

Topics on Tournaments

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<https://www.gutenberg.org/ebooks/42833> (version 2013)

Errata

Errata and comments

All but four of the errors below were found by GPT-5.5, but verified by myself.

- **page 1, Section 1:** "(symbolically, $p_i \rightarrow_j$)" should be "(symbolically, $p_i \rightarrow p_j$)". It appears that this is due to a LaTeX bug in the source:

$\backslash\newcommand{\Dom}[1]{\to}$
 should be $\backslash\newcommand{\Dom}{\to}$ (no [1]).

- **page 2, Section 1, Exercise 2:** "the labels of its node" should be "the labels of its nodes".
- **page 2, Section 2:** Again, replace " $p_i \rightarrow_j$ " by " $p_i \rightarrow p_j$ ".
- **page 2, Section 2:** "every node in $T^{(j)}$ dominates every node in $T^{(i)}$ if $1 \leq i \leq j \leq l$ " is only true when $i < j$, not when $i = j$.
- **page 6, Section 3, Exercise 1:** Replace "all the entries" by "all the off-diagonal entries".
- **page 7, Section 4, proof of Theorem 3:** "replacing the arcs \overrightarrow{ab} and \overrightarrow{be} " should be "replacing the arcs \overrightarrow{ab} and \overrightarrow{bc} ".
- **page 11, Section 5, proof of Theorem 5:** "To show that $s(n, k) < n - k + 1$ " should be "To show that $s(n, k) \leq n - k + 1$ ".
- **page 16, Section 6, Theorem 8:** In the formula for μ' , replace " $\binom{n}{8}$ " by " $\binom{n}{2}$ ". (This formula actually gives the exact mean of $c(m, n)$, not just a limit.)
- **page 18, Section 7, proof of Theorem 10:** "and theorem is proved" should be "and the theorem is proved".
- **page 22, Section 7, Exercise 6:** Remove the period after the question mark.
- **page 26, Section 9, proof of Theorem 14, equality (1):** The upper limit of the sum should be n , not m .

- **page 27, Section 9, proof of Theorem 14, explanation after (3):** "a term in the expansion of $|A^T|$ " should be "a term in the expansion of $|\overline{A}^T|$ ".
- **page 27, Section 9, proof of Theorem 14, equality (6):** Replace the " $e_1 \subset e_1$ " under the summation sign by " $e_1 \subset e$ ".
- **page 32, Section 10, proof of Theorem 15:** "If we let $h(n) = \frac{t(n)}{n}$ " should be "If we let $h(n) = \frac{t(n)}{n!}$ ".
- **page 33, Section 10, Exercise 4:** "approximately $\frac{2\pi}{n} \left(\frac{n}{2e}\right)^n$ " should be "exactly $\frac{(n-1)!}{2^n}$ and approximately $\sqrt{\frac{2\pi}{n}} \left(\frac{n}{2e}\right)^n$ ".
- **page 41, Section 13:** "if and only if there exists a path $P(p_i, p_j)$ " should be "if and only if there exists a walk $P(p_i, p_j)$ ", if modern graph-theoretical terminology is to be followed. (Paths nowadays are required to have distinct vertices.) The same change is probably needed throughout this section.
- **page 43, Section 13:** "subtournaments T^i " should be "subtournaments $T^{(i)}$ ".
- **page 44, Section 13, Corollary:** Remove the " $\eta \geq 7$ " case; it is rendered redundant by the " $\eta > 4$ " case.
- **page 45, Section 13, proof of Theorem 23:** "where $T^{(i)}$ is" \rightarrow "where $T^{(1)}$ is".
- **page 56, Section 16, Theorem 24:** Replace " 2^{i-1} " by " 2^{t-1} ".
- **page 56, Section 16, proof of Theorem 24:** Again, replace " 2^{i-1} " by " 2^{t-1} ".
- **page 59, Section 17, proof of Theorem 25:** "then $q_{1,A} \rightarrow q_{j,B}$ " should be "then $q_{i,A} \rightarrow q_{j,B}$ ".
- **page 59, Section 17, proof of Theorem 25:** In the proof of the n -universality of H , replace " $p_i \in A(j)$ " by " $y_i \in A(j)$ ". Likewise, replace " $p_i \notin A(j)$ " by " $y_i \notin A(j)$ ".
- **page 62, Section 18, proof of Lemma 1:** Here you write: "The number N of sets B of b nodes for which there exists some node y_i ($1 \leq i \leq t$) such that y_i dominates every node of B is given by the formula

$$N = \sum_{i=1}^t \binom{s_i}{b}.$$

"

This is imprecise: The formula $N = \sum_{i=1}^t \binom{s_i}{b}$ gives the number of **pairs** (i, B) , where $i \in \{1, 2, \dots, t\}$ and where B is a set of b nodes for which y_i dominates every node of B . Different pairs (i, B) can have the same set B .

- **page 62, Section 18, Lemma 2:** Expressions of the form " u/vw " are inherently ambiguous; thus, " $(n \log n) / (r + 3) 2^{r+11}$ " should be rewritten as " $\frac{n \log n}{(r + 3) 2^{r+11}}$ ".
- **page 64, Section 18, proof of Lemma 4:** In " $e = e_1 + e_2 + \dots + e_i$ ", replace " e_i " by " e_t ".
- **page 66, Section 19, McGarvey's construction:** "the orders $(i, j, 1, 2, \dots, n)$ and $(n, n - 1, \dots, 2, 1, i, j)$ " should be "the orders $(i, j, r_1, r_2, \dots, r_{n-2})$ and $(r_{n-2}, r_{n-3}, \dots, r_1, i, j)$, where r_1, r_2, \dots, r_{n-2} are the elements of $\{1, 2, \dots, n\} \setminus \{i, j\}$ in increasing order".
- **page 72, Section 21, proof of Theorem 29:** The definition of the s'_j would greatly profit from a picture of the Ferrers diagram of the partitions $(s_n, s_{n-1}, \dots, s_1)$ and $(s'_{n-1}, s'_{n-2}, \dots, s'_1)$ (with the horizontal strip between the two being marked).
- **page 73, Section 21, proof of Theorem 29:** "that $s'_1 = s_i$ for s_n values of i " should be "that $s'_i = s_i$ for s_n values of i ".
- **page 73, Section 21, proof of Theorem 29:** The inequality (3) needs explanation.
- **page 74, Section 21, Exercise 2:** Replace " $\sum_k s_i$ " by " $\sum_i s_i$ ".
- **page 75, Section 22, Theorem 30:** The displayed equality must be an inequality:

$$\sum_{i=1}^n \sum_{j=1}^{k_i} s_{ij} \geq \sum_{i=1}^{n-1} \sum_{j=i+1}^n k_i k_j.$$

- **page 75, Section 22, first corollary:** "where $r_1 \leq r_2 \dots \leq r_m$ " needs an extra " \leq " in front of the " \dots ". Furthermore, " $\sum_{j=1}^i (m - c_j)$ " should be " $\sum_{j=1}^l (m - c_j)$ ".

- **page 78, Section 23:** In the formula

$$[t, l]^1 = \begin{cases} 1 & \text{if } t = l, \\ 0 & \text{otherwise,} \end{cases}$$

replace " $t = l$ " by " $t = l$ ".

- **page 79, Section 23:** In (7), replace "if $\frac{1}{2}m < m \leq m$ " by "if $\frac{1}{2}m < n \leq m$ " probably.
- **page 80, Section 23, Theorem 33:** On the right hand side, replace " c_1 " by " c_2 ".
- **page 80, Section 23, proof of Theorem 33:** After "we may suppose that n is even, say $n = 2m$ ", add "(since we can then handle the case of odd n by adding a new vertex that dominates all existing vertices, which shows that $s(n) \leq s(n + 1)$)".
- **page 80, Section 23, proof of Theorem 33:** "the node p_{2m+i-1} " should be "the node p_{2m+1-i} ".
- **page 81, Section 23, proof of Theorem 33:** "integers s_1, s_2, \dots, s_n and l_1, l_2, \dots, l_n " should be "integers s_1, s_2, \dots, s_m and l_1, l_2, \dots, l_m ".
- **page 82, Section 23, proof of Theorem 33:** Replace " $\left(\frac{1}{m^2(m+1)^2} \binom{2m}{m}\right)^2 m^2$ " by " $\frac{1}{m^2} \left(\frac{1}{m+1} \binom{2m}{m}\right)^2$ " (it looks similar, but is a factor of $(m+1)^2$ away).
- **page 87, Section 25, Theorem 35:** It would be better not to speak of score vectors (which is often sorted into nondecreasing order), but rather be explicit about the preservation of vertices:

If T_n and T'_n are two tournaments on n vertices $1, 2, \dots, n$ such that each vertex i has the same score in T_n as it has in T'_n , then T_n can be transformed into T'_n by successively reversing the orientation of appropriate 3-cycles.

In this formulation, it is clear that the result of the transformation will be literally T'_n , not just a tournament isomorphic to T'_n .

I don't know if your proof of Theorem 35 yields this stronger formulation, but the proof I gave in the solution to Exercise 6 from Math 5707 Spring 2017 at UMN does. (Please excuse my awkward writing...)

- **page 95, Section 27, proof of Theorem 37:** "a permutation α of $G(R \circ T)$ " should be "a permutation α in $G(R \circ T)$ " or "an element α of $G(R \circ T)$ ".

- **page 96, Section 27, Exercise 5:** Is " $R \circ T = T \circ R$ " meant as a literal equality or as an isomorphism?
- **page 107, Appendix:** The labels in the table are floating in random places.