, \ldots, \alpha_{k+1}, \alpha_k, \ldots, \alpha_N)$" by "$\alpha' = (\alpha_1, \ldots, \alpha_{k-1}, \alpha_{k+1}, \alpha_k, \alpha_{k+2}, \ldots, \alpha_N)$" (to avoid misunderstanding as $\left(\underbrace{\alpha_1, \ldots, \alpha_{k+1}}_{k+1 \text{ entries}}, \underbrace{\alpha_k, \ldots, \alpha_N}_{N-k+1 \text{ entries}}\right)$).

- **page 32, Lemma 4.4:** The claim that "$S_k E_k : \mathrm{SSYT}_{k+1}(\mu, \alpha) \to \mathrm{SSYT}_{k+1}(\mu, \alpha' - \epsilon(k))$ is an involution" is misstated: An involution must be a bijection from a set to itself, not to another set. What you mean, of course, is that if you combine the maps $S_k E_k : \mathrm{SSYT}_{k+1}(\mu, \alpha) \to \mathrm{SSYT}_{k+1}(\mu, \alpha' - \epsilon(k))$ for all α into one large map $S_k E_k : \mathrm{SSYT}_{k+1}(\mu) \to \mathrm{SSYT}_{k+1}(\mu)$, where $\mathrm{SSYT}_{k+1}(\mu) = \bigsqcup_\alpha \mathrm{SSYT}_{k+1}(\mu, \alpha)$, then this large map $S_k E_k$ is an involution.

- **page 32, proof of Lemma 4.4:** You write: "If t' is not semistandard then $t(a-1, b) = k$". This requires proof. A priori, it is clear that if t' is not semistandard, then either $t(a-1, b) = k$ or $t(a, b-1) = k+1$ (or both). To obtain your claim, we need to rule out that $t(a, b-1) = k+1$. Fortunately, this is easy: If we had $t(a, b-1) = k+1$, then the letter $k+1$ of $\mathrm{w}(t)$

corresponding to the entry $k+1$ in position $(a, b-1)$ of t would be a k-unpaired $k+1$ (indeed, the letter immediately following it is a k-unpaired $k+1$, but there is a fact (easily proven using Definition 4.1) that if a letter p in a word w is a $k+1$, and if the letter immediately following it is a k-unpaired $k+1$, then the letter p must also be a k-unpaired $k+1$), but this would contradict the fact that the leftmost unpaired $k+1$ in $\mathrm{w}(t)$ is the letter corresponding to the entry $t(a,b)$ (which is further right than the letter we are talking about).

For some reason, every argument I make about coplactic maps degenerates into a run-on sentence like this...

- **page 32, proof of Lemma 4.4:** I'd add a sentence somewhere in this proof (after proving that E_k and F_k are well-defined) saying something like "For each $t \in \mathrm{SSYT}_k(\mu, \alpha)$, the tableau $S_k(t)$ is semistandard again (since it can be written either in the form $(E_k)^\ell(t)$ or in the form $(F_k)^\ell(t)$ for some $\ell \in \mathbf{N}_0$), and belongs to $\mathrm{SSYT}_k(\mu, \alpha')$ (since the operation S_k switches the number of unpaired ks with the number of unpaired $(k+1)$s, whereas the numbers of paired ks and of paired $(k+1)$s were equal to begin with).".

- **page 33, proof of Lemma 4.4:** You say "S_k and $S_k E_k$ are involutions". Well, almost... In order to be able to say that S_k is an involution, you need to extend S_k to a map $\mathrm{SSYT}(\mu, \alpha) \to \mathrm{SSYT}(\mu, \alpha)$ (rather than merely $\mathrm{SSYT}_k(\mu, \alpha) \to \mathrm{SSYT}_{k+1}(\mu, \alpha)$). Fortunately, this is easy (just use the same definition as before).

- **page 33, §4.3:** After "the action is not well-defined", I'd add "(at least not as a right action)".

- **page 33, Definition 4.5:** Add "infinite" before "integer sequence" (your notion of "sequence", per se, allows finite tuples just as well).

- **page 34, proof of Lemma 4.7:** The wording "differ only in the positions of the k-unpaired entries of $\mathrm{w}(t)$" is ambiguous: It can be interpreted both as "differ only in the positions $i_1, i_2, \ldots, i_s$, where $i_1, i_2, \ldots, i_s$ are the positions of the k-unpaired entries of $\mathrm{w}(t)$" and as "their only difference is where the k-unpaired entries are placed". I assume that you mean the first interpretation.

- **page 34, proof of Lemma 4.7:** "Let the rightmost $k+1$ in $\mathrm{w}(t)$" $\to$ "Let the rightmost k-unpaired $k+1$ in $\mathrm{w}(t)$".

- **page 34, proof of Lemma 4.7:** "For (ii), if $J(t) = t$" $\to$ "For (ii), if t is latticed".

- **page 34, proof of Lemma 4.7:** "$t \in \mathrm{SSYT}(\mu, \lambda)$ by Question 22" $\to$ "$\sigma = \mathrm{id}_{\mathrm{Sym}_N}$ by Question 22 (b) and therefore $t \in \mathrm{SSYT}(\mu, \lambda)$".

- **page 34, proof of Lemma 4.7:** "equal to i" $\to$ "are equal to i".
- **page 34, proof of Lemma 4.7:** At the very end of this proof, it wouldn't hurt to explicitly mention that t is the unique element of $\operatorname{SSYT}(\lambda, \lambda)$ because $|\operatorname{SSYT}(\lambda, \lambda)| = K_{\lambda\lambda} = 1$ by Question 11 (c).
- **page 34:** After "J has a unique fixed point in $\mathcal{T}$", add "if $\mu = \lambda$, and no fixed points if $\mu \neq \lambda$".

  Also, this is not quite obvious. In order to prove that an unlatticed tableau $t \in \mathcal{T}$ cannot be a fixed point of J, you need to observe that the content of $J(t)$ is different from the content of t (because Question 22 (c) shows that $\lambda \cdot (\sigma(k, k+1)) \neq \lambda \cdot \sigma$).
- **page 34, Example 3.8:** I don't see how Question 22 shows that "there is no need to carry on once a negative entry appears". Is this a statement about the weak or the strong Bruhat order? (I.e., in what exact way do we follow the arrows?)
- **page 35, Example 3.8:** "Young's Rule (Theorem 3.8)" $\to$ "Young's Rule (Corollary 3.9)".
- **page 35, Example 3.8:** A closing parenthesis is missing in "$|\operatorname{SSYT}(\mu, (2, 0, 7)|$". (One gets mindful of such things after a few pages on coplactic maps.)
- **page 35, Example 3.8:** "the coefficient of $\operatorname{ev}_3 a_{\delta+\mu}$" $\to$ "the coefficient of $a_{\delta+\mu}$".
- **page 36, proof of Theorem 4.10:** Replace "$\sum_{\mu \vdash n} \operatorname{SSYT}(\mu, \lambda \cdot \sigma) s_\mu$" by "$\sum_{\substack{\mu \vdash n; \\ \ell(\mu) \leq N}} |\operatorname{SSYT}(\mu, \lambda \cdot \sigma)| a_{\mu+\delta} / a_\delta$".
- **page 36, proof of Theorem 4.10:** I'm not sure what you want to say by "(Recall that the results in § 3 apply to the antisymmetric Schur polynomials defined by $a_{\delta+\mu}/a_\delta$.)". This sentence is neither necessary nor quite correct (the polynomials $a_{\delta+\mu}/a_\delta$ are symmetric, not antisymmetric).
- **page 36, proof of Theorem 4.10:** Replace "$\sum_{\mu \vdash n} c_\mu a_{\delta+\mu}$" by "$\sum_{\substack{\mu \vdash n; \\ \ell(\mu) \leq N}} c_\mu a_{\delta+\mu}$".
- **page 36, proof of Theorem 4.10:** When speaking of "contributions to c_μ", you again tacitly use the fact that the union $\bigcup_{\sigma \in \operatorname{Sym}_N} \operatorname{SSYT}(\mu, \lambda \cdot \sigma)$ is a disjoint union. This is a consequence of Question 22 (c), and should probably be stated as such.

I would actually go as far as adding further detail: I'd define the *sign* $\operatorname{sgn}(t)$ of a tableau $t \in \mathcal{T}$ to be $\operatorname{sgn}\sigma$, where σ is the unique permutation in Sym_N satisfying $t \in \mathrm{SSYT}(\mu, \lambda \cdot \sigma)$. (The uniqueness here follows from Question 22 (c).) Then, I would rewrite the definition of c_μ as $c_\mu = \sum_{t \in \mathcal{T}} \operatorname{sgn}(t)$; this makes it clear that a sign-reversing involution would help in simplifying c_μ. And Lemma 4.7 (i) shows precisely that the involution J is sign-reversing on the unlatticed tableaux $t \in \mathcal{T}$.

- **page 36, proof of Theorem 4.10:** After "is the unique element of $\mathrm{SSYT}(\lambda, \lambda)$", add "when $\mu = \lambda$, and does not exist otherwise".
- **page 36, proof of Theorem 4.10:** I would replace "Therefore $c_\mu = 0$ unless $\mu = \lambda$ and $c_\lambda = 1$" by "Therefore $c_\lambda = 1$ and $c_\mu = 0$ unless $\mu = \lambda$" for clarity.
- **page 37, §4.6:** "Murnaghan Nakayama" → "Murnaghan–Nakayama".
- **page 37, §5:** Is your inner product bilinear or sesquilinear, and in the latter case, which argument is it linear in? This is probably not particularly important for what you do (although Lemma 5.1 at least needs $\langle \cdot, \cdot \rangle$ to be linear in its first argument), but it might help to clear up the confusion that readers might have.
- **page 37, §5.1:** The period in "otherwise." should be a comma.
- **page 37, proof of Theorem 5.2:** I'd replace "By the combinatorial definition of s_λ" by "By (10)".
- **page 37, proof of Theorem 5.2:** After the first displayed equation in this proof, add "for all $\lambda \vdash n$".
- **page 38, proof of Theorem 5.2:** After the second displayed equation in this proof, add "for all $\nu \vdash n$".
- **page 38:** After "and let $\pi^\mu(\alpha)$ be the number of ordered set partitions $(P_1, \ldots, P_k)$", add "of $\{1, 2, \ldots, n\}$ with $|P_i| = \mu_i$".
- **page 38, proof of Theorem 5.3:** "an $k \times n$ matrix" → "a $k \times n$ matrix" (or do you pronounce "k" differently in Britain?).
- **page 38, proof of Theorem 5.3:** In Claim 1, it might be better to replace "$\dfrac{a_j!}{C_{1j}! \cdots C_{kj}!}$" by "$\dbinom{a_j}{C_{1,j}, \ldots, C_{k,j}}$" (after perhaps reminding the reader of the definition of multinomial coefficients). After all, you always write it as a multinomial coefficient later on.
- **page 38, proof of Theorem 5.3:** In Claim 1, "all packing" should be "all α-packing".

- **page 38, proof of Theorem 5.3:** In the proof of Claim 2, the second factor "$(x_1 + \cdots + x_N)^{a_2}$" should be "$\left(x_1^2 + \cdots + x_N^2\right)^{a_2}$".

- **page 38, proof of Theorem 5.3:** In the proof of Claim 2, "the product $\left(x_1^j + \cdots + x_N\right)^{a_j}$" should be "the product $\left(x_1^j + \cdots + x_N^j\right)^{a_j}$".

- **page 38, proof of Theorem 5.3:** In the proof of Claim 2, replace "$\binom{a_j}{C_{1j} \ldots C_{kj}}$" by "$\binom{a_j}{C_{1j}, \ldots, C_{Nj}}$". Yes, I have not just added commas but also replaced "k" by "N" since you can't yet restrict yourself to $k \times n$ matrices.

- **page 39, proof of Theorem 5.3:** I'd replace "By Claim 2 we have" by "By Claim 2 and Theorem 5.2 we have".

- **page 39, proof of Theorem 5.3:** In (18), replace each "ν" by "α", because it's called α both in Claim 2 and in Claim 4.

- **page 40, proof of Lemma 5.4:** In the last computation of this proof, you are tacitly using the identity $\langle s_\lambda, h_\mu \rangle = K_{\lambda\mu}$ (for any partitions λ and μ). This is probably worth stating earlier on.

- **page 40, Definition 5.5:** "ring homomorphism" $\to$ "$\mathbb{C}$-algebra homomorphism".

- **page 40, §5.3:** After "$\omega(p_n) = (-1)^{n-1} p_n$", add "for $n \in \mathbf{N}$".

- **page 41, proof of Lemma 5.6:** This proof seems to be built upon the illusion that
  $$\{\lambda \vdash n \mid \lambda \trianglerighteq \mu^\star\} = \{\lambda \vdash n \mid \lambda \trianglerighteq \mu\} \cup \{\mu^\star\}$$
  (or else, I am not sure how you get the second displayed equality of the proof). But this is false. What you probably want to do instead is forget about $\mu^\star$, and just derive $\omega(s_\mu) = s_{\mu'}$ after assuming that every $\lambda \triangleright \mu$ (not $\lambda \trianglerighteq \mu$) satisfies $\omega(s_\lambda) = s_{\lambda'}$. This strategy would also have less notational ballast.

  Notice that this is a strong induction, so the base case is not required.

  (Notice also that the solution of Question 25 (a) is more or less the same proof. Rather than sketching it twice, maybe it's worth showing it once in more detail?)

- **page 41, §5.3, Alternative proof:** After "By the Murnaghan–Nakayama rule", I would add "(Corollary 3.13, rewritten using Theorem 4.10)".

- **page 41, §5.3, Alternative proof:** "weighted sum" $\to$ "sum". (You are summing the signs; you need no further weights here.)

- **page 41, §5.3, Alternative proof:** "Hence" $\to$ "Hence, if λ is a partition of n, then Theorem 5.3 yields".
- **page 41, §5.3, Alternative proof:** I don't know how detailed this all is supposed to be, but I feel like there are a lot of silent steps here. In particular, it would help pointing out (probably somewhere in §3) how the abacus of a partition λ is related to the abacus of its conjugate λ'; this is beautiful and explains why the border-strip tableaux for λ are in bijection with those of λ'.
- **page 41, §5.4:** Have you ever defined what a skew-partition is, and how its Young diagram is defined?
- **page 41, §5.5:** "Let $n \in \mathbf{N}$" $\to$ "Let $n \in \mathbf{N}_0$" (since you later take the direct sum $\bigoplus_{n \in \mathbf{N}_0} \mathrm{Cl}\left(\mathrm{Sym}_n\right)$).
- **page 41, §5.5:** "indicator functions f_α" $\to$ "indicator functions $\mathbb{1}_\alpha$".
- **page 41, §5.5, and 3 other places in the text:** "cycle type" $\to$ "cycle-type" (in order to keep your notations consistent).
- **page 41, §5.5:** It can't possibly hurt to say somewhere that the "ω-involution" means the involution ω.
- **page 42, proof of Proposition 5.7:** In the last sentence, "the image of s_λ" should be "the image of s_μ". But more importantly, I am not sure how you conclude that this irreducible constituent is actually the image of s_μ. (This is not hard to check – e.g., there is a standard trick that uses $\langle \chi^\mu, \chi^\mu \rangle = \langle s_\mu, s_\mu \rangle = 1$ to show that χ^μ is $\pm$ an irreducible character, and then we can use $\langle \chi^\mu, \pi^\mu \rangle = 1 > 0$ to conclude that the $\pm$ is in fact a $+$.)
- **page 42:** "is the signed sum" $\to$ "is the sum of the signs".
- **page 42:** "and content α" $\to$ "and type α".
- **page 42:** Replace "Let $\mathrm{ch} : \Lambda \to \bigoplus_{n \in \mathbf{N}_0} \mathrm{Cl}\left(\mathrm{Sym}_n\right)$" by "Let $\mathrm{ch} : \bigoplus_{n \in \mathbf{N}_0} \mathrm{Cl}\left(\mathrm{Sym}_n\right) \to \Lambda$".
- **page 42:** "The right-hand side" $\to$ "The left-hand side".
- **page 43, Theorem 5.8:** Replace "$\mathrm{ch}\left(\phi \, \mathrm{sgn}_{\mathrm{Sym}_n}\right) = \omega\left(\phi\right)$" by "$\mathrm{ch}\left(\phi \, \mathrm{sgn}_{\mathrm{Sym}_n}\right) = \omega\left(\mathrm{ch}\,\phi\right)$".
- **page 43, proof of Theorem 5.8:** Replace "$h_\lambda h_v$" by "$h_\lambda h_\mu$" in the displayed equation.
- **page 43, Remark 5.9:** In "by $s_\lambda\left(x_1, \ldots, x_N\right) \to \chi^\lambda$", the "\to" arrow should be a "\mapsto".

- **page 43, proof of Corollary 5.10:** "$\left(\chi_\mu \times \chi_\nu\right)$" $\to$ "$\left(\chi^\mu \times \chi^\nu\right)$".
- **page 43, proof of Corollary 5.10:** "$\uparrow_{S_m \times S_n}^{S_{m+n}}$" $\to$ "$\uparrow_{\operatorname{Sym}_m \times \operatorname{Sym}_n}^{\operatorname{Sym}_{m+n}}$".
- **page 43, proof of Corollary 5.10:** Strictly speaking, you have not shown that all irreducible characters of Sym_n are of the form χ^λ, so the proof is incomplete. (I am not saying that this is difficult, but it needs a couple more lines.)
- **page 44, Question 2:** Part (a) of this Question is false. A counterexample follows by observing that the Young diagram $[(2,2,2,1)]$ can be obtained from $[(3,2,2)]$ by moving the single box $(1,3)$ into the first available position below it, but $(3,2,2)$ is not a dominance neighbor of $(2,2,2,1)$ (indeed, $(3,2,2) \rhd (3,2,1,1) \rhd (2,2,2,1)$).

  The proper condition for dominance neighbors is subtler. Fortunately, part (b) of the Question can be easily solved without ever passing through these slippery places. One such solution appears in the solution to Exercise 2.2.9 in

  Darij Grinberg and Victor Reiner,

  Hopf Algebras in Combinatorics,

  version of 11 May 2018,

  `http://www.cip.ifi.lmu.de/~grinberg/algebra/HopfComb-sols.pdf`
  (also available at arXiv:1409.8356v5)

  (beware that the numbering on my website might have changed by the time you're reading this, but the numbering on arXiv:1409.8356v5 will never change).

  Incidentally, a generalization of your Question 2 appears in Propositions 1.1 and 1.2 of

  C. DeConcini, David Eisenbud, and C. Procesi,

  Young Diagrams and Determinantal Varieties,

  Inventiones math. 56 (1980), pp. 129–165.
- **page 45, Question 4:** Replace "Lemma 1.3" by "Lemma 1.2".
- **page 45, Question 7:** In the first line of this exercise, in "$v(\alpha) = (1, \ldots, 1, \ldots, n \ldots, n)$", a comma is missing after the first "n".
- **page 46, Question 7:** "Work with symmetric functions" $\to$ "Work with symmetric polynomials".
- **page 46, Question 7:** Here is an easier way to solve part (g) (which also shows that you can replace "$\ell \in \mathbf{N}$" by "$\ell \in \mathbf{N}_0$"):

Step 1: We observe that every $N \geq 0$ satisfies

$$\sum_{i=0}^{N} \binom{N}{i} d_{(1^i)} = N!. \tag{1}$$

(This follows by noticing that $\binom{N}{i} d_{(1^i)}$ is the number of permutations $\sigma \in \operatorname{Sym}_N$ that have exactly i fixed points.)

Step 2: Now, fix $n \in \mathbf{N}_0$. For each $\ell \in \{0, 1, \ldots, n\}$, we set

$$w_\ell = (-1)^{\ell+1} \sum_{m=\ell+1}^{n} \binom{m-1}{\ell} \binom{n}{m} d_{(1^{n-m})}. \tag{2}$$

Thus, our goal is to prove that

$$d_{(1^n)} = \frac{n!}{\ell!} d_{(1^\ell)} + w_\ell \qquad \text{for each } \ell \in \{0, 1, \ldots, n\}.$$

This we shall prove by induction over $n - \ell$. The base case ($n - \ell = 0$) is obvious (since $w_n = 0$). For the induction step, it suffices to prove that

$$\frac{n!}{\ell!} d_{(1^\ell)} + w_\ell = \frac{n!}{(\ell+1)!} d_{(1^{\ell+1})} + w_{\ell+1} \tag{3}$$

for each $\ell \in \{0, 1, \ldots, n-1\}$. Thus we shall focus on proving (3).

Step 3: Fix $\ell \in \{0, 1, \ldots, n-1\}$. The definition of $w_{\ell+1}$ yields

$$\begin{aligned} w_{\ell+1} &= (-1)^{\ell+2} \sum_{m=\ell+2}^{n} \binom{m-1}{\ell+1} \binom{n}{m} d_{(1^{n-m})} \\ &= (-1)^{\ell+2} \sum_{m=\ell+1}^{n} \binom{m-1}{\ell+1} \binom{n}{m} d_{(1^{n-m})} \\ &\qquad \left(\begin{array}{c} \text{here, we have extended the range of the sum by one} \\ \text{extra addend, which is zero} \\ \text{(since } \binom{m-1}{\ell+1} = 0 \text{ when } m = \ell+1\text{)} \end{array} \right) \\ &= -(-1)^{\ell+1} \sum_{m=\ell+1}^{n} \binom{m-1}{\ell+1} \binom{n}{m} d_{(1^{n-m})}. \end{aligned}$$

Subtracting this equality from (2), we find

$$\begin{aligned}
& w_\ell - w_{\ell+1} \\
&= (-1)^{\ell+1} \sum_{m=\ell+1}^{n} \binom{m-1}{\ell} \binom{n}{m} d_{(1^{n-m})} \\
&\qquad - \left(-(-1)^{\ell+1} \sum_{m=\ell+1}^{n} \binom{m-1}{\ell+1} \binom{n}{m} d_{(1^{n-m})} \right) \\
&= (-1)^{\ell+1} \sum_{m=\ell+1}^{n} \underbrace{\left(\binom{m-1}{\ell} + \binom{m-1}{\ell+1} \right)}_{\substack{= \binom{m}{\ell+1} \\ \text{(by the recursion of the} \\ \text{binomial coefficients)}}} \binom{n}{m} d_{(1^{n-m})} \\
&= (-1)^{\ell+1} \sum_{m=\ell+1}^{n} \underbrace{\binom{m}{\ell+1} \binom{n}{m}}_{\substack{= \binom{n}{\ell+1} \binom{n-(\ell+1)}{n-m} \\ \text{(by straightforward manipulations)}}} d_{(1^{n-m})} \\
&= (-1)^{\ell+1} \binom{n}{\ell+1} \underbrace{\sum_{m=\ell+1}^{n} \binom{n-(\ell+1)}{n-m} d_{(1^{n-m})}}_{\substack{= \sum_{i=0}^{n-(\ell+1)} \binom{n-(\ell+1)}{i} d_{(1^i)} \\ \text{(here, we have substituted } i \\ \text{for } n-m \text{ in the sum)}}} \\
&= (-1)^{\ell+1} \binom{n}{\ell+1} \underbrace{\sum_{i=0}^{n-(\ell+1)} \binom{n-(\ell+1)}{i} d_{(1^i)}}_{\substack{= (n-(\ell+1))! \\ \text{(by (1) (applied to } N=n-(\ell+1)))}} \\
&= (-1)^{\ell+1} \underbrace{\binom{n}{\ell+1} (n-(\ell+1))!}_{= \frac{n!}{(\ell+1)!}} = (-1)^{\ell+1} \frac{n!}{(\ell+1)!}.
\end{aligned}$$

Comparing this with

$$\begin{aligned}
&\frac{n!}{(\ell+1)!} \underbrace{d_{(1^{\ell+1})}}_{\substack{=(\ell+1)d_{(1^\ell)}+(-1)^{\ell+1} \\ \text{(by the well-known recursion} \\ \text{for derangement numbers)}}} - \frac{n!}{\ell!} d_{(1^\ell)} \\
&= \frac{n!}{(\ell+1)!} \left((\ell+1)\, d_{(1^\ell)} + (-1)^{\ell+1} \right) - \frac{n!}{\ell!} d_{(1^\ell)} \\
&= \underbrace{\frac{n!}{(\ell+1)!} (\ell+1)}_{=\frac{n!}{\ell!}} d_{(1^\ell)} + \frac{n!}{(\ell+1)!} (-1)^{\ell+1} - \frac{n!}{\ell!} d_{(1^\ell)} \\
&= \frac{n!}{\ell!} d_{(1^\ell)} + \frac{n!}{(\ell+1)!} (-1)^{\ell+1} - \frac{n!}{\ell!} d_{(1^\ell)} = \frac{n!}{(\ell+1)!} (-1)^{\ell+1} = (-1)^{\ell+1} \frac{n!}{(\ell+1)!},
\end{aligned}$$

we obtain

$$w_\ell - w_{\ell+1} = \frac{n!}{(\ell+1)!} d_{(1^{\ell+1})} - \frac{n!}{\ell!} d_{(1^\ell)}.$$

This is clearly equivalent to (3). Thus, (3) is proven. This completes the induction step.

- **pages 46 and 47, Question 8:** Replace every "r" by "n" here (since part (a) of the question speaks of Sym_n).
- **page 47, Question 10:** "ring homomorphism" → "$\mathbb{C}$-algebra homomorphism".
- **page 48, Question 12:** "symmetric function" → "ev_N of the symmetric function".
- **page 48, Question 12:** "the signed weight" → "the sum of the signed weights".
- **page 48, Question 12:** "tuples $(P_1, \ldots, P_M)$" → "tuples $(P_M, \ldots, P_1)$" (remember that you started them with P_3 in Example 2.2).
- **page 48, Question 13:** "path triples (P_1, P_2, P_3)" → "path triples (P_3, P_2, P_1)".
- **page 48, Question 14:** Isn't it too early at this point to speak of the ω involution, let alone use Lemma 5.6?
- **page 49, Question 17:** I assume w should be an arbitrary element of $\mathbf{N}_0$?

- **page 50, Question 22:** The "Of course" sentence in part (c) looks out of place here: what coefficients, and why should we care about them? (Example 4.8, which it probably is referring to, is far away.) Maybe it is better to say that "Thus, the union in Definition 4.6 is a union of disjoint sets" instead.

- **page 50, Question 23:** I know it's a stupid remark, but you have never actually defined the notion of a "coplactic map".

- **page 50, Question 24:** "and let" $\to$ "Let".

- **page 51, Question 25:** In part (a), the two equalities need to be switched, since the first one follows from Pieri's rule and the second from Young's (and you do say "respectively").

- **page 52, Question 31:** "of content ν" $\to$ "of content μ".

- **page 53, solution to Question 3:** In the second displayed equation, replace "$p_k t^k$" by "$p_k t^{k-1}$".

- **page 53, solution to Question 3:** The left hand side of the last displayed equality in this solution should be "$tH'(t)E(t)$", not "$tH'(t)E(-t)$".

- **page 53, solution to Question 3:** The right hand side of the last displayed equality in this solution should be "$tQ'(t)$", not "$tQ(t)$".

- **page 54, solution to Question 5:** "for each $k \in \mathbf{N}_0$" $\to$ "for each $k \in \mathbf{N}$".

- **page 54, solution to Question 5:** "we have $R_{\mu(1^n)} = 1$" $\to$ "we have $R_{(1^n)\mu} = 1$".

- **page 54, solution to Question 7:** "$d_{(1^{n-m)}}$" $\to$ "$d_{(1^{n-m})}$" (wrong placement of the closing parenthesis).

- **page 55, solution to Question 8:** "S_{a_i}" $\to$ "Sym_{a_i}" on the third line of the solution.

  Actually, there is one more imprecision here: "$\prod_i C_i \wr S_{a_i}$" is not a wreath product, but a direct product of wreath products :)

- **page 56, solution to Question 10:** There is no such thing as "Question 33(f)". You probably mean "Question 3(f)".

- **page 56, solution to Question 10:** After "$\omega(p_\lambda) = (-1)^{n-\ell(\lambda)}$", add "$p_\lambda$".

- **page 56, solution to Question 10:** "$\lambda \in \operatorname{Sym}_n$" $\to$ "$\sigma_\lambda \in \operatorname{Sym}_n$".

- **page 56, solution to Question 12:** "the Lindström" $\to$ "Lindström".

- **page 57, solution to Question 14:** Before the first sentence, it would be helpful to inform the reader that you are again defining an involution on M-tuples of paths as in Example 2.2, although now you will have different starting points and ending points and the weights too will be defined differently.
- **page 57, solution to Question 14:** "involution defining" $\to$ "involution defined".
- **page 57, solution to Question 14:** "*intersect then, they*" $\to$ "*intersect, then they*".
- **page 57, solution to Question 14:** "paths $(P_1, \ldots, P_M)$" $\to$ "paths $(P_M, \ldots, P_1)$".
- **page 57, solution to Question 14:** "tabloid" $\to$ "tableau".
- **page 57, solution to Question 14:** In "therefore P_i and P_{i+1} meet on or before step a", you probably mean "right step a" when you say "step a".
- **page 57, solution to Question 15 (b):** It took me a while to understand what you mean by "θ^n". (You mean the map $\operatorname{Sym}^n V \to \mathbf{C}$ sending each $v_1 v_2 \cdots v_n$ to $\theta(v_1) \cdots \theta(v_n)$.)
- **page 58, solution to Question 15 (b):** Replace "$g(u, \ldots, u) = \theta(u)^n$" by "$g(u) = f(u, \ldots, u) = \theta(u)^n$".
- **page 58, Remark after the solution to Question 15:** "polynomial" $\to$ "n-multilinear map".
- **page 58, solution to Question 19:** I am not personally fond of references to talk slides, but let me make an exception here: Drew Armstrong's slides from FPSAC 2017 (`http://www.math.miami.edu/~armstrong/Talks/RCC_FPSAC_17.pdf`) give gorgeous picture proofs of all parts of Question 19 (as well as mentioning further results).
- **page 58, solution to Question 19:** In part (a), replace "$b = 0$" by "$a = 0$".
- **page 59, solution to Question 21:** "By Question 3(e)" $\to$ "By Question 3(f)".
- **page 59, solution to Question 21:** After "μ/λ-tableaux", insert "with entries from $\{1, 2\}$".
- **page 59, solution to Question 21:** After condition (c), add the condition "(d) Each row and each column are weakly increasing.".
- **page 59, solution to Question 21:** Maybe explain what "disjoint union" means (in "disjoint union of hooks").

- **page 59, solution to Question 21:** "such that $(i+1, j) \notin [\nu/\lambda]$ and $(i, j-1) \notin [\nu/\lambda]$" $\rightarrow$ "such that $(i-1, j) \notin [\nu/\lambda]$ and $(i, j+1) \notin [\nu/\lambda]$". (This is if I am understanding you right, that if I regard as a rim hook as a snake crawling to the northwest, then its terminal point is its head. Your examples suggest this, at least.)

- **page 59, solution to Question 21, Example:** "$\nu = (3,2)$" $\rightarrow$ "$\mu = (3,2)$".

- **page 59, solution to Question 21, Example:** "$\nu = (3,2,2)$" $\rightarrow$ "$\mu = (3,2,2)$".

- **page 61, solution to Question 26:** I'd replace the "We have" at the beginning by "By Claim 3 in the proof of Theorem 5.3, we have".

- **page 61, solution to Question 26:** Replace "$\alpha(1), \ldots, \alpha(k)$" by "$\beta(1), \ldots, \beta(k)$".

- **page 61, solution to Question 26:** Replace "$|\alpha(i)|$" by "$|\beta(i)|$".

- **page 61, solution to Question 26:** In the second displayed equality, replace each "b" by a "B", and also add commas into the multinomial coefficients.

- **page 61, solution to Question 27:** "$\langle s_{\lambda/\nu}, f\rangle = \langle s_\nu f, s_\lambda\rangle$" $\rightarrow$ "$\langle s_{\lambda/\nu}, f\rangle = \langle s_\lambda, s_\nu f\rangle$". (This is important if your form $\langle\cdot,\cdot\rangle$ is sesquilinear; but even if it is bilinear, it is probably better to avoid moving f from right to left argument without purpose.)

- **page 61, solution to Question 28:** This doesn't answer the question about "a version of the Murnaghan–Nakayama rule for skew-Schur functions", as no skew Schur functions appear here!

  (I know only one version of Murnaghan–Nakayama for skew Schur functions, and it involves skewing operators, which you haven't introduced. So maybe you didn't mean to say "skew-Schur" in the exercise.)

- **page 62, solution to Question 29:** Replace every "μ" by "ν" here.

- **page 62, solution to Question 29:** "at $(\lambda_{j\tau} + M - j\tau)$ and $(\lambda_{i\tau} + M - i\tau)$" $\rightarrow$ "at $(\lambda_{j\tau} + M - j\tau, N)$ and $(\lambda_{i\tau} + M - i\tau, N)$".

- **page 62, solution to Question 30:** I don't think you want to say "with k minimal" here. As I understand, the k in "k-unlatticed" is chosen by looking at which of the records is broken first when reading $\mathrm{w}(t)$ from right to left, not by minimality of k.