Algebras, Rings and Modules: Lie Algebras and Hopf Algebras

Michiel Hazewinkel, Nadiya Gubareni, V. V. Kirichenko

[`hazewinkel gubareni kirichenko - algebras rings and modules 3.pdf`]

Darij's list of errata and comments

- **Page 232:** "counably" → "countably".
- **Page 232:** "Frobenious" → "Frobenius".
- **Page 234, (6.1.10):** Replace "Q_{n-2}" by "$Q_{n-2}(Z)$".
- **Page 235:** In the second displayed equality on this page, replace "$\sum_{r+s=m} Z_r \otimes Z_r$" by "$\sum_{r+s=m} Z_r \otimes Z_s$".
- **Page 237, (6.3.9):** Add an equality sign after "$\beta \times_{osh} \gamma$".
- **Page 237:** It would be better to replace "the RHS of (6.3.8) has a contribution 1 for each pair $(\alpha, \beta \otimes \gamma)$ such that α is a column sum of a matrix in $\mathcal{M}_{\beta,\gamma}$" by "the RHS of (6.3.8) has a contribution 1 from each matrix in $\mathcal{M}_{\beta,\gamma}$ having column sum α".
- **Page 238:** On the first line of this page, replace "$[r_1, ...r_m]$" by "$[r_1, ..., r_m]$". Similarly, on the second line of this page, replace "$[s_1, ...s_m]$" by "$[s_1, ..., s_m]$".
- **Page 241, Definition 6.5.3:** Replace "$\alpha'' = [a_i, a_{i+l}, ...a_m]$" by "$\alpha'' = [a_i, a_{i+1}, ..., a_m]$".
- **Page 241, Definition 6.5.3:** Replace "A word is **Lyndon** iff it is" by "A word is **Lyndon** if it is nonempty and". (Otherwise, the empty word would be Lyndon, which would break uniqueness of CFL factorization.)
- **Page 242, Theorem 6.5.5:** The word "nonincreasing" is very confusing here: it suggests (falsely) that each of the $\lambda_1, \lambda_2, ..., \lambda_s$ itself is nonincreasing, whereas it is supposed to mean that the sequence $(\lambda_1, \lambda_2, ..., \lambda_s)$ is nonincreasing.
- **Page 242, proof of Theorem 6.5.5:** Replace "for some prefix β" by "for some nonempty prefix β". (Otherwise, $\lambda_1 <_{\text{lex}} \beta$ would be false.)
- **Page 243, footnote [13]:** You don't prove the claim of this footnote, but you later use it in the proof of Lemma 6.7.15.

- **Page 243, proof of Theorem 6.5.8, Case A:** Replace "The element $a_{i,1}$" by "The element $a_{1,1}$".

- **Page 244, proof of Theorem 6.5.8, Case A2:** I think the condition for this case should be "There exists some $i \in \{1, 2, ..., \min\{n_1, n_2\}\}$ such that $a_{1,i} > a_{2,i}$ and ($a_{1,j} = a_{2,j}$ for all $j = 1, 2, ..., i-1$)" rather than "$[a_{1,1}, ..., a_{1,n_1}] >_{\text{lex}} [a_{2,1}, ..., a_{2,n_2}]$ and $n_1 \leq n_2$". Otherwise, for example, the situation when $\alpha_1 = 132$ and $\alpha_2 = 12$ does not fit in any of the Cases A1, A2 and A3.

- **Page 244, proof of Theorem 6.5.8, Case A3:** Replace "the equal prefixes $[a_{1,1}, ..., , a_{1,n}] = [a_{2,1}, ..., , a_{2,n_2}]$" by "the equal prefixes $[a_{1,1}, ..., a_{1,n_2}] = [a_{2,1}, ..., a_{2,n_2}]$".

- **Page 244, proof of Theorem 6.5.8, Case A3:** In "$[b_1, ..., b_s] >_{\text{lex}} [a_{1,1}, ..., a_{1,n}] >_{\text{lex}} [a_{2,1}, ..., a_{2,n_2}]$", replace "$a_{1,n}$" by "$a_{1,n_1}$". But you might want to remove this relation altogether, since I don't think you ever use $[b_1, ..., b_s] >_{\text{lex}} [a_{2,1}, ..., a_{2,n_2}]$.

- **Page 244, proof of Theorem 6.5.8, Case A3:** Replace "the CFL factorization of $[\beta_1, ..., \beta_s]$" by "the CFL factorization of $[b_1, ..., b_s]$".

- **Page 244, proof of Theorem 6.5.8, Case A3:** I would replace "$\alpha >_{\text{lex}} \beta$" by "$\alpha \geq_{\text{lex}} \beta$" since the latter is easier to prove and just as fine.

- **Page 244, proof of Theorem 6.5.8, Case B:** In the second displayed equation of Case B, replace "i_2" by "i_1".

- **Page 244, proof of Theorem 6.5.8, Case B1:** I would replace "$\alpha >_{\text{lex}} \beta$" by "$\alpha \geq_{\text{lex}} \beta$" since the latter is easier to prove and just as fine.

- **Page 244, proof of Theorem 6.5.8, Case B3:** I would replace "$\alpha >_{\text{lex}} \beta$" by "$\alpha \geq_{\text{lex}} \beta$" since the latter is easier to prove and just as fine.

- **Page 244, proof of Theorem 6.5.8, Case B3:** I have my doubts about this argument. You write that "relabel the latter $a_{1,j}$ with the equal letters $a_{2,j}$, $j = 1, ..., i_1$". But can't it happen that, after the relabelling, the letters of α_2 will be out of order in β'', which means β'' might no longer be a shuffle of $\beta_1, \beta_2, ..., \beta_t, \alpha_2, ..., \alpha_m$? For an example, let $\alpha = 12141213$, so that $m = 2$, $\alpha_1 = 1214$ and $\alpha_2 = 1213$. Let β be the shuffle $a_{2,1}a_{2,2}a_{2,3}a_{1,1}a_{2,4}a_{1,2}a_{1,3}a_{1,4} = 12113214$ of α_1

and α_2. Then, we are in Case B3, with $i_1 = 3$ and $[a_{1,1}, a_{1,2}, a_{1,3}] = [a_{2,1}, a_{2,2}, a_{2,3}] = 121$. But removing this prefix $[a_{2,1}, a_{2,2}, a_{2,3}]$ from β leaves us with $a_{1,1}a_{2,4}a_{1,2}a_{1,3}a_{1,4} = 13214$, which cannot be obtained as a shuffle of $[a_{1,4}]$ with α_2.

- **Page 245, §6.5.17:** Replace "$Shuffle \otimes_Z \mathbf{Z}/(2)$" by "$Shuffle \otimes_{\mathbf{Z}} \mathbf{Z}/(2)$".

- **Page 247, proof of Lemma 6.5.32:** Typo: "the the".

- **Page 248, proof of Construction and Lemma 6.5.33:** I don't see how you conclude that $\beta >_{\text{lex}} \alpha'$. In my opinion, some more arguments are needed here.

- **Page 249:** Remove the opening parenthesis at the beginning of "(from the left **Symm** of (6.6.1) to the right **Symm**" (since no matching closing parenthesis exists).

- **Page 249, footnote [19]:** Replace "h_l" by "h_1".

- **Page 250:** Replace "a word as in (6.7.2)" by "a word as in (6.7.3)".

- **Page 251, between (6.7.7) and (6.7.8):** Replace "Ψ'" by "Ψ^i".

- **Page 251, one line below (6.7.8):** Replace "([**23**], p 49ff), and section 4.9 above." by "([**23**], p 49ff, and section 4.9 above).".

- **Page 252, proof of Lemma 6.7.15:** Replace "$\lambda_n(\alpha)$" by "$\lambda^n(\alpha)$" (throughout the proof, for different values of n).

- **Page 253, proof of Theorem 6.7.5:** Replace "$\beta = [b_1, b_2, ..., b_n]$" by "$\beta = [b_1, b_2, ..., b_m]$" (since the letter n will soon be used for something different). In the same vein, replace "$\alpha = \beta_{red} = \left[g(\beta)^{-1} b_1, g(\beta)^{-1} b_2, ..., g(\beta)^{-1} b_n\right]$" by "$\alpha = \beta_{\text{red}} = \left[g(\beta)^{-1} b_1, g(\beta)^{-1} b_2, ..., g(\beta)^{-1} b_m\right]$".

- **Page 253, proof of Theorem 6.7.5:** It would clarify things if you replace "and formula (6.7.18)" by "and the first equation of (6.7.18) (applied to r_1 instead of n) we have".

- **Page 255:** In the first displayed formula on page 255, I believe the summation index "$\text{wt}(\alpha)$" should be "$\text{wt}(\alpha) = j$".

- **Page 255, Theorem 6.9.4 (ii):** Replace "$\varphi_*(h(t))$" by "$\varphi_*(Z(t))$".

- **Page 256, proof of Theorem 6.9.4:** Remove the period after "**NSymm**".

- **Page 257, (6.9.12):** Replace "$\mathbf{v}_n$ (?**Symm**)" by "$\mathbf{v}_n$ (?$\mathbf{Symm}_k$)".

- **Page 258:** Remove the word "a" in "Now there exist a natural lifts".

- **Page 258, (6.9.15):** Replace "$\mathbf{f}_n^{\mathbf{QSymm}}$" by "$\mathbf{f}_p^{\mathbf{QSymm}}$".

- **Page 258, (6.9.15):** Replace the equality sign by an "$\equiv$" sign.

- **Page 259, (6.9.23):** Replace the "**Z**" by a "**Q**".

- **Page 259, footnote** [26]**:** Here you conjecture that "The corresponding ideals in **NSymm** are most likely the iterated commutator ideals". If by "the corresponding ideals" you mean the orthogonal spaces $(F_i(\mathbf{QSymm}))^\perp$ of **QSymm** with respect to the bilinear form, then I think the conjecture is false. It is true that $\left(\underbrace{F_1(\mathbf{QSymm})}_{=\mathbf{Symm}}\right)^\perp = \mathbf{Symm}^\perp = [\mathbf{NSymm}, \mathbf{NSymm}]$, but it is not true that $(F_2(\mathbf{QSymm}))^\perp = [\mathbf{NSymm}, [\mathbf{NSymm}, \mathbf{NSymm}]]$. For example, $M_{12}^2 = e_1(M_{12}) \cdot e_1(M_{12})$ (I write M_α for the monomial quasisymmetric function you call α) has scalar product 2 with $[H_2, [H_3, H_1]]$, as the following sage code shows:

```
sage:  QSym = QuasiSymmetricFunctions(QQ)
sage:  M = QSym.M()
sage:  NSym = NonCommutativeSymmetricFunctions(QQ)
sage:  S = NSym.S()
sage:  print (M[1,2]**2).duality_pairing(S[2,3,1]-S[2,1,3]-S[3,1,2]+S[1,3,2])
2
```

- **Page 260, Theorem 6.9.27 (i):** Replace "**v**" by "$\mathbf{v}_n$".

- **Page 260, Theorem 6.9.27 (i):** Replace "$[a_1, ...a_m]$" by "$[a_1, ..., a_m]$". Similarly, replace "$[n^{-1}a_1, ...n^{-1}a_m]$" by "$[n^{-1}a_1, ..., n^{-1}a_m]$".

- **Page 260, Theorem 6.9.27 (iv):** Replace the equality sign by an "$\equiv$" sign.