The Littlewood-Richardson rule, and related combinatorics

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Errata by Darij Grinberg - I

Page 2, §1.1: "the the action" → "the action".

Page 2, §1.2: Here, you write: "The Schur polynomials of degree d form yet another $\mathbb{Z}$-basis of Λ_n parametrized by $\mathcal{P}_{d,n}$." Methinks the Λ_n here must be replaced by Λ_n^d.

Page 4, §1.2: In "the fact that $\{m_\lambda(n) \mid \lambda \in \mathcal{P}_{d,n}\}$ is a $\mathbb{Z}$-basis of Λ_n", the Λ_n again should be replaced by Λ_n^d.

Page 4, §1.3: You claim that "this representation is not unique in general (although in some cases it is, for instance when the sets of rows and columns meeting D are both initial intervals of $\mathbb{N}$)". This is not literally correct, since the representation is not unique when D is the empty skew diagram, whereas it is clear that the sets of rows and columns meeting the empty skew diagram are both initial intervals of $\mathbb{N}$. However, this is the only exception. In fact, something more general holds: If the set of rows meeting a skew diagram D is an initial interval of $\mathbb{N}$, and the set of columns meeting D contains 0, then the representation of D as a difference of two Young diagrams is unique. (Of course, the same holds with "rows" and "columns" switched. These two cases, however, don't cover all cases where the difference representation is unique; and in general, whether or not the representation is unique is not determined by the sets of rows and of columns meeting D.)

Page 8, §1.5: I think that "another a valid reading order" should be "another valid reading order".

Page 9, §2.1: You haven't defined the relation "$\subseteq$" in "$(\mathcal{P}, \subseteq)$". (Or did you say that you identify Young diagrams with partitions?)

Page 9, §2.2: On the first line of §2.2, remove the word "of" from the formulation "Among the skew shapes λ/μ of with $|\lambda/\mu| = 2$".

Page 10, §2.2: Shortly before Definition 2.2.1, you write:

"It follows that if values $\lambda^{[i,j]}$ are prescribed for indices $[i, j]$ traversing some "lattice path" going from $[k, m]$ to $[l, n]$ (a zig-zag path in which at each step either i or j increases by 1) by any skew standard tableau, then there is a unique way to extend these values to tableau switching family on $I \times J$".

I think, "tableau switching family" should be "a tableau switching family" here.

On a less trivial note, it took me a while to figure out why this extension exists and is unique. A posteriori, it was merely a psychological difficulty: since there are many ways to actually compute the extension of the values to the rectangle, my intuition suggested that there would be some "holonomy" that would need to be proven trivial. Of course, these apparent troubles dissolve when one doesn't try to prove uniqueness and existence at one dash. Maybe it's worth explicitly saying that the proofs of both existence and uniqueness are each just a straightforward induction?

Page 11, §2.2: There is a typo in the paragraph succeeding Theorem 2.2.2: "exactly that of in inward *jeu de taquin* slide" should be "exactly that of an inward *jeu de taquin* slide".

Page 11, §2.2: I think it would add value to the exposition in the paper if the notion of a jeu de taquin slide would be defined (particularly the "inward slide into" and "outward slide into" terminology). While it is possible to define jeu de taquin slides through tableau switching, I don't think you make it very clear how to do that, and you do use "sliding" metaphors further in the text. You kind-of define jeu de taquin slides in your `pictures.pdf` paper, but for some reason you call them "glissements" there...

Page 12, §2.3: In the Young tableau one line above Proposition 2.3.2, why are the entries $1, 2, 3$ called rather than $0, 1, 2$?

Page 13, §2.3: One line above Proposition 2.3.3, "consequence proposition 2.2.5" should be "consequence of proposition 2.2.5".

Page 17, Proposition 3.1.1: Remove one of the words "to" from "associating to w to the sequence".

Page 17: When you say "connected component", it might be useful to point out that these words mean a connected component of the **undirected** graph obtained by forgetting the directions of the edges in the coplactic graph.

Page 21, proof of Corollary 3.3.3: There is a "the the" typo here.

Page 28, §4.1: Typo: "estabishes".
