

A New Degree Bound for Vector Invariants of Symmetric Groups

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Errata and addenda by Darij Grinberg

1. Errata

- **page 1704:** In the sentence “An element of $A_R(n, k)_a$ is called homogeneous of multidegree \underline{m} .”, replace \underline{a} by \underline{m} .

- **page 1704:** In the formula

$$\begin{aligned} \nu(F) &= \frac{1}{n}(f(g_1(w_1), g_1(w_2), \dots, g_1(w_k)) + \dots + f(g_n(w_1), g_n(w_2), \dots, g_n(w_k))) \\ &= \frac{1}{n}(g_1 f(w_1, w_2, \dots, w_k) + \dots + g_n f(w_1, w_2, \dots, w_k)) = f, \end{aligned}$$

two closing parentheses (one in the first line, and one in the second line) are missing.

- **page 1704:** Replace “exceedes” by “exceeds”.
- **page 1705:** Replace “Since $(e_i)_{\underline{m}} = 0$ ” by “Since $Pol(e_i)_{\underline{m}} = 0$ ” in the first line of page 1705.
- **page 1705:** Replace the comma in front of “In this case Weyl’s” by a period.
- **page 1705:** In the sentence “[...] and suppose we are given a a cross-section $[U : G : V]$ ” (in the last paragraph of page 1705), there is an “a” too much.
- **page 1705:** In the last paragraph of page 1705, you write “[...] and for each $g \in [U : G : V]$ a cross-section $[UgV : V] \subseteq U$ of V -cosets with $UgV = \bigsqcup_{t \in [UgV : V]} tgV$ ”. This is not completely to the point, in my opinion – in fact the elements of $[UgV : V]$ represent the gV -cosets rather than V -cosets, though (of course) $UgV = \bigsqcup_{t \in [UgV : V]} tgV$ is a decomposition of UgV into V -cosets. It is probably better to define $[UgV : V]$ as a cross-section $[U : U \cap {}^gV]$ of $U \cap {}^gV$ -cosets in U , and then to prove that $UgV = \bigsqcup_{t \in [UgV : V]} tgV$.
- **page 1705:** In the last paragraph of page 1705, you write “ $U = \bigsqcup_{t \in [UgV : V]} tU \cap {}^gV$.” It would be better to have parentheses around the $U \cap {}^gV$ here, because otherwise $tU \cap {}^gV$ is easily misread as $(tU) \cap {}^gV$.

- **page 1706:** In the proof of Proposition 2.1, you write:

$$T_{U \cap ugvV}^G(a(ugvb)) = T_{U \cap ugV}^G((ua)(ugvb)).$$

It took me some time to see how you did this:

$$\underbrace{T_{U \cap ugvV}^G}_{=T_{U \cap ugV}^G} \left(\underbrace{a}_{=ua}(ugvb) \right) = T_{U \cap ugV}^G((ua)(ugvb)).$$

Probably an intermediate step wouldn't hurt.

- **page 1706:** “stabilizer” is misspelt “stablizer”.
- **pages 1706–1707:** You write: “ $a = (a_{ij}) < b = (b_{ij})$ if and only if $d(a) < d(b)$ or $d(a) = d(b)$ and $md(a) < md(b)$ or $md(a) = md(b)$ and there is a row index $i_0 \in \mathbb{N}$ such that the row vectors $a_i = b_i$ for all $i < i_0$ and $a_{i_0} < b_{i_0}$ ”. This sentence is somewhat ambiguous due to the different precedences the “or” and “and” operators could possibly have. For example, does “ \mathcal{A} or \mathcal{B} and \mathcal{C} ” mean “ $(\mathcal{A}$ or $\mathcal{B})$ and \mathcal{C} ” or does it mean “ \mathcal{A} or $(\mathcal{B}$ and $\mathcal{C})$ ”? I think the ambiguities can be resolved if I rewrite your definition of row-lex order as follows: $a = (a_{ij}) < b = (b_{ij})$ if and only if one of the following three statements holds:
 - We have $d(a) < d(b)$.
 - We have $d(a) = d(b)$ and $md(a) < md(b)$.
 - We have $md(a) = md(b)$ and there is a row index $i_0 \in \mathbb{N}$ such that the row vectors $a_i = b_i$ for all $i < i_0$ and $a_{i_0} < b_{i_0}$.
- **page 1707:** You write: “This amounts to the total order of variables x_{ij} given by $x_{11} > x_{21} > \cdots > x_{n1} > x_{12} > x_{22} \cdots > x_{(n-1)n} > x_{nn}$.” This sounds somewhat misleading to me; your order $>$ does *yield* the total order of variables x_{ij} given by $x_{11} > x_{21} > \cdots > x_{n1} > x_{12} > x_{22} > \cdots > x_{(n-1)n} > x_{nn}$, but it is not *determined* by this total order of variables, as it is not a lexicographic order. (Note also that a $>$ sign is missing in “ $x_{11} > x_{21} > \cdots > x_{n1} > x_{12} > x_{22} \cdots > x_{(n-1)n} > x_{nn}$ ”.)
- **page 1707:** In Lemma 3.1, why do you write $:=$ rather than $=$ in “ $a := \max a^G$ and $b := \max b^G$ ”? The elements a and b are already defined by “Let $a, b \in \mathcal{M}$ ”.
- **page 1707:** In the last line of page 1707, replace “ $a \in M$ ” by “ $a \in \mathcal{M}$ ”.
- **page 1709:** In the first line of page 1709, “three distinct entries” is to be understood as “at least three distinct entries”, while “two distinct entries” means “exactly two distinct entries”. This could use some clarification.

- **page 1709:** In the second paragraph of page 1709, you assume that $a_{11} \geq a_{21} \geq \dots a_{s1} > c = a_{(s+1)1} = \dots = a_{t1} > 0 = a_{(t+1)1} = \dots = a_{n1}$. There is a \geq sign missing here (before the a_{s1}). Also, to be maximally finicky, you seem to be using both \dots and \dots in this chain of inequalities.
- **page 1709:** When considering the first case (i. e., the case when the first column has at least three different entries), you write: “and we get $a^+ - a'i^+e^+ = (a'e)^+ - a'^+e^+ < a^+$ ”. The i here should be removed, id est, $a'i^+e^+$ should be replaced by a'^+e^+ .
- **page 1709:** When considering the first case (i. e., the case when the first column has at least three different entries), you write: “ $a^+ \in \mathcal{B}^1 \cap A_{\underline{d}}^G + R(d)$ ”. You never define what \underline{d} means here (though it is pretty clear that \underline{d} means $md(a)$).
- **page 1710:** Shortly before Theorem 4.6, you write: “On \mathcal{M} this action is described by right multiplication with $k \times n$ -permutation matrices”. You mean $k \times k$ here, not $k \times n$.
- **page 1711:** In the proof of Theorem 4.6, you write: “So suppose that a^+ has multidegree (a_1, \dots, a_n) ”. The n here should be a k .
- **page 1712:** In the first line of page 1712, you write: “It is easy to see from 4.5 that [...]”. In my opinion, this is yet easier to see from the main theorem on symmetric functions (which you have already used on page 1711).