Notes for Math 740 (Symmetric Functions)
Steven V. Sam
version of 27 April 2018
`https://www.math.wisc.edu/~svs/740/notes.pdf`
Errata and addenda by Darij Grinberg

This is a (slightly haphazard) list of corrections and comments I have to the "Notes for Math 740 (Symmetric Functions)". I will refer to the results appearing in these notes by the numbers under which they appear in the notes (specifically, in their version linked above).

(I have read most of the notes, minus the geometric parts of §5, a part of §8 that got too confusing for me, and §9.)

13. Errata

- **page 1, §1.1:** On the last line of page 1, you use the notation Λ, but you don't define it until later. It might be better to first define the homomorphism $\pi_n : R \to \mathbf{Z}[x_1, x_2, \ldots, x_n]$, and only later restrict this π_n to Λ (once Λ is defined).
- **page 2, Remark 1.1.1:** "inverse limit of the $\Lambda(n)$" $\to$ "inverse limit of the rings $\Lambda(n)$".

  In fact, Λ is the inverse limit of the **graded** rings $\Lambda(n)$ (that is, the inverse limit of the $\Lambda(n)$ in the category whose objects are the graded rings and whose morphisms are the degree-preserving homomorphisms of graded rings). This is actually important, as you later use it to define the Hall-Littlewood symmetric function $P_\lambda(x;t)$ on page 53.
- **page 3, Example 1.2.3, second bullet point:** "h_n" $\to$ "h_d".
- **page 3, Example 1.2.3, third bullet point:** "e_n" $\to$ "e_d".
- **page 4, footnote [1]:** After "In the **English convention**", add "(which is the one we will use)".
- **page 5, §1:** Before defining the three partial orders, you need to explain that if $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_k)$ is a partition, then λ_i is understood to mean 0 for each $i > k$. (This is sort-of forced by the second sentence of §1.3, but this might be not explicit enough to clear up reader-side confusion.)
- **page 6, §2.1:** In the (displayed) formula for $m_{3,2,1}$, replace "$x_i x_j x_k$" by "$x_i^3 x_j^2 x_k$".
- **page 7, proof of Theorem 2.2.3:** "Then there is $(0,1)$-matrix" $\to$ "Then there is a $(0,1)$-matrix A".

- **page 7, proof of Theorem 2.2.3:** "as least as many" $\to$ "at least as many".
- **page 7, proof of Theorem 2.3.1:** Directly after (2.3.2), erplace "$\prod_{n\geq 1}$" by "$\prod_{i\geq 1}$". And do the same on the second-to-last-line of page 7.
- **page 8, §2.5:** After "For a partition $\lambda = (\lambda_1, \ldots, \lambda_k)$", add "(with all of $\lambda_1, \ldots, \lambda_k$ positive)". The definition of p_λ doesn't tolerate trailing zeroes (unless you set $p_0 = 1$, which is somewhat artificial).
- **page 9, proof of Theorem 2.5.1:** "be reordering" $\to$ "by reordering".
- **page 9, proof of Theorem 2.5.1:** I know how this is proven, but I must say I don't understand your argument (beginning with "For each λ_j with $j \leq i$").
- **page 11, §2.6:** You write: "we need to work in two sets of variables x and y and in the ring $\Lambda \otimes \Lambda$ where the x's and y's are separately symmetric".

  I think you want to work in the ring $\mathbf{Z}[[x_1, x_2, \ldots, y_1, y_2, \ldots]]$ instead. The ring $\Lambda \otimes \Lambda$ does not contain **infinite** sums such as $\sum_\lambda u_\lambda(x) v_\lambda(y)$; they only exist in its completion, which is a whole new can of worms to open. Doing it justice would require showing that the $s_\lambda(x) \otimes s_\mu(y)$ for distinct pairs (λ, μ) of partitions are linearly independent in this completion, in an appropriate sense (i.e., even infinite linear combinations don't vanish). Meanwhile, in $\mathbf{Z}[[x_1, x_2, \ldots, y_1, y_2, \ldots]]$, everything is fairly simple.
- **page 11, Lemma 2.6.1:** Be careful with this – convergence isn't guaranteed! For example, $(1 + s_\lambda)_{\lambda \text{ is a partition}}$ is certainly a basis of $\Lambda_{\mathbf{Q}}$, but if you take both (u_λ) and (v_λ) to be this basis, then the sum $\sum_\lambda u_\lambda(x) v_\lambda(y) = \sum_\lambda (1 + s_\lambda(x))(1 + s_\lambda(y))$ is not well-defined (it has infinitely many 1 addends once you expand the parentheses). The safest way to dispel this problem is to require the bases (u_λ) and (v_λ) to be **graded** (i.e., for each partition λ, both u_λ and v_λ should be homogeneous symmetric functions of degree $|\lambda|$). Then, the sum $\sum_\lambda u_\lambda(x) v_\lambda(y)$ converges (in the formal sense).
- **page 11, proof of Lemma 2.6.1:** On the first line of this proof, you write "$u_\lambda = \sum_\alpha a_{\lambda,\rho} m_\alpha$". Replace "$a_{\lambda,\rho}$" by "$a_{\lambda,\alpha}$" here.
- **page 11, proof of Lemma 2.6.1:** The matrices A and B^T are infinite; thus, it is not immediately clear why $AB^T = I$ is equivalent to $B^T A = I$ (and why AB^T and $B^T A$ are well-defined to begin with). If you have required the bases (u_λ) and (v_λ) to be graded in the lemma, then this is easy to resolve (just notice that A and B are block-diagonal matrices, with each block being

a finite square matrix corresponding to a certain graded component of Λ or $\Lambda_{\mathbf{Q}}$). This also shows why all sums appearing in this proof converge.

- **page 12, proof of Proposition 2.6.4:** On the third line of the computation, "$\exp\left(\dfrac{p_n(x)\,p_n(y)}{n}\right)$" should be "$\exp\left(\sum\limits_{n\geq 1}\dfrac{p_n(x)\,p_n(y)}{n}\right)$".

- **page 12, proof of Proposition 2.6.4:** On the fourth line of the computation, "$\sum\limits_{d\geq 0}\dfrac{p_n(x)^d\,p_n(y)^d}{d!n^d}$" should be "$\prod\limits_{n\geq 1}\sum\limits_{d\geq 0}\dfrac{p_n(x)^d\,p_n(y)^d}{d!n^d}$".

- **page 12, proof of Corollary 2.6.5:** At the end of the displayed equation, "$= \varepsilon_\lambda\varepsilon_\mu\delta_{\lambda,\mu}$" should be "$= z_\lambda\varepsilon_\lambda\varepsilon_\mu\delta_{\lambda,\mu}$". Also, remove the period at the end of this equation, since the sentence goes on after it.

- **page 12, proof of Corollary 2.6.5:** "is the same as $\delta_{\lambda,\mu}$" $\to$ "is the same as $z_\lambda\delta_{\lambda,\mu}$".

- **page 13, §2.7:** Add a period at the end of the second displayed equation of §2.7.

- **page 14, §3.1:** "A **semistandard Young tableaux**" $\to$ "A **semistandard Young tableau**". Note that "tableaux" is the plural form.

- **page 14, §3.1:** It is best to explain what a "natural number" is. I suspect you don't count 0 as a natural number.

- **page 14, §3.1:** "The type of a SSYT" $\to$ "The **type** of a SSYT" (this is a definition).

- **page 14, §3.1:** "natural numbers of this Young diagram" $\to$ "natural numbers to the boxes of this Young diagram".

- **page 15, proof of Theorem 3.1.4:** "Let T be a SSYT of shape α" $\to$ "Let T be a SSYT of shape λ/μ and type α".

- **page 17, proof of Proposition 3.2.2:** "If not, then $T_{i+1,j} < T_{i,j}$ since b cannot bump the number in position $(i+1,j)$" is a bit confusing. I suggest "If not, then $T_{i+1,j} < T_{i,j}$ since otherwise b would bump the number in position $(i+1,j)$ or further left instead of bumping the number in position $(i+1,j')$".

- **page 17, proof of Proposition 3.2.2:** On the second-to-last line of this proof, "$b = T_{i,j} > T_{i,j'}$" should be "$b = T_{i,j} \leq T_{i,j'}$".

- **page 17:** Somewhere you should say that your matrix A is infinite, with both rows and columns indexed by positive integers; but in the examples, you are only showing a northwestern corner of it that contains all the nonzero entries.

- **page 17:** "This value gets added to some new box" $\to$ "The tableau $P(t+1)$ has a new box that $P(t)$ does not have". (Otherwise, what is "this value"? It isn't $(w_A)_{2,t+1}$.)

- **page 18, proof of Lemma 3.2.5:** You write: "i.e., that $(w_A)_{1,k} = (w_A)_{1,k+1}$". But it doesn't suffice to consider only two consecutive insertion steps here; perhaps a column of Q has two equal values coming from $(w_A)_{1,k}$ and $(w_A)_{1,k+3}$? It is probably best to argue not by contradiction, but instead say something like "We shall prove that if $(w_A)_{1,k} = (w_A)_{1,k+1} = \cdots = (w_A)_{1,m}$, then the boxes that get added to $P(k-1)$ to obtain $P(k), P(k+1), \ldots, P(m)$ (in this order) move further and further right (i.e., if $k \leq i < j \leq m$, then the box added to $P(i-1)$ to get $P(i)$ is strictly left than the box added to $P(j-1)$ to get $P(j)$); this will show that no two of these boxes lie in the same column. This will follow from the following lemma:".

- **page 18, proof of Lemma 3.2.6:** On the last line of the proof, "$r = \#((T \leftarrow j) \leftarrow k)$" should be "$r = \# I((T \leftarrow j) \leftarrow k)$".

- **page 18, proof of Theorem 3.2.7:** "That gives the last entry in the second row of w_A" is not quite correct: We need to reverse-bump it first, and whatever gets bumped out of the first row will be the last entry in the second row of w_A.

- **page 19, Remark 3.2.9:** Remove the period at the end of the displayed equation.

- **page 19, Remark 3.2.9:** An alternative reference is Corollary 3.4 of D. Eisenbud, D. DeConcini, C. Procesi, Young Diagrams and Determinantal Varieties, *Inventiones mathematicae*, vol. **56** (1980), pp. 129–166. (It is more explicit than Howe and does what it can in arbitrary characteristic.)

- **page 19, proof of Corollary 3.2.11:** After "by definition", add "and Corollary 2.6.3".

- **page 20, Theorem 3.3.1:** The concept of a "tableau" (without the "semistandard" accompanying it) should be defined.

- **page 20, Theorem 3.3.1:** The transpose tableau $P^\dagger$ should be defined.

- **page 20, Remark 3.3.3:** "The Cauchy identity" $\to$ "The dual Cauchy identity".

- **page 20, Remark 3.3.3:** The period at the end of the displayed equation should be a comma.

- **page 20, proof of Corollary 3.3.5:** Before the displayed equation, I'd add "Corollary 3.2.8 yields".

- **page 21, §3.4:** I'd say at the very start of this section that an $n \in \mathbb{N}$ shall be fixed for the whole section.
- **page 21, §3.4:** "by a permutation σ" $\to$ "by a permutation $\sigma \in \Sigma_n$".
- **page 21, Lemma 3.4.2:** Please say here that λ and ν should be two partitions of length $\leq n$ (with λ fixed), and that e_μ is understood to mean $e_\mu(x_1, x_2, \ldots, x_n)$ here.
- **page 21, Lemma 3.4.2:** "a_λ" should be "$a_{\lambda+\rho}$".
- **page 21, proof of Lemma 3.4.2:** I'd say right away here that all symmetric functions are understood to be specialized to n indeterminates $x_1, x_2, \ldots, x_n$ throughout this proof.
- **page 21, proof of Lemma 3.4.2:** In the second sentence of the proof, "a_ρ" should be "$a_{\nu+\rho}$".
- **page 21, proof of Lemma 3.4.2:** "have distinct exponents" $\to$ "has distinct exponents".
- **page 21, proof of Lemma 3.4.2:** In the first displayed equation of this proof, "$\gamma(n)$" should be "$\gamma(k)$".
- **page 21, proof of Lemma 3.4.2:** "only has boxes" $\to$ "is a size-μ_{r+1} diagram that only has boxes".
- **page 21, proof of Lemma 3.4.2:** "SSYT of $\lambda^\dagger/\nu^\dagger$" should be "SSYT of shape $\lambda^\dagger/\nu^\dagger$".
- **page 21, proof of Lemma 3.4.2:** After the second displayed equation of this proof, I would add "This difference is a polynomial (since $K_{\lambda^\dagger/\nu^\dagger,\mu} = 0$ unless $|\lambda| = |\mu| + |\nu|$) and is skew-symmetric.".
- **page 22, proof of Lemma 3.4.2:** "If $\lambda' \neq \lambda$" $\to$ "If λ' is a partition of length $\leq n$ such that $\lambda' \neq \lambda$".
- **page 22, proof of Lemma 3.4.2:** "of this difference" $\to$ "in this difference".
- **page 22, proof of Lemma 3.4.2:** "function of degree $|\lambda| + \binom{n}{2}$" $\to$ "polynomial". (The degree is irrelevant for the argument, and it makes no sense to speak of $|\lambda|$ when λ is not fixed anyway.)
- **page 22, Corollary 3.4.3:** After "a partition λ", add "of length $\leq n$".
- **page 22, proof of Corollary 3.4.3:** Again, you should say that all symmetric functions are understood to be specialized to n indeterminates $x_1, x_2, \ldots, x_n$ throughout this proof.

- **page 22, proof of Corollary 3.4.3:** "The s_λ and the e_μ are both bases" should be expanded to "The s_λ (for λ ranging over all partitions of length $\leq n$) and the e_μ (for μ ranging over all partitions with $\mu_1 \leq n$) are both bases of $\Lambda(n)$ (by Corollary 3.1.7 and Theorem 2.2.4)".

- **page 22, proof of Corollary 3.5.1:** Replace "e_μ" by "$e_\mu(x_1, \ldots, x_n)$" in the displayed equation.

- **page 22, proof of Corollary 3.5.1:** At the end of the displayed equation, add "for all sufficiently large n".

- **page 23, proof of Theorem 3.5.3:** "Since $\langle h_\nu, h_\mu \rangle = \delta_{\nu,\mu}$" $\to$ "Since $\langle h_\mu, m_\alpha \rangle = \delta_{\mu,\alpha}$".

- **page 24, proof of Theorem 3.5.7:** "exactly when no two boxes of $\lambda^\dagger / \nu^\dagger$ are in the same column" $\to$ "exactly when $|\lambda / \nu| = k$ and no two boxes of $\lambda^\dagger / \nu^\dagger$ are in the same column".

- **page 24, Example 3.5.8:** "all of the ways" $\to$ "all such ways".

- **page 24, proof of Corollary 3.5.9:** Add "$\lambda^{(0)} \subset$" before "$\lambda^{(1)} \subset \lambda^{(2)} \subset \cdots \subset \lambda^{(n)}$", since otherwise $\lambda^{(i)} / \lambda^{(i-1)}$ makes no sense for $i = 1$.

- **page 24, Remark 3.5.10:** In "The Pieri rule describes the decomposition of the tensor product of $\mathbf{S}_\lambda$", I would replace "$\mathbf{S}_\lambda$" by "$\mathbf{S}_\nu$", just to match the notations of Theorem 3.5.7.

- **page 25, proof of Theorem 3.6.1:** Again, "work in $\Lambda(N) \otimes \Lambda(N)$" isn't what you are doing, since your infinite sums don't exist in $\Lambda(N) \otimes \Lambda(N)$. It is best to work in $\mathbf{Z}[[x_1, x_2, \ldots, y_1, y_2, \ldots]]$ instead.

- **page 25, proof of Theorem 3.6.1:** On the last line of the displayed computation, replace "$m_\mu(y)$" by "$m_\nu(y)$".

- **page 26, proof of Theorem 3.6.1:** Worth saying that ρ here means $(N-1, N-2, \ldots, 1, 0)$, and we're using N instead of n here when applying results of §3.4.

- **page 26, proof of Theorem 3.6.1:** On the first line of the displayed computation, replace "$m_\mu(y)$" by "$m_\nu(y)$".

- **page 26, Remark 3.6.3:** Do you have a readable source to recommend for these complexes? Does Olver's *Differential Hyperforms* do them in this generality?

- **page 27, §4.1:** In the equivalent definition of a representation, after "$g \cdot (v + v') = g \cdot v + g \cdot v'$", add " and $g \cdot (\lambda v) = \lambda (g \cdot v)$".

- **page 27, §4.1:** Before you speak of a "nonzero representation", it is worth explaining that a representation $\rho : G \to \mathbf{GL}(V)$ is often called "the representation V" (with ρ being implicit), so various properties of V like dimension and nonzeroness are attributed to the representation.

- **page 27, §4.1:** "conjugacy classes of g" $\to$ "conjugacy classes of G".

- **page 27, §4.1:** "and define a bilinear pairing" $\to$ ". We define a sesquilinear pairing". These two sentences should not be one; otherwise it sounds like the functions $G \to \mathbf{C}$ are required to define a bilinear pairing! And yes, the pairing as you define it is sesquilinear. I prefer the bilinear variant, but that's not what you define :)

- **page 27, Theorem 4.1.2:** After "the number of irreducible representations of G", add "(up to isomorphism)".

- **page 27, §4.1:** The word "class function" should be defined before you use it.

- **page 28, §4.2:** On the second line of §4.2, "all of its" $\to$ "all of their".

- **page 28, footnote [5]:** Before "are roots of a monic", add "its values".

- **page 28, §4.2:** When defining the cycle type of a permutation, it should probably be said that length-1 cycles are counted into it.

- **page 28, §4.2:** "given positive integers n, m" $\to$ "given nonnegative integers n, m" (you need the $n = 0$ and $m = 0$ cases to build a ring out of it).

- **page 29, proof of Proposition 4.3.2:** The partition $\lambda + \mu$ should be defined (it is **not** the same as the $\alpha + \beta$ in §3.4; I would not even call it $\lambda + \mu$).

- **page 29, proof of Proposition 4.3.2:** In the displayed equation, I would put parenthesis in "$\operatorname{Ind}_{\Sigma_n \times \Sigma_m}^{\Sigma_{n+m}} 1_\lambda \otimes 1_\mu$" to make it clear that the whole tensor product is subject to induction.

- **page 29, proof of Proposition 4.3.2:** The space CF (without the subscript $_n$) needs to be defined.

- **page 29, proof of Proposition 4.3.2:** "map to a basis for Λ" should be "map to a basis for $\Lambda_{\mathbf{Q}}$".

- **page 29:** In "Let $R_n \subset \mathrm{CF}_n$ be the subspace", replace "subspace" by "$\mathbf{Z}$-submodule".

- **page 29:** You define "$R = \bigoplus_{n \geq 0} R_n$"; this notation conflicts with the "R" from page 1 (§1.1).

- **page 29, Proposition 4.3.3:** After "the irreducible characters", add "of Σ_n".

- **page 30:** The equality "$\eta^\alpha = 1_{\alpha_1} \circ \cdots \circ 1_{\alpha_k}$" relies on the shorthand notation $1_n := 1_{\Sigma_n}$. This should be explained.
- **page 30, proof of Lemma 4.3.4:** Add comma before "by Theorem 2.5.5".
- **page 30, proof of Corollary 4.3.5:** The first sentence of the second paragraph is maybe going too fast. I'd first say that the first paragraph entails $\chi^\lambda \in R$; then, the orthonormality of the χ^λ (which follows from Proposition 4.3.1) lets us apply Proposition 4.3.3.
- **page 31, proof of Theorem 4.4.2:** The notation "$(\alpha_1, \ldots, \alpha_i, \alpha_{i-1}, \ldots, \alpha_n)$" is ambiguous: It could mean both an n-tuple and an $(n+2)$-tuple (obtained by concatenating the i-tuple $(\alpha_1, \ldots, \alpha_i)$ with the $(n-i+2)$-tuple $(\alpha_{i-1}, \ldots, \alpha_n)$). Of course you mean the former; it's best to clarify it by writing "$(\alpha_1, \ldots, \alpha_{i-2}, \alpha_i, \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_n)$".

  Likewise, "$(\alpha_1, \ldots, \alpha_i - 1, \alpha_{i-1} + 1, \ldots, \alpha_n)$" should be "$(\alpha_1, \ldots, \alpha_{i-2}, \alpha_i - 1, \alpha_{i-1} + 1, \alpha_{i+1}, \ldots, \alpha_n)$".
- **page 31, proof of Theorem 4.4.2:** "and also $a_{\alpha+\rho} = 0$ if α has any repeating entries" → "and also $a_{\alpha+\rho} = 0$ if $\alpha + \rho$ has any repeating entries".
- **page 31, proof of Theorem 4.4.2:** "Suppose that $\mu + r\varepsilon_j$ has no repeating entries" → "Suppose that $\mu + r\varepsilon_j + \rho$ has no repeating entries".
- **page 32:** After "a **border-strip tableau** of shape λ/μ", add "and type α".
- **page 32:** In "sequence of partitions $\mu = \lambda^0 \subseteq \lambda^1 \subseteq \cdots \lambda^k = \lambda$", add a "$\subseteq$" sign after the "$\cdots$".
- **page 32:** When defining the height of a border-strip tableau, add the remark that the height of an empty border-strip tableau (i.e., a border-strip tableau of size 0) is defined to be 0 (not -1, as the definition might suggest).
- **page 32, proof of Corollary 4.4.6:** After "as a standard Young tableau", add "of size n (encoded as a sequence of partitions, as in the proof of Corollary 3.5.9)".
- **page 32, proof of Corollary 4.4.6:** "is always 1" → "is always 0".
- **page 33, proof of Corollary 4.4.7:** "the the" should be removed.
- **page 33, §4.5:** The definition of the Σ_d-action on $(\mathbf{C}^n)^{\otimes d}$ is wrong: you want σ to send $v_1 \otimes \cdots \otimes v_1$ to $v_1 \otimes \cdots \otimes v_1$ itself, not to $v_{\sigma(1)} \otimes \cdots \otimes v_{\sigma(1)}$.
- **page 34, §5.1:** "a matrix is not injective if and only if all of its maximal size square submatrices have determinant 0" → "an $n \times r$-matrix is not injective if and only if all its $r \times r$ submatrices have determinant 0". This

is the right statement, whereas "maximal size" means two different things depending on whether the matrix has more rows than columns or the other way round.

- **page 35, §5.2:** "$B \subset \mathbf{GL}_n(\mathbf{C})$" → "$B \subseteq \mathbf{GL}_n(\mathbf{C})$". (This is not a proper containment unless $n > 1$.)

- **page 35, §5.2:** The word "clear" in "it is clear that this is a unique way" is an exaggeration. This claim is equivalent to the uniqueness of the row reduced echelon form for a matrix (at least for a surjective matrix, but I don't think this case is any easier than the general case), and is one of the harder results in a standard linear algebra course.

- **page 35, Example 5.2.1:** "shorthand for an arbitrary complex number" → "shorthand for arbitrary complex numbers".

- **page 36, proof of Proposition 5.2.4:** You are only proving $\bigcup_{\mu \supseteq \lambda} X_\mu^\circ \subseteq \overline{X_\lambda^\circ}$ here. The reverse inclusion also needs to be proven.

- **page 36, proof of Proposition 5.2.4:** After "is in the closure", add "of X_λ°".

- **page 36:** In the definition of a "complete flag", replace "$F_1 \subset F_2 \subset \cdots \subset F_{n-1} \subset V$" by "$0 = F_0 \subset F_1 \subset F_2 \subset \cdots \subset F_{n-1} \subset F_n = V$". You do need F_0 and F_n, as you refer to them several times below.

- **page 37, proof of Lemma 5.2.6:** "where $\mu_{r-j+1} \geq \lambda_{r-j+1}$" → "where $\mu_j \geq \lambda_j$" (since i_j has nothing to do with λ_{r-j+1}).

- **page 37, proof of Lemma 5.2.7:** "From what we've shown" → "From the definition".

- **page 37, proof of Lemma 5.2.7:** Replace "$E, F \subset \mathbf{C}^n$" by "$E, F \subseteq \mathbf{C}^n$" (since the $\subset$ sign suggests proper containedness).

- **page 38, proof of Theorem 5.3.2:** At the beginning of the proof of Claim 1, I'd add the following sentence: "Both C and $\bigcap_{i=0}^{r} (A_i + B_{r-i})$, as well as each of the spaces $A_i + B_{r-i}$, are spans of some of the standard basis vectors $e_1, e_2, \ldots, e_n$." (This justifies focussing on the basis vectors e_p contained in these spaces when proving their mutual inclusions.)

- **page 38, proof of Theorem 5.3.2:** In the proof of Claim 1, "Pick $e_p \in C_j$" should be "Pick $j \in \{1, 2, \ldots, r\}$ and $e_p \in C_j$".

- **page 38, proof of Theorem 5.3.2:** In the proof of Claim 1, in the second displayed equation, "$n - r + j - \lambda_j \leq p$" should be "$n - r + j - \lambda_j \geq p$".

- **page 38, proof of Theorem 5.3.2:** In the proof of Claim 1, you write: "Pick j minimal so that $p \leq n - r + j - \lambda_j$". First of all, I'd replace "Pick j" by "Pick $j \in \{1, 2, \ldots, r\}$" here. Also, maybe you should say a few words about why such a j exists (namely, we have $e_p \in \bigcap_{i=0}^{r} (A_i + B_{r-i}) \subseteq A_r + B_0 = A_r$, so that $p \leq n - r + r - \lambda_r$).

- **page 39, proof of Theorem 5.3.2:** In the proof of Claim 2, replace the "$\subset$" sign in "$W \subset A_i + B_{r-i}$" by "$\subseteq$" (otherwise, it sounds like "proper subset", which you probably don't mean).

- **page 39, proof of Theorem 5.3.2:** In the proof of Claim 2, replace "$W \subseteq A_i + B_{i+1}$ again" by "$W \subseteq A_i + B_{r-i}$ again".

- **page 39, proof of Theorem 5.3.2:** In Claim 3, it is better to remove the words "$\lambda \subseteq \mu$ and". After all, you've already assumed that $\lambda_i \leq \mu_i$ for all i (which means that $\lambda \subseteq \mu$). Better to say "Recall that $\lambda \subseteq \mu$ by our above assumption.". (If you hadn't assumed $\lambda \subseteq \mu$, then Claim 3 would be false – for example, $C = C_1 + \cdots + C_r$ is also a direct sum if $\mu_i < \lambda_i$ for all i, because in this case all of the C_i are 0.)

- **page 39, proof of Theorem 5.3.2:** In the proof of Claim 3, I suggest replacing "if and only if $C_i \cap C_j = 0$ for all i, j" by "if and only if no two of the spaces $C_1, C_2, \ldots, C_r$ have a basis vector in common".

- **page 39, proof of Theorem 5.3.2:** The comma before "let c_i be any nonzero vector" should be a semicolon.

- **page 39, proof of Theorem 5.3.2:** After "If $W \in X(L_\bullet)_k$ as well", add "(where $L_\bullet$ is any flag such that $L_{n-k-r+1} = L$)".

- **page 39, proof of Theorem 5.3.2:** I am finding the last paragraph of this proof rather confusing. Why, for example, must the projection of c to C_i be a multiple of c_i ? (Why does it lie in W in the first place?)

- **page 40, §5.4:** Here, you suddenly start denoting $\mathbf{Gr}_r(\mathbf{C}^n)$ by $\mathbf{Gr}(r, \mathbf{C}^n)$.

- **page 42, proof of Theorem 6.1.1:** After "Recall from §3.4", add "(where our k is taking the role of the n from §3.4)".

- **page 42, proof of Theorem 6.1.1:** In

  $$\sum_{\sigma \in \Sigma_k} \operatorname{sgn}(\sigma) \frac{n!}{(\ell_1 - k + \sigma(1))! \cdots (\lambda_k - k + \sigma(k))!},$$

  replace "$(\lambda_k - k + \sigma(k))!$" by "$(\ell_k - k + \sigma(k))!$".

- **page 42, proof of Theorem 6.1.1:** "the binomial coefficients make sense" $\to$ "the multinomial coefficient makes sense".

- **page 42, proof of Theorem 6.1.1:** After the matrix in the last displayed equation, add a period.
- **page 42, proof of Theorem 6.1.1:** "and reduce it to the matrix $a_\rho(\ell_1, \ldots, \ell_k)$" $\to$ "and reduce it to the Vandermonde determinant $a_\rho(\ell_1, \ldots, \ell_k)$".
- **page 42, proof of Theorem 6.1.1:** "its hook is the set" $\to$ "its **hook** is the set".
- **page 42, proof of Theorem 6.1.1:** "in the book" $\to$ "in the hook".
- **page 43, Example 6.1.3:** After "in Theorem 6.1.1", add "(where we take $k = 3$)".
- **page 43, proof of Theorem 6.1.2:** "in the other boxes" $\to$ "of the other boxes".
- **page 44, Theorem 6.2.1:** At the beginning of this theorem, add "Let $k \geq \ell(\lambda)$. Then".
- **page 44, proof of Theorem 6.2.1:** "let's us get" $\to$ "lets us get".
- **page 44, proof of Theorem 6.2.1:** Replace "$\dfrac{\det\left(q^{i(\lambda_j+k-j)}\right)_{i,j=1}^k}{\det\left(q^{i(k-j)}\right)_{i,j=1}^k}$" by "$\dfrac{\det\left(q^{(i-1)(\lambda_j+k-j)}\right)_{i,j=1}^k}{\det\left(q^{(i-1)(k-j)}\right)_{i,j=1}^k}$".
- **page 44, Corollary 6.2.2:** Have you defined $n\lambda$?
- **page 44, Theorem 6.2.4:** At the beginning of this theorem, add "Let $k \in \mathbf{N}$. Then".
- **page 45, proof of Theorem 6.2.4:** Your argument here works only when $k \geq \ell(\lambda)$. The case $k < \ell(\lambda)$ requires a different (but simpler) argument: In this case, $s_\lambda(1, \ldots, 1) = 0$ (since there are no semistandard tableaux of shape λ with entries $1, 2, \ldots, k$) and also $\prod_{(i,j) \in Y(\lambda)} \dfrac{k + c(i,j)}{h(i,j)} = 0$ (since the cell $(k+1, 1)$ in the $(k+1)$-st row of $Y(\lambda)$ yields a factor of $\dfrac{k + c(k+1,1)}{h(k+1,1)} = 0$ in the product).
- **page 45, §6.3:** It is worth pointing out that the "$w_1 w_2 \cdots w_n$" in "Let $w = w_1 w_2 \cdots w_n$" is shorthand for $(w_1, w_2, \ldots, w_n)$.

- **page 45, §6.3:** "Littlewood–Richardson tableau" should be boldfaced the first time it appears.
- **page 46, Remark 6.3.3:** After "if ν/μ is a horizontal strip", add "of size d". Same after "if ν/μ is a vertical strip".
- **page 47, Lemma 7.1:** The comma before "let μ_i" should be a semicolon.
- **page 47, proof of Lemma 7.1.1:** "$\#\left\{\lambda_j \mid \lambda_j \geq i\right\}$" should be "$\#\left\{j \mid \lambda_j \geq i\right\}$" (if you only count the distinct λ_j, then you often undercount).
- **page 47:** "number of submodules" → "number of **Z**-submodules" (or of abelian subgroups).
- **page 47, Theorem 7.1.2:** After "such that", add "every prime p satisfies".
- **page 48, proof of Proposition 7.1.6:** At the beginning of this proof, add the sentence "Let r be the length of λ". (This r is used on the third line of the proof.)
- **page 48, proof of Proposition 7.1.6:** "abelian group of type λ" → "abelian p-group of type λ".
- **page 48, proof of Proposition 7.1.6:** On the first line of the proof, "$N \subset M$" → "$N \subseteq M$".
- **page 48, proof of Proposition 7.1.6:** "Now we count" → "Now we assume that λ/μ is a vertical strip of size k, and we count".
- **page 48, proof of Proposition 7.1.6:** "submodules" → "subgroups" (twice).
- **page 48, proof of Proposition 7.1.6:** The claim "$N/N_i \cong \left(N + p^i M\right)/p^i M$" took me a while to justify. Here is how I prove it: We have

  $$N_i = N \cap \underbrace{S_i}_{=S \cap p^i M} = \underbrace{N \cap S}_{\substack{=N \\ (\text{since } N \subseteq S)}} \cap p^i M = N \cap p^i M$$

  and thus

  $$N/\underbrace{N_i}_{=N \cap p^i M} = N/\left(N \cap p^i M\right) = \left(N + p^i M\right)/p^i M$$

  (by the second isomorphism theorem).
- **page 48–49, proof of Proposition 7.1.6:** The second display on page 49 says

  $$\ell\left(W_{i-1}\right)\ell\left(S_{i-1}/N_{i-1}\right) = \left(\lambda_i^\dagger - \mu_i^\dagger\right)\left(\sum_{j \geq i} \mu_j^\dagger - \sum_{j \geq i+1} \lambda_j^\dagger\right).$$

It should instead say

$$\ell\left(W_{i-1}\right)\ell\left(S_i/N_i\right)=\left(\lambda_i^\dagger-\mu_i^\dagger\right)\left(\sum_{j\geq i+1}\mu_j^\dagger-\sum_{j\geq i+2}\lambda_j^\dagger\right).$$

More importantly: The construction of N needs more details. First of all, you should say that each $i \geq \lambda_1$ satisfies $S_i = 0$, and therefore N_i must necessarily be 0. It thus remains to construct N_i for all $i \in \{0, 1, \ldots, \lambda_1 - 1\}$ (because then, N is determined by $N = N_0$). You intend to do this by recursion in the order of decreasing i. You want to construct them in such a way that for each i, we have $N_i \subseteq N_{i-1}$ and $N_{i-1} \cap S_i = N_i$, and the image of the canonical map $N_{i-1}/N_i \to S_{i-1}/S_i$ (which, by the way, is injective because of $N_{i-1} \cap S_i = N_i$) is W_{i-1} (which then automatically entails $\ell\left(N_{i-1}/N_i\right) = \ell\left(W_{i-1}\right) = \lambda_i^\dagger - \mu_i^\dagger$).

So you fix some positive integer i, and assume that N_i (and W_{i-1}) are already chosen; now you need to construct N_{i-1}. You say that "we take any preimages of a basis for W_{i-1} under the map S_{i-1}/S_i and take its span with N_i". I am not convinced that I understand this; different bases might lead to identical spaces. Instead, I argue as follows:

First, we observe that if V is a finite abelian p-group, and if U is a subgroup of V such that $p\left(V/U\right) = 0$, then

$$\left(\text{the number of complements to } U \text{ in } V\right) = p^{\ell(U)\cdot\ell(V/U)}. \tag{1}$$

[1] Using this fact, it is easy to see that if V is a finite abelian p-group, and if U and W are two subgroups of V satisfying $W \subseteq U$ and $p\left(V/U\right) = 0$, then

$$\left(\text{the number of subgroups } U' \text{ of } V \text{ satisfying } U' \cap U = W \text{ and } U' + U = V\right) = p^{\ell(U/W)\cdot\ell(V/U)}. \tag{2}$$

(Indeed, such subgroups U' are in bijection with the complements to the subgroup U/W in the p-group V/W; therefore, (2) follows from (1).)

Now, we want to choose a subgroup N_{i-1} of S_{i-1} such that $N_i \subseteq N_{i-1}$ and $N_{i-1} \cap S_i = N_i$ and the image of the canonical map $N_{i-1}/N_i \to S_{i-1}/S_i$ is W_{i-1}. Such a subgroup N_{i-1} will be called a *helpful* subgroup. Note

[1] *Proof of (1):* Fix a basis $(b_1, b_2, \ldots, b_k)$ of the $\mathbf{Z}/p$-vector space V/U; then, any complement to U in V has a unique basis $(\beta_1, \beta_2, \ldots, \beta_k)$ with the property that the projection of each β_i onto V/U is b_i. Conversely, every k-tuple $(\beta_1, \beta_2, \ldots, \beta_k)$ of vectors in V with this property is a basis of a unique complement to U in V. Thus, the number of complements to U in V equals the number of ways to pick k elements $\beta_1, \beta_2, \ldots, \beta_k$ of V such that the projection of each β_i onto V/U is b_i. But the latter number is $|U|^k = p^{\ell(U)\cdot\ell(V/U)}$ (since $|U| = p^{\ell(U)}$ and $k = \ell\left(V/U\right)$).

that the requirement $N_i \subseteq N_{i-1}$ in the definition of a helpful subgroup is redundant, since it follows from $N_{i-1} \cap S_i = N_i$. Thus, a subgroup N_{i-1} of S_{i-1} is helpful if and only if it satisfies $N_{i-1} \cap S_i = N_i$ and the image of the canonical map $N_{i-1}/N_i \to S_{i-1}/S_i$ is W_{i-1}.

In order to count all helpful subgroups, we first let $\widehat{W}_{i-1}$ denote the preimage of W_{i-1} under the canonical projection $S_{i-1} \to S_{i-1}/S_i$. Note that $S_i \subseteq \widehat{W}_{i-1} \subseteq S_{i-1}$; thus, every subgroup of $\widehat{W}_{i-1}$ is a subgroup of S_{i-1}. Also, from $\widehat{W}_{i-1} \subseteq S_{i-1}$, we obtain $\widehat{W}_{i-1}/S_i \subseteq S_{i-1}/S_i$, so that $p\left(\widehat{W}_{i-1}/S_i\right) \subseteq p\left(S_{i-1}/S_i\right) = 0$ and thus $p\left(\widehat{W}_{i-1}/S_i\right) = 0$.

But a subgroup N_{i-1} of S_{i-1} satisfies $N_{i-1} + S_i = \widehat{W}_{i-1}$ if and only if the image of the canonical map $N_{i-1}/N_i \to S_{i-1}/S_i$ is W_{i-1}. Hence, a subgroup N_{i-1} of S_{i-1} is helpful if and only if it satisfies $N_{i-1} \cap S_i = N_i$ and $N_{i-1} + S_i = \widehat{W}_{i-1}$. Thus, the helpful subgroups N_{i-1} of S_{i-1} are precisely those subgroups of $\widehat{W}_{i-1}$ that satisfy $N_{i-1} \cap S_i = N_i$ and $N_{i-1} + S_i = \widehat{W}_{i-1}$ (indeed, every helpful subgroup N_{i-1} of S_{i-1} must be a subgroup of $\widehat{W}_{i-1}$, since it satisfies $N_{i-1} \subseteq N_{i-1} + S_i = \widehat{W}_{i-1}$; conversely, every subgroup of $\widehat{W}_{i-1}$ is a subgroup of S_{i-1}). But (2) (applied to $V = \widehat{W}_{i-1}$ and $U = S_i$ and $W = N_i$) yields that the number of the latter subgroups is $p^{\ell(S_i/N_i)\cdot\ell\left(\widehat{W}_{i-1}/S_i\right)}$. Hence, the number of helpful subgroups of S_{i-1} is

$$p^{\ell(S_i/N_i)\cdot\ell\left(\widehat{W}_{i-1}/S_i\right)} = p^{\ell(S_i/N_i)\cdot\ell(W_{i-1})} \qquad \left(\text{since } \widehat{W}_{i-1}/S_i \cong W_{i-1}\right)$$
$$= p^{\ell(W_{i-1})\ell(S_i/N_i)}.$$

In other words, the number of ways to choose N_{i-1} is $p^{\ell(W_{i-1})\ell(S_i/N_i)}$.

I don't see a way to make this shorter (and it took me 2 hours to figure out)...

- **page 49, proof of Proposition 7.1.6:** "The binomial coefficient" $\to$ "The p-binomial coefficient".
- **page 49, proof of Proposition 7.1.6:** Every "m" in the last paragraph of this proof should be a "k".
- **page 50, Proposition 7.1.8:** After "there exist unique polynomials $g^{\lambda}_{\mu,\nu}(t)$", add "$\in \mathbf{Z}[t]$ independent of p".
- **page 50, proof of Proposition 7.1.8:** "such that M is an abelian p-group of type μ" $\to$ "such that M is a fixed abelian p-group of type μ (chosen once and for all)".
- **page 50, proof of Proposition 7.1.8:** In "the change of basis matrix between u_λ and v_λ is lower-unitriangular", replace "v_λ" by "v_{λ^t}".

- **page 50, proof of Proposition 7.1.8:** After "there exists a polynomial $a_{\lambda,\mu}(t)$", add "$\in \mathbf{Z}[t]$ independent of p".

- **page 50, proof of Proposition 7.1.8:** Replace "such that $A_{\lambda,\mu}(p) = a_{\lambda,\mu}(p)$ and $a_{\lambda,\lambda^\dagger}(t) = 1$" by "such that $A_{\lambda,\mu}(p) = a_{\lambda,\mu}(p)$ for all primes p. Since there are infinitely many primes, we conclude that $a_{\lambda,\lambda^\dagger}(t) = 1$ (because $A_{\lambda,\lambda^\dagger}(p) = 1$ for all primes p), and similarly $a_{\lambda,\mu}(t) = 0$ whenever we don't have $\mu^\dagger \geq \lambda$".

- **page 50:** In the definition of the universal Hall algebra, replace "$\mathbf{H} = H \otimes \mathbf{Z}[t]$ where" by "$\mathbf{H}$, which is $H \otimes \mathbf{Z}[t]$ as a $\mathbf{Z}[t]$-module, but where". Or, better: "$\mathbf{H}$; this is the $\mathbf{Z}[t]$-algebra defined as the free $\mathbf{Z}[t]$-module with basis u_λ (with λ ranging over all partitions) endowed with the same multiplication law as H except that".

- **page 51, proof of Theorem 7.1.10:** On the first two lines of this proof, replace "by Lemma 7.1.4 and Lemma 7.1.1" by "by Lemma 7.1.4 (b)". (I don't see where you are using Lemma 7.1.1 here.)

- **page 51, proof of Theorem 7.1.10:** On the third line of this proof, replace "$\lambda^{(i)^\dagger}$" by "$\lambda^{(i)\dagger}$" (there should not be any nested superscripts here).

- **page 51, §7.2:** "This is a polynomial in t which does not" $\to$ "These are polynomials in t which do not".

- **page 52, §7.2:** In the first display on page 52, add a "$\operatorname{sgn}(\sigma)$" factor immediately after the summation sign.

- **page 52, §7.2:** After "and of the same degree", add "in the variables $x_1, \ldots, x_n$".

- **page 52, proof of Lemma 7.2.1:** You should say that you are treating t as a constant here, so that "coefficient of $x_1^{n-1} x_2^{n-2} \cdots x_{n-1}$" does not mean that powers of t get discarded.

- **page 52, proof of Lemma 7.2.1:** "from a permutation in $\tau \in \Sigma_{n-1}$" has an "in" too much.

- **page 52, proof of Lemma 7.2.1:** "insert if" $\to$ "insert it".

- **page 52, Lemma 7.2.2:** I'd add "For every partition $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$" at the beginning of this lemma.

- **page 52, proof of Lemma 7.2.2:** In the last displayed equation on page 52, insert "$\beta_{\lambda_1} \beta_{\lambda_1 - 1} \cdots \beta_0$" immediately after the "$\sum\limits_{(\beta_{\lambda_1}, \ldots, \beta_0)}$" sign.

Also, I'd insert an extra step between the middle-hand side and the right-hand side of this computation (for the sake of clarity), namely

$$\left(\prod_{u=0}^{\lambda_1}\sum_{\beta\in\Sigma_{m_u(\lambda)}}\beta_u\left(\prod_{\substack{1\leq i<j\leq n;\\ \lambda_i=\lambda_j=u}}\frac{x_i-tx_j}{x_i-x_j}\right)\right)\cdot\prod_{\substack{1\leq i<j\leq n;\\ \lambda_i>\lambda_j}}\frac{x_i-tx_j}{x_i-x_j}.$$

- **page 53, Proposition 7.2.3:** It is worth adding a fifth claim, **(e)**, saying that P_λ is homogeneous of degree $|\lambda|$ in the variables $x_1, x_2, \ldots, x_n$. (This is easy to check from either expression in Lemma 7.2.2; but it's crucial to the construction of $P_\lambda(x;t) \in \Lambda[t]$ later. If the degree of $P_\lambda(x_1, \ldots, x_n; t)$ could grow with n, then there wouldn't be a $P_\lambda(x;t) \in \Lambda[t]$ that projects down to all of these $P_\lambda(x_1, \ldots, x_n; t)$.)
- **page 53, proof of Proposition 7.2.3 (b):** Lemma 7.2.1 yields a simpler reason why $v_n(0) = 1$.
- **page 53, proof of Proposition 7.2.3 (d):** Why is this "clear"?
- **page 53:** The definition of the Hall-Littlewood symmetric function $P_\lambda(x;t) \in \Lambda[t]$ has the consequence that a Hall-Littlewood polynomials $P_\lambda(x_1, x_2, \ldots, x_n; t)$ also becomes defined when $n < \ell(\lambda)$ (although Lemma 7.2.2 does not hold in this case). It is worth mentioning that these polynomials aren't very interesting: Namely, for every partition λ and any $n < \ell(\lambda)$, we have

$$P_\lambda(x_1, x_2, \ldots, x_n; t) = 0. \tag{3}$$

(This is easy to prove: Just notice that $P_\lambda\left(x_1, x_2, \ldots, x_{\ell(\lambda)}; t\right)$ is a multiple of $x_1 x_2 \cdots x_{\ell(\lambda)}$ (this follows from either of the two expressions in Lemma 7.2.2), and thus becomes 0 when $x_{\ell(\lambda)}$ is set to 0.)

- **page 54, proof of Lemma 7.2.7:** At the beginning of this proof, add "We WLOG assume that q is a prime power.". (Otherwise there is no $\mathbf{F}_q$.)
- **page 54, Proposition 7.2.8:** "If" $\to$ "If λ/μ is a vertical strip and" at the beginning of this proposition.

  Also, add "Otherwise, $f^{\lambda}_{\mu,1^m}(t) = 0$." at the end.
- **page 54, proof of Proposition 7.2.8:** "in finitely many variables n" $\to$ "in finitely many variables $x_1, x_2, \ldots, x_n$".
- **page 54, proof of Proposition 7.2.8:** "with $n \geq \ell(\mu) + m$" $\to$ "with $n \geq |\mu| + m$" (at least this is safer; maybe $n \geq \ell(\mu) + m$ is sufficient too).
- **page 54, proof of Proposition 7.2.8:** Remove the period at the end of the first displayed equation in this proof.

- **page 54, proof of Proposition 7.2.8:** "If $X_i = \{y_1, \ldots, y_{r_i}\}$" should be "If $X_i = \left\{y_1, \ldots, y_{m_i(\mu)}\right\}$".
- **page 54, proof of Proposition 7.2.8:** In the second-to-last display on page 54, replace "$\operatorname{Aut}(\{1, \ldots, r_i\})$" by "$\operatorname{Aut}(\{1, \ldots, m_i(\mu)\})$" (though I'm also wondering why you don't just say "$\Sigma_{m_i(\mu)}$").
- **pages 54–55, proof of Proposition 7.2.8:** In the last display on page 54, replace "$v_{m_i(\mu)}(t)$" by "$v_{m_i(\mu)-r_i}(t)$". The same typo also appears 5 times on page 55.
- **page 55, proof of Proposition 7.2.8:** The equality

$$P_\mu(x;t)\, e_{r_0}(X_0) \cdots e_{r_k}(X_k) = \left(\prod_{i=0}^{k} v_{r_i}(t)\, v_{m_i(\mu)}(t)\right)^{-1} \sum_{\sigma \in \Sigma_n} \sigma\left(x_1^{\lambda(\mathbf{r})_1} \cdots x_n^{\lambda(\mathbf{r})_n} \prod_{i<j} \frac{x_i - t x_j}{x_i - x_j}\right)$$

is not true as stated (even after correcting "$v_{m_i(\mu)}(t)$" to "$v_{m_i(\mu)-r_i}(t)$"); the left hand side is not a symmetric polynomial while the right hand side is a symmetric polynomial. It only becomes symmetric after summing over all

r. More precisely, the following holds:

$$P_\mu(x;t)\, e_m(x_1,\ldots,x_n)$$

$$= \sum_{\sigma\in\Sigma_n/\Sigma_n^\mu} \sigma\left(x_1^{\mu_1}\cdots x_n^{\mu_n} \prod_{\substack{i,j;\\ \mu_i>\mu_j}} \frac{x_i - tx_j}{x_i - x_j} \right) \underbrace{e_m(x_1,\ldots,x_n)}_{=\sigma(e_m(x_1,\ldots,x_n))}$$

$$\left(\text{by the second expression for } P_\mu(x;t) \text{ in Lemma 7.2.2}\right)$$

$$= \sum_{\sigma\in\Sigma_n/\Sigma_n^\mu} \sigma\left(x_1^{\mu_1}\cdots x_n^{\mu_n} \prod_{\substack{i,j;\\ \mu_i>\mu_j}} \frac{x_i - tx_j}{x_i - x_j} \right) \sigma\left(\underbrace{e_m(x_1,\ldots,x_n)}_{=\sum_{\mathbf{r}=(r_0,\ldots,r_k)} e_{r_0}(X_0)\cdots e_{r_k}(X_k)} \right)$$

$$= \sum_{\mathbf{r}=(r_0,\ldots,r_k)} \sum_{\sigma\in\Sigma_n/\Sigma_n^\mu} \sigma\left(x_1^{\mu_1}\cdots x_n^{\mu_n} \prod_{\substack{i,j;\\ \mu_i>\mu_j}} \frac{x_i - tx_j}{x_i - x_j} \right)$$

$$\sigma\left(\underbrace{e_{r_0}(X_0)\cdots e_{r_k}(X_k)}_{\substack{=\left(\prod_{i=0}^k v_{r_i}(t)v_{m_i(\mu)-r_i}(t)\right)^{-1} \sum_{\tau\in\Sigma_n^\mu} \tau\left(x_1^{\lambda(\mathbf{r})_1-\mu_1}\cdots x_n^{\lambda(\mathbf{r})_n-\mu_n} \prod_{\substack{i<j;\\ \mu_i=\mu_j}} \frac{x_i-tx_j}{x_i-x_j} \right) \\ \text{(by the previous formula)}}} \right)$$

$$= \sum_{\mathbf{r}=(r_0,\ldots,r_k)} \left(\prod_{i=0}^k v_{r_i}(t)\, v_{m_i(\mu)-r_i}(t) \right)^{-1} \sum_{\sigma\in\Sigma_n/\Sigma_n^\mu} \sum_{\tau\in\Sigma_n^\mu} \sigma\left(\underbrace{x_1^{\mu_1}\cdots x_n^{\mu_n} \prod_{\substack{i,j;\\ \mu_i>\mu_j}} \frac{x_i - tx_j}{x_i - x_j}}_{=\tau\left(x_1^{\mu_1}\cdots x_n^{\mu_n} \prod_{\substack{i,j;\\ \mu_i>\mu_j}} \frac{x_i-tx_j}{x_i-x_j} \right)} \right)$$

$$\sigma\left(\tau\left(x_1^{\lambda(\mathbf{r})_1-\mu_1}\cdots x_n^{\lambda(\mathbf{r})_n-\mu_n} \prod_{\substack{i<j;\\ \mu_i=\mu_j}} \frac{x_i - tx_j}{x_i - x_j} \right)\right)$$

$$= \sum_{\mathbf{r}=(r_0,\ldots,r_k)} \left(\prod_{i=0}^{k} v_{r_i}(t)\, v_{m_i(\mu)-r_i}(t) \right)^{-1} \sum_{\sigma \in \Sigma_n / \Sigma_n^\mu} \sum_{\tau \in \Sigma_n^\mu}$$

$$\sigma \left(\tau \left(\underbrace{ x_1^{\mu_1} \cdots x_n^{\mu_n} \prod_{\substack{i,j;\\ \mu_i > \mu_j}} \frac{x_i - t x_j}{x_i - x_j} x_1^{\lambda(\mathbf{r})_1 - \mu_1} \cdots x_n^{\lambda(\mathbf{r})_n - \mu_n} \prod_{\substack{i<j;\\ \mu_i = \mu_j}} \frac{x_i - t x_j}{x_i - x_j} }_{= x_1^{\lambda(\mathbf{r})_1} \cdots x_n^{\lambda(\mathbf{r})_n} \prod_{i<j} \frac{x_i - t x_j}{x_i - x_j}} \right) \right)$$

$$= \sum_{\mathbf{r}=(r_0,\ldots,r_k)} \left(\prod_{i=0}^{k} v_{r_i}(t)\, v_{m_i(\mu)-r_i}(t) \right)^{-1}$$

$$\underbrace{ \sum_{\sigma \in \Sigma_n / \Sigma_n^\mu} \sum_{\tau \in \Sigma_n^\mu} \sigma \left(\tau \left(x_1^{\lambda(\mathbf{r})_1} \cdots x_n^{\lambda(\mathbf{r})_n} \prod_{i<j} \frac{x_i - t x_j}{x_i - x_j} \right) \right) }_{= \sum_{\sigma \in \Sigma_n} \sigma \left(x_1^{\lambda(\mathbf{r})_1} \cdots x_n^{\lambda(\mathbf{r})_n} \prod_{i<j} \frac{x_i - t x_j}{x_i - x_j} \right)}$$

$$= \sum_{\mathbf{r}=(r_0,\ldots,r_k)} \left(\prod_{i=0}^{k} v_{r_i}(t)\, v_{m_i(\mu)-r_i}(t) \right)^{-1} \underbrace{ \sum_{\sigma \in \Sigma_n} \sigma \left(x_1^{\lambda(\mathbf{r})_1} \cdots x_n^{\lambda(\mathbf{r})_n} \prod_{i<j} \frac{x_i - t x_j}{x_i - x_j} \right) }_{\substack{= v_{\lambda(\mathbf{r})}(t) P_{\lambda(\mathbf{r})}(x;t) \\ \text{(by the first expression for } P_{\lambda(\mathbf{r})}(x;t) \\ \text{in Lemma 7.2.2)}}}$$

$$= \sum_{\mathbf{r}=(r_0,\ldots,r_k)} \left(\prod_{i=0}^{k} v_{r_i}(t)\, v_{m_i(\mu)-r_i}(t) \right)^{-1} v_{\lambda(\mathbf{r})}(t)\, P_{\lambda(\mathbf{r})}(x;t).$$

From then on, conclude (as you say) that

$$f_{\mu,1^m}^{\lambda(\mathbf{r})}(t) = \left(\prod_{i=0}^{k} v_{r_i}(t)\, v_{m_i(\mu)-r_i}(t) \right)^{-1} v_{\lambda(\mathbf{r})}(t)$$

for every $\mathbf{r}$, and also that $f_{\mu,1^m}^{\lambda}(t) = 0$ whenever λ/μ is not a vertical strip of size m.

- **page 55, proof of Proposition 7.2.8:** The very last equality sign in the proof, between the $\prod_{i \geq 0}$ and the $\prod_{i \geq 1}$ products, relies on the (easy) fact that $m_0(\lambda(\mathbf{r})) = m_0(\mu) - r_0$. This might be worth mentioning, since otherwise it may appear that some factors just mysteriously disappear.

- **page 55, proof of Corollary 7.2.9:** Add "We WLOG assume that λ/μ is a vertical m-strip, since otherwise (by Proposition 7.2.8 and Proposition 7.1.6) both $g^{\lambda}_{\mu,1^m}$ and $f^{\lambda}_{\mu,1^m}$ are zero." at the beginning of this proof.
- **page 55, proof of Corollary 7.2.9:** "product of binomial coefficients" $\rightarrow$ "product of t-binomial coefficients".
- **page 55, proof of Theorem 7.2.10:** "any submodule of type 1^m of a module of type λ" $\rightarrow$ "any subgroup of type 1^m of an abelian p-group of type λ".
- **page 55, proof of Theorem 7.2.10:** "if ν/μ is a vertical strip" $\rightarrow$ "if ν/μ is a vertical strip of size m".
- **page 56, proof of Theorem 7.2.10:** In the second displayed equation on this page, "$\sum\limits_{\nu}$" should be "$\sum\limits_{\nu<\lambda}$".
- **page 56, proof of Theorem 7.2.10:** Replace "gives $\psi'(u_\lambda) = p^{n(\mu)+n(1^m)} P_\lambda\left(x; p^{-1}\right)$" by "gives $p^{n(\mu)+n(1^m)} \psi'(u_\lambda) = P_\lambda\left(x; p^{-1}\right)$".
- **page 56, Corollary 7.2.13:** Worth explaining what $\lambda + \alpha$ means (you have previously only introduced the sum of two n-tuples, not the sum of two arbitrary partitions).
- **page 56, Corollary 7.2.13:** I'd replace "$\beta + \mu$" by "$\mu + \beta$" just for the sake of consistency.
- **page 56, proof of Corollary 7.2.13:** "Let p be prime" $\rightarrow$ "Let p be a prime".
- **page 56, proof of Corollary 7.2.13:** The union $\alpha \cup \beta$ of two partitions α, β should be defined.
- **page 57, §8:** "and $Q_r(x; 0) = s_\lambda(x)$" should be "and $Q_\lambda(x; 0) = s_\lambda(x)$".
- **page 57, §8:** After "In finitely many variables", add "$x_1, x_2, \ldots, x_n$".
- **page 57, (8.1):** This holds only for $r > 0$. When $r = 0$, the right hand side is $1 - t^2$, whereas the left hand side is 1.
- **page 58, Lemma 8.1.2:** This only holds for $\lambda \neq \varnothing$.
- **page 58, proof of Lemma 8.1.2:** At the beginning of this proof, add "We WLOG assume that $n \geq \ell(\lambda)$, since otherwise both sides equal 0 (as can be easily shown using (3))."
- **page 58, proof of Lemma 8.1.2:** "the leading coefficient" $\rightarrow$ "the fraction before the summation sign".

- **page 58, proof of Lemma 8.1.2:** I don't understand where the last equality sign on page 58 comes from. Here is an argument I would suggest instead (starting with the first line of the last display on page 58, slightly modified to make it more obvious):

$$\begin{aligned}
&Q_\lambda\left(x_1,\ldots,x_n;t\right)\\
&=(1-t)^{k-1}\sum_{\sigma\in\Sigma_n/\Sigma_{n-k}}\sigma_1\left(x_1^{\lambda_1}g_1\sigma_2\left(x_2^{\lambda_2}\cdots x_k^{\lambda_k}\prod_{i=2}^{k}\prod_{j>i}\frac{x_i-tx_j}{x_i-x_j}\right)\right)\\
&=(1-t)^{k-1}\sum_{\sigma_1\in\Sigma_n/\Sigma_{n-1}}\\
&\qquad\sigma_1\left(x_1^{\lambda_1}g_1\sum_{\sigma_2\in\Sigma_{n-1}/\Sigma_{n-k}}\sigma_2\left(x_2^{\lambda_2}\cdots x_k^{\lambda_k}\prod_{i=2}^{k}\prod_{j>i}\frac{x_i-tx_j}{x_i-x_j}\right)\right). \qquad (4)
\end{aligned}$$

Let us now agree to treat μ as the $(n-1)$-tuple $(\mu_1,\mu_2,\ldots,\mu_{n-1})=(\lambda_2,\lambda_3,\ldots,\lambda_n)$. Then, $m_0(\mu)=n-k$. Now,

$$\begin{aligned}
b_\mu(t)&=\prod_{i\geq1}\underbrace{\varphi_{m_i(\mu)}(t)}_{=(1-t)^{m_i(\mu)}\cdot[m_i(\mu)]_t!}=\prod_{i\geq1}\left((1-t)^{m_i(\mu)}\cdot[m_i(\mu)]_t!\right)\\
&=\underbrace{(1-t)^{\sum\limits_{i\geq1}m_i(\mu)}}_{=(1-t)^{k-1}}\cdot\underbrace{\prod_{i\geq1}[m_i(\mu)]_t!}_{=\left(\prod\limits_{i\geq0}[m_i(\mu)]_t!\right)/[m_0(\mu)]_t!}\\
&=(1-t)^{k-1}\cdot\left(\prod_{i\geq0}\underbrace{[m_i(\mu)]_t!}_{=v_{m_i(\mu)}(t)}\right)\Big/\underbrace{[m_0(\mu)]_t!}_{\substack{=v_{m_0(\mu)}(t)=v_{n-k}(t)\\ (\text{since } m_0(\mu)=n-k)}}\\
&=(1-t)^{k-1}\cdot\underbrace{\left(\prod_{i\geq0}v_{m_i(\mu)}(t)\right)}_{=v_\mu(t)}\Big/v_{n-k}(t)=(1-t)^{k-1}\frac{v_\mu(t)}{v_{n-k}(t)}. \qquad (5)
\end{aligned}$$

But every $\sigma_1 \in \Sigma_n / \Sigma_{n-1}$ satisfies

$$\sum_{\sigma_2 \in \Sigma_{n-1}/\Sigma_{n-k}} \sigma_2 \left(x_2^{\lambda_2} \cdots x_k^{\lambda_k} \prod_{i=2}^{k} \prod_{j>i} \frac{x_i - tx_j}{x_i - x_j} \right)$$

$$= \sum_{\sigma_2 \in \Sigma_{n-1}/\Sigma_{n-k}} \sigma_2 \left(x_2^{\lambda_2} \cdots x_k^{\lambda_k} \prod_{i=2}^{k} \prod_{j>i} \frac{x_i - tx_j}{x_i - x_j} \cdot \frac{1}{v_{n-k}(t)} \sum_{\tau \in \Sigma_{n-k}} \tau \left(\prod_{k<i<j\leq n} \frac{x_i - tx_j}{x_i - x_j} \right) \right)$$

$$\left(\begin{array}{c} \text{since } v_{n-k}(t) = \sum_{\tau \in \Sigma_{n-k}} \tau \left(\prod_{k<i<j\leq n} \frac{x_i - tx_j}{x_i - x_j} \right) \\ \text{(by the definition of } v_{n-k}(t) \text{)}, \\ \text{so we have inserted 2 factors whose product is 1} \end{array} \right)$$

$$= \frac{1}{v_{n-k}(t)} \sum_{\sigma_2 \in \Sigma_{n-1}/\Sigma_{n-k}} \sum_{\tau \in \Sigma_{n-k}} \underbrace{\sigma_2 \left(x_2^{\lambda_2} \cdots x_k^{\lambda_k} \left(\prod_{i=2}^{k} \prod_{j>i} \frac{x_i - tx_j}{x_i - x_j} \right) \cdot \tau \left(\prod_{k<i<j\leq n} \frac{x_i - tx_j}{x_i - x_j} \right) \right)}_{\substack{= (\sigma_2 \tau) \left(x_2^{\lambda_2} \cdots x_k^{\lambda_k} \left(\prod_{i=2}^{k} \prod_{j>i} \frac{x_i - tx_j}{x_i - x_j} \right) \left(\prod_{k<i<j\leq n} \frac{x_i - tx_j}{x_i - x_j} \right) \right) \\ \left(\text{since } \tau \text{ fixes } x_2^{\lambda_2} \cdots x_k^{\lambda_k} \left(\prod_{i=2}^{k} \prod_{j>i} \frac{x_i - tx_j}{x_i - x_j} \right) \right)}}$$

$$= \frac{1}{v_{n-k}(t)} \sum_{\sigma_2 \in \Sigma_{n-1}/\Sigma_{n-k}} \sum_{\tau \in \Sigma_{n-k}} (\sigma_2 \tau) \left(x_2^{\lambda_2} \cdots x_k^{\lambda_k} \left(\prod_{i=2}^{k} \prod_{j>i} \frac{x_i - tx_j}{x_i - x_j} \right) \left(\prod_{k<i<j\leq n} \frac{x_i - tx_j}{x_i - x_j} \right) \right)$$

$$= \underbrace{\frac{1}{v_{n-k}(t)}}_{= \frac{v_\mu(t)}{v_{n-k}(t)} \cdot \frac{1}{v_\mu(t)}} \sum_{\gamma \in \Sigma_{n-1}} \gamma \left(\underbrace{x_2^{\lambda_2} \cdots x_k^{\lambda_k}}_{= x_2^{\mu_1} \cdots x_k^{\mu_{k-1}}} \underbrace{\left(\prod_{i=2}^{k} \prod_{j>i} \frac{x_i - tx_j}{x_i - x_j} \right) \left(\prod_{k<i<j\leq n} \frac{x_i - tx_j}{x_i - x_j} \right)}_{= \prod_{2<i<j\leq n} \frac{x_i - tx_j}{x_i - x_j}} \right)$$

$$\left(\begin{array}{c} \text{since any } \gamma \in \Sigma_{n-1} \text{ can be written uniquely as } \sigma_2 \tau \\ \text{for some } \sigma_2 \in \Sigma_{n-1}/\Sigma_{n-k} \text{ and } \tau \in \Sigma_{n-k} \end{array} \right)$$

$$= \frac{v_\mu(t)}{v_{n-k}(t)} \cdot \underbrace{\frac{1}{v_\mu(t)} \sum_{\gamma \in \Sigma_{n-1}} \gamma \left(x_2^{\mu_1} \cdots x_k^{\mu_{k-1}} \prod_{2<i<j\leq n} \frac{x_i - tx_j}{x_i - x_j} \right)}_{\substack{= P_\mu(x_2, \ldots, x_n; t) \\ \text{(by the first expression in Lemma 7.2.2)}}}$$

$$= \frac{v_\mu(t)}{v_{n-k}(t)} P_\mu(x_2, \ldots, x_n; t).$$

Thus, (4) becomes

$$Q_\lambda(x_1, \ldots, x_n; t)$$
$$= (1-t)^{k-1} \sum_{\sigma_1 \in \Sigma_n / \Sigma_{n-1}} \sigma_1 \left(x_1^{\lambda_1} g_1 \underbrace{\sum_{\sigma_2 \in \Sigma_{n-1}/\Sigma_{n-k}} \sigma_2 \left(x_2^{\lambda_2} \cdots x_k^{\lambda_k} \prod_{i=2}^{k} \prod_{j>i} \frac{x_i - t x_j}{x_i - x_j} \right)}_{= \frac{v_\mu(t)}{v_{n-k}(t)} P_\mu(x_2,\ldots,x_n;t)} \right)$$
$$= \underbrace{(1-t)^{k-1} \frac{v_\mu(t)}{v_{n-k}(t)}}_{\substack{=b_\mu(t) \\ \text{(by (5))}}} \underbrace{\sum_{\sigma_1 \in \Sigma_n/\Sigma_{n-1}} \sigma_1 \left(x_1^{\lambda_1} g_1 P_\mu(x_2, \ldots, x_n; t) \right)}_{= \sum_{i=1}^n x_i^{\lambda_1} g_i P_\mu(x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_n;t)}$$
$$= b_{\mu'}(t) \sum_{i=1}^n x_i^{\lambda_1} g_i \underbrace{P_\mu(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n; t)}_{\substack{= P_\mu^{(i)}(x_1,\ldots,x_n;t) \\ \text{(by the symmetry of } P_\mu \text{ and by} \\ \text{Proposition 7.2.3 (d))}}}$$
$$= b_{\mu'}(t) \sum_{i=1}^n x_i^{\lambda_1} g_i P_\mu^{(i)}(x_1, \ldots, x_n; t) = \sum_{i=1}^n x_i^{\lambda_1} g_i \underbrace{b_{\mu'}(t) P_\mu^{(i)}(x_1, \ldots, x_n; t)}_{\substack{= Q_\mu^{(i)}(x_1,\ldots,x_n;t) \\ \text{(since } Q_\mu(x_1,\ldots,x_n;t) = b_\mu(t) P_\mu(x_1,\ldots,x_n;t))}}$$
$$= \sum_{i=1}^n x_i^{\lambda_1} g_i Q_\mu^{(i)}(x_1, \ldots, x_n; t).$$

This completes the proof of Lemma 8.1.2.

- **page 59, Proposition 8.1.3:** It should be said here that you are working in the ring
$$\Lambda[t]\left[\left[u_1, u_1^{-1}u_2, u_2^{-1}u_3, u_3^{-1}u_4, \ldots\right]\right]$$
(so the power series $\underline{Q}(u_i)$ and $F\left(u_i^{-1}u_j\right)$ are actually well-defined), and that elements of $\Lambda[t]$ are treated as scalars (so that a coefficient can include t's and x_i's).

- **page 59, proof of Proposition 8.1.3:** Strictly speaking, you need an extra induction base case $\ell(\lambda) = 0$ here.

- **page 59, proof of Proposition 8.1.3:** "and so this follows from Lemma 8.1.1" $\to$ "and so this follows from the definition of $\underline{Q}(u)$".
- **page 59, proof of Proposition 8.1.3:** Before "By our induction hypothesis", add the sentence "We work with finitely many variables $x_1, \ldots, x_n$ from now on.".
- **page 59, proof of Proposition 8.1.3:** In the third display in this proof, and every time from there onwards until the end of page 59, you're using the letter "λ_1" as a summation index; but it already has a different meaning (it stands for the first entry of λ). The summation index should be renamed to avoid this collision.
- **page 59, proof of Proposition 8.1.3:** The second equality sign in the last display looks suspicious to me: It uses (8.1), but (8.1) holds only for $r > 0$, so we would somehow get an error term from the $(p, m) = (0, 0)$ addend, right? (Here I have renamed your summation index λ_1 as p, so as to avoid the collision mentioned above.) The third equality sign also feels wrong. I understand that

$$\underline{Q}(u_1) \sum_{m \geq 0} f_m u_1^{-m} = \sum_{m \geq 0} \sum_{p \geq -m} u_1^p f_m q_{p+m}(x_1, \ldots, x_n; t).$$

  But $\sum_{p \geq -m}$ is not $\sum_{p \geq 0}$. Also I'm not sure if $\sum_{m \geq 0} f_m u_1^{-m}$ is actually well-defined in the ring $\Lambda[t]\left[\left[u_1, u_1^{-1}u_2, u_2^{-1}u_3, u_3^{-1}u_4, \ldots\right]\right]$; it is easy to get contradictions when computing with power series in u_1 and power series in u_1^{-1} at the same time.
- **page 60:** I don't understand the raising operators. When computing $R_{i_1,j_1} R_{i_2,j_2} q_\alpha$, it may happen that $R_{i_2,j_2}\alpha$ has a negative entry, but $R_{i_1,j_1} R_{i_2,j_2} q_\alpha$ has no negative entries (specifically, this tends to happen when $j_2 = i_1$). In this case, should $R_{i_1,j_1} R_{i_2,j_2} q_\alpha$ be understood as 0 because $R_{i_2,j_2} q_\alpha = 0$, or should it be nonzero? In other words, should I first have the R's act formally on the subscripts and only then evaluate to a symmetric function, or should I apply the R operators one by one? In the first case, how do you get, e.g., the first displayed equation in the proof of Theorem 8.1.6 (i.e., why does $\prod_{i<j}\left(1 + (t-1) R_{ij} + (t^2 - t) R_{ij}^2 + \cdots\right)$ undo the action of $\prod_{i<j} \frac{1 - R_{ij}}{1 - tR_{ij}}$)? In the second case, the R_{ij} don't commute, so how is the product in Corollary 8.1.4 defined? (I have the same issue every time I see any author applying raising operators. I am not sure if raising operators exist at all...)
- **page 61, proof of Theorem 8.1.6:** "with $\mu \geq \lambda$" $\to$ "with $\mu > \lambda$".
- **page 61, Proposition 8.2.1:** "if $\langle u_\lambda, v_\lambda \rangle_t$" $\to$ "if $\langle u_\lambda, v_\mu \rangle_t$".

- **page 63:** In the first display on page 63, the first "$\prod_{i\geq 1}$" sign should be "$\prod_{i=1}^{\ell(\lambda)}$", since any factors with $i > \ell(\lambda)$ are undefined (they involve division by 0). However, the second "$\prod_{i\geq 1}$" sign is more complicated, since the numerators should be multiplied over all $i \geq 1$, whereas the denominators should only be multiplied over $i \in \{1, 2, \ldots, \ell(\lambda)\}$. So I suggest replacing the right hand side by "$\dfrac{\prod_{i\geq 1}\left(m_i(\lambda)! i^{m_i(\lambda)}\right)}{\prod_{i=1}^{\ell(\lambda)}(1-t^{\lambda_i})}$".

- **page 64, proof of Lemma 9.1.2:** After "and that", add "Lemma 2.5.3 yields".

- **page 64, proof of Lemma 9.1.2:** I think it's worth saying a couple words about why the q_r with odd r are algebraically independent, too.

- **page 65, proof of Lemma 9.1.4:** "$\prod_{i\geq 1}\dfrac{1}{1-t^{2i+1}}$" should be "$\prod_{i\geq 0}\dfrac{1}{1-t^{2i+1}}$".

- **page 65, Proposition 9.1.5:** You're using the notion of a "strict partition", so you should define it. (Of course, it just means a partition in DPar, so you may also avoid using the word altogether.)

- **page 65, Proposition 9.1.5:** "the coefficient of q_μ is divisible by $2^{\ell(\lambda)-\ell(\mu)}$" $\to$ "the coefficient of q_λ is divisible by $2^{\ell(\mu)-\ell(\lambda)}$" (note both changes here). Alternatively, you can avoid talking about coefficients, by just saying "each q_μ can be written as a $\mathbf{Z}$-linear combination of $2^{\ell(\mu)-\ell(\lambda)} q_\lambda$ with $\lambda \in$ DPar and $\lambda \geq \mu$".

- **page 65, proof of Proposition 9.1.5:** After "with the required divisibility condition", add "and $\lambda \geq \mu$". (Or, again, you can avoid talking about divisibility and just put the powers of 2 in front of the q_μ's.)

- **page 65, proof of Proposition 9.1.5:** After "such that $\mu_i = \mu_{i+1} = m$", add "> 0".

- **page 65, proof of Proposition 9.1.5:** After "The coefficient of each q_{ν^i} is", add "a multiple of".

- **page 65, proof of Proposition 9.1.5:** "divisiblity" $\to$ "divisibility" (twice).

- **page 65, Proposition 9.1.7:** Shouldn't you somehow hint to the fact that the $\langle\cdot,\cdot\rangle$ form is not the usual bilinear form on Λ but rather the $t = -1$ specialization of $\langle\cdot,\cdot\rangle_t$? Maybe call it $\langle\cdot,\cdot\rangle_{-1}$?

- **page 67:** You say: "So this is a finite group of order $2n!$". This is far from obvious at this point, I believe; the simplest proof(?) comes from the existence of negative representations. (I am currently fighting a similar problem at MathOverflow question #285263.)
- **page 72, §9.5:** Once again, I suggest writing $\langle \cdot, \cdot \rangle_{-1}$ instead of $\langle \cdot, \cdot \rangle$.
- **page 72, Theorem 9.5.1:** A comma too much in "$f, g, \in$".