On the straightening law for minors of a matrix
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`http://www.math.uchicago.edu/~swan/strLaw.pdf`
version of 18 February 2003
Errata and addenda by Darij Grinberg

I will refer to the results appearing in the preprint "On the straightening law for minors of a matrix" by the numbers under which they appear in this preprint (specifically, in its version of 18 February 2003, published on `http://www.math.uchicago.edu/~swan/strLaw.pdf`).

11. Errata

- **Page 1:** The name "Doubilet" is misspelt as "Doubillet" twice (once in the abstract, and once again in §1).,
- **Page 2, proof of Lemma 2.3:** Replace "etc. ,each" by "etc., each".
- **Page 2, proof of Theorem 2.6:** The "σ" comes out of the blue in the proof, as there is no σ in the statement of the theorem. To clarify where it comes from, I suggest adding the following sentence at the beginning of the proof: "By Corollary 2.5, we can WLOG assume that X is a permutation matrix $P\left(\sigma^{-1}\right)$ for some $\sigma \in \mathcal{S}_n$.".
- **Page 3, §3:** Replace "say that S is good" by "say that S is good". (I have replaced the text-"S" by a math-"S" here.)
- **Page 3, Theorem 3.1:** It would be helpful to clarify that some of the (A_i, B_i) may be equal.
- **Page 3, proof of Theorem 3.1:** Replace "$B_i \leq B$,and" by "$B_i \leq B$, and".
- **Page 3, proof of Theorem 3.1:** Replace "all other terms" by "all other nonzero terms".
- **Page 4, proof of Theorem 4.1:** This proof is confusing due to some of its parts being out of order. The determinant $Y(P \mid Q)$ is not well-defined before the sets I and J are totally ordered; I even would not call Y a "square matrix" until the indexing set I for its rows and the indexing set J for its columns have been identified. I suggest modifying the proof as follows:
  - At the beginning of the proof, add the following sentence: "We WLOG assume that $|S'| = |T'|$ and $|S''| = |T''|$, as otherwise the left hand side is 0.".

  – Move the second paragraph of the proof (the paragraph that begins with "Order I by setting" and ends with "and similarly for subsets of J") to before the sentence that begins with "Define a square matrix Y".

  – Before the sentence that begins with "Define a square matrix Y", add the following sentence: "Now the sets I and J are totally ordered. Let $k = |I| = |J|$. Then, we have order-preserving bijections $I \to \{1, 2, \ldots, k\}$ and $J \to \{1, 2, \ldots, k\}$. Use these bijections to identify I and J with $\{1, 2, \ldots, k\}$. For every subset Q of $\{1, 2, \ldots, k\}$, we will write $\widetilde{Q}$ for the complement $\{1, 2, \ldots, k\} - Q$.".

  Once this is done, Y actually becomes a honest square matrix (of size $k \times k$).

- **Page 4, proof of Theorem 4.1:** Replace "$(S' \mid T')(S'' \mid T'') = Y\{I' \mid J'\}$" by "$(S' \mid T')(S'' \mid T'') = \pm Y\{I' \mid J'\}$". (At least I don't see a reason why the $\pm$ must always be a $+$. Maybe it is?)

- **Page 5, proof of Lemma 4.2:** Add "WLOG assume that $|K| = |I'|$ (since otherwise, $\varphi(K) \neq S'$ is obvious)." after the first sentence of the proof.

- **Page 5, proof of Lemma 4.2:** Remove ", $S' = \{s_1 < \cdots < s_p\}$" (as this is unnecessary).

- **Page 5, proof of Lemma 4.2:** The argument "But i_v lies in I' so k_v is in I'' and therefore $k_v > i_v$" goes a bit too fast for me. I suggest adding a few details, e.g., as follows:

  "Recall that φ is injective on I' and on K. Thus, from $I' = \{i_1 < \cdots < i_p\}$ and $K = \{k_1 < \cdots < k_p\}$, we obtain $\varphi(I') = \{\varphi(i_1) < \cdots < \varphi(i_p)\}$ and $\varphi(K) = \{\varphi(k_1) < \cdots < \varphi(k_p)\}$. Hence, $\varphi(K) = S' = \varphi(I')$ shows that $\varphi(k_v) = \varphi(i_v)$. Since $k_v \neq i_v$ and $i_v \in I'$, this entails $k_v \in I''$ and therefore $k_v > i_v$".

- **Page 5, §5:** A nitpick: Replace "whose entries are indeterminates" by "whose entries are distinct indeterminates".

- **Page 5, Theorem 5.3:** In this theorem, you probably want to state that two standard monomials which only differ by factors of the form $(\varnothing, \varnothing)$ are considered to be identical. (You can always add $(\varnothing, \varnothing)$ to the end of a standard monomial without changing the value of this monomial.)

- **Page 6, proof of Theorem 5.3:** Replace all appearances of "leading form" by "leading term" (or is "leading form" really a synonym for "leading term"?).

- **Page 6, proof of Theorem 5.3:** Add a period before "Similarly, if".

- **Page 6, proof of Theorem 5.3:** After "$z(B) = z_{b_1}^{(1)} \cdots z_{b_p}^{(p)}$", add "(if $N \geq |A| = |B|$)".
- **Page 6, References:** "Desarmenian" $\to$ "Desarmenien" in [4].