Drexel University, Department of Mathematics

Math 235: Mathematical Problem Solving, Fall 2021: Homework 4

Darij Grinberg

October 5, 2021

---

1 Exercise 1

1.1 Problem

Let n be a positive integer. Let u be the number of pairs (j,k) of positive integers satisfying $\frac{1}{j}+\frac{1}{k}=\frac{1}{n}$.

Prove that u is the number of all positive divisors of n^2.

1.2 Solution

...

---

2 Exercise 2

2.1 Problem

Let $n \geq 2$ be an integer. Simplify the product $\prod\limits_{k=2}^{n} \frac{k^3-1}{k^3+1}$.

2.2 Solution

...

---

3 Exercise 3

3.1 Problem

Prove that

$$a\left(a-b\right)\left(a-c\right)+b\left(b-c\right)\left(b-a\right)+c\left(c-a\right)\left(c-b\right)\geq 0$$

for any three nonnegative reals a,b,c.

[**Hint:** Don't look for a factorization! This polynomial does not have a nontrivial factorization.]

3.2 Solution

...

---

4 Exercise 4

4.1 Problem

Let a,b,c be three real numbers such that $\left(a+b+c\right)^3=a^3+b^3+c^3$. Prove that $\left(a+b+c\right)^n=a^n+b^n+c^n$ for each odd positive integer n.

4.2 Solution

...

---

5 Exercise 5

5.1 Problem

Let $n>1$ be an integer. Factor the polynomial

$$\left(1+x+x^2+\cdots+x^n\right)^2-x^n$$

as a product of two non-constant polynomials.

5.2 Solution

...

---

6 Exercise 6

6.1 Problem

Let n be a positive integer. Let $g(x)$ denote the polynomial $-(x^1 + x^2 + \cdots + x^n)$. Let $h(x)$ denote the polynomial

$$\sum_{k=0}^{n} (g(x))^k = (g(x))^0 + (g(x))^1 + (g(x))^2 + \cdots + (g(x))^n .$$

Prove that the coefficients of the powers $x^2, x^3, \ldots, x^n$ in $h(x)$ are 0.

6.2 Solution

...

References