Algebra
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Errata and comments by Darij Grinberg

The page numbers below refer to the printed page numbers in the PDF dated August 31, 2006. This list is not claimed to be exhaustive. Items marked "substantive" change a mathematical assertion or supply a missing hypothesis; other items are local corrections, proof gaps, or clarifications.

Almost all of the errata below were found by GPT-5.6 Sol; I have merely verified them (and, in some cases, improved the wording).

A. Corrections and comments

1. **Page 5, proof of Proposition 1.2.5:** The parenthetical assertion that "if Z has more than one element, then g' is not unique" is false when f is surjective: in that case, g' is forced to be $g \circ f^{-1}$. Replace the assertion by the following:

   "If Z has more than one element and f is not surjective, then g' is not unique: its values on $Y \setminus \operatorname{im} f$ can be changed".

   Likewise, the paragraph preceding Proposition 1.2.5 should say that the factorization is *not always* unique.

2. **Page 8, proof of Corollary 1.4.3:** The proof gives only the forward implication of Corollary 1.4.3. For the converse, if $E(x) = E(y)$, then reflexivity gives $x \in E(x) = E(y)$, and hence $x \in E(x) \cap E(y)$; thus this intersection is nonempty.

3. **Page 10, proof of Proposition 1.4.11:** *Proof gap.* The argument after the commutative diagram does not handle the case where $X = \varnothing$ and Y has one element: then f is not surjective, but there is only one map $Y \to Y$, so $g' = 1_Y$ cannot be altered.

   Here is a uniform argument for the converse direction (i.e., for statement 2 $\Longrightarrow$ statement 1): Let $Z = \{0,1\}$ and let $g : X \to Z$ be the constant-0 map. By statement 2, there is a unique map $g' : Y \to Z$ such that $g' \circ f = g$. If f were not surjective, choose $y_0 \in Y \setminus \operatorname{im} f$ and change the value of g' at y_0. This would give a second such factorization, a contradiction.

4. **Page 19, first sentence of Section 2.2:** Delete one occurrence of "about" in "about about a group."

5. **Page 20, proof of Lemma 2.2.6:** The proof of

$$(g^m)^n = g^{mn}$$

does not justify the assertion that "we may assume $n \geq 0$". Let us do so here: If $n = -k < 0$, then the already-proved first power law gives $g^m \cdot g^{-m} = g^{m+(-m)} = g^0 = e$, so that $(g^m)^{-1} = g^{-m}$, and the positive-n case gives $(g^{-m})^k = g^{-mk} = g^{mn}$ (since $-mk = m\underbrace{(-k)}_{=n} = mn$), so that

$$(g^m)^n = (g^m)^{-k} = \left((g^m)^{-1}\right)^k = \left(g^{-m}\right)^k = g^{mn}.$$

6. **Page 21, proof of Lemma 2.2.7:** After "$(-1)^k m \cdot (-1)^l n = (-1)^{k+l} mn$", add "for $k, l \in \mathbb{Z}$".

7. **Page 21, paragraph after Definition 2.2.10:** Insert "the" in "Cyclic groups are simplest kind of group there is."

8. **Page 22, Definitions 2.2.11:** Replace "$k \geq 1$" by "$k \geq 0$" (and interpret the empty product $g_1^{n_1} \cdots g_0^{n_0}$ as e). Otherwise, $\langle S \rangle$ comes out empty when $S = \varnothing$, but a subgroup of G cannot be empty.

9. **Page 24, Exercise 2.2.18 (11):** The hypothesis

$$aba^{-1} \in \langle b \rangle$$

does not imply that

$$H = \{a^i b^j \mid i, j \in \mathbb{Z}\}$$

is a subgroup. For example, let a and b be the affine transformations of $\mathbb{R}$ given by

$$a(t) = 2t \qquad \text{and} \qquad b(t) = t + 1.$$

Then $aba^{-1} = b^2$, but

$$ba(t) = 2t + 1, \qquad a^i b^j(t) = 2^i t + 2^i j,$$

so ba is not of the form $a^i b^j$.

There are two ways to fix this. One is to replace the hypothesis $aba^{-1} \in \langle b \rangle$ by

$$a\langle b \rangle a^{-1} = \langle b \rangle,$$

i.e., that a normalizes $\langle b \rangle$. Another way is to add the additional hypothesis $a^{-1}ba \in \langle b \rangle$. In either case, the claim of the exercise becomes true.

10. **Page 25, proof of Lemma 2.3.9:** Replace "The inverse of $rn + sm$ is $(-r)m + (-s)n$" by "The inverse of $rm + sn$ is $(-r)m + (-s)n$."

11. **Page 32, proof of Corollary 2.5.13:** Before invoking the order of g, note that g has finite order. Indeed, since $(m, n) = 1$, at least one of m and n is nonzero; a nonzero exponent of g yields a positive exponent by changing its sign if necessary.

12. **Page 34, Exercise 2.5.21 (11):** Require d to be positive. As divisibility was defined for arbitrary integers, a negative divisor d of n is otherwise included, although neither a subgroup of order d nor "exactly d elements" in part (d) makes sense.

13. **Page 37, proof of Proposition 2.7.2:** The polar-coordinate argument does not cover the zero vector, for which the angle ϕ is not defined. Begin by observing that $R_\theta \mathbf{0} = \mathbf{0}$, and then assume that $\mathbf{v} \neq \mathbf{0}$ when choosing ϕ.

14. **Page 38, Proposition 2.7.7:** In "$\theta = 2\pi k/n$ for some integers k and n," require $n > 0$. Otherwise $n = 0$ makes the expression undefined, while a negative n makes the displayed order $n/(k, n)$ inappropriate.

15. **Page 40, Exercises 2.8.5 (5) and (7):** In parts (5) and (7), require $k > 0$, since D_{2k} was defined only for positive k.

16. **Page 42, Exercises 2.9.7 (4) and (7):** In parts (4) and (7), require $k > 0$, since Q_{4k} was defined only for positive k.

17. **Page 45, Exercises 2.10.12 (1) and (2):** *Missing convention.* The order of an element was allowed to be ∞, but the least common multiple of a list containing ∞ has not been defined. The best way to fix this is to explicitly extend the least-common-multiple convention by declaring that the least common multiple of several "numbers" is ∞ when at least one of the "numbers" is ∞.

18. **Pages 49–50, proof of Lemma 3.1.12:** Replace each "$Y - X$" in this proof by "$X - Y$". (There are four of them in total.) Also, on page 49, replace "fixes the elements of of $X - Y$" by "fixes the elements of $X - Y$".

19. **Page 53, Remarks 3.2.7:** *Proof correction.* The displayed inverse

$$(x^{-1}y)^{-1} = y^{-1}x$$

establishes symmetry, not reflexivity. Reflexivity, instead, follows from $x^{-1}x = e \in H$.

Later in the same paragraph, "takes each $x \in H$ to its equivalence class" should read "takes each $x \in G$ to its equivalence class".

20. **Page 63, Exercise 3.3.23 (24) (a):** Both occurrences of $\mathbf{S}_n$ in the parenthetical sentence should be $\mathbf{S}_p$. The displayed p-cycle generates a subgroup of $\mathbf{S}_p$, and the action from Exercise 3.3.23 (23) being restricted here is the $\mathbf{S}_p$-action on G^p.

21. **Page 67, proof of Proposition 3.5.3:** Replace "has the same effect on Y_i is σ does" by "has the same effect on Y_i as σ does".

22. **Page 74, proof of Corollary 3.6.20:** Insert a period after "Let $\sigma \in \mathbf{S}_n$".

23. **Page 75, Exercise 3.6.24 (10):** The assertion "Suppose that $\mathbf{S}_n$ has an element of order k. Show that D_{2k} embeds in $\mathbf{S}_n$" is false for small k. For example, $\mathbf{S}_2$ has an element of order 2, but $\mathrm{D}_4 \cong \mathbb{Z}_2 \times \mathbb{Z}_2$ does not embed in $\mathbf{S}_2$. Add the hypothesis $k \geq 3$. With this hypothesis, one can choose an involution that reverses each cycle of a permutation of order k; this involution conjugates the permutation to its inverse and is not contained in the cyclic subgroup it generates.

24. **Page 77, Corollary 3.7.5:** The statement that the number of conjugates of H "divides $[G : H]$" requires $[G : H]$ to be finite. This finiteness condition appears in the proof but not in the statement.

25. **Page 84, Notation 4.1.6:** Replace "and have $\overline{g} \cdot \overline{g'} = \overline{gg'}$" by "and we have $\overline{g} \cdot \overline{g'} = \overline{gg'}$".

26. **Page 86, Corollary 4.1.11:** This holds only for $n \geq 2$. For $n = 1$, the sign homomorphism is not surjective, $\mathbf{A}_1 = \mathbf{S}_1$, and the formula $|\mathbf{A}_n| = n!/2$ is false.

27. **Page 87, Example 4.1.15:** "Let n and k be positive numbers" should be "Let n and k be positive integers."

28. **Page 87, proof of Lemma 4.1.18:** "and hence KH a subgroup of G" should be "and hence KH is a subgroup of G".

29. **Page 88, Corollary 4.1.21:** The divisibility assertion

$$[K : H \cap K] \mid [G : H]$$

    requires $[G : H]$ to be finite. The proof invokes finite group orders and Lagrange's theorem for the quotient G/H.

30. **Page 90, Definition 4.2.1:** The definition should begin "A *nontrivial* group G is simple if ...". As printed, the trivial group is simple, contradicting Lemma 4.2.2 (and later Corollary 5.2.4).

31. **Page 92, proof of Lemma 4.2.11:** Replace "Suppose the first conditions holds" by "Suppose the first condition holds".

32. **Page 101, Proposition 4.4.11:** Replace "Let G be a finite group" by "Let G be a finite *abelian* group." For a nonabelian group, the elements whose orders divide a specified integer need not form a subgroup. The proof itself uses additive notation, Lemma 4.4.3, and commutativity.

33. **Page 101, proof of Proposition 4.4.12:** The last sentence only shows that p divides $o(x)$; it does not yet produce an element of order p. To obtain the latter, we write $o(x) = pq$. Then qx has order p, which completes the induction.

34. **Page 102, proof of Lemma 4.4.17:** The claim that "$y = x - tg$ has order k" could use some more explanation. Namely,

$$ky = k(x - tg) = \underbrace{kx}_{=sg} - \underbrace{kt}_{=s} g = sg - sg = 0$$

shows that $o(y)$ divides k. But k also divides $o(y)$, since the image $\overline{y} = \overline{x}$ in $G/\langle g \rangle$ has order k. These two divisibilities give $o(y) = k$.

35. **Page 103, proof of Proposition 4.4.18:** The induction omits the trivial p-group, whose identity has order $1 = p^0$. Begin with $|G| = 1$, where G is the empty direct product, and then treat $|G| = p$ as the next case.

36. **Page 104, proof of Proposition 4.4.22:** After quotienting by the subgroups of order p, the proof writes factors $\mathbb{Z}_{p^{r_i - 1}}$. If $r_i = 1$, this is the trivial factor $\mathbb{Z}_{p^0} = \mathbb{Z}_1$, whereas the induction hypothesis was stated using positive exponents. Delete the trivial factors before applying induction, and use the already established equality $k = l$ to keep track of how many factors disappeared.

37. **Page 105, Lemma 4.5.1:** Replace "$n > 1$" by "$n > 0$". (There is no need to discriminate against 1 here.)

38. **Page 105, Definition 4.5.2:** Replace "$n > 1$" by "$n > 0$". (The case $n = 1$ is used in Corollary 4.5.5.)

39. **Page 105, Lemma 4.5.3:** Again, replace "$n > 1$" by "$n > 0$".

40. **Page 106, Lemma 4.5.6:** Lemma 4.5.6 should assume that G is finite; otherwise the phrase "the primes dividing the order of G" and the asserted primary decomposition are not available in the stated form.

41. **Page 110, Proposition 4.5.18:** *Missing hypothesis.* Require

$$1 \leq s \leq r;$$

when $p = 2$, retain also the hypothesis $s > 1$. This is needed to ensure that p^{r-s} is an integer.

42. **Page 116, proof of Proposition 4.6.11:** Lowercase "There" in "Thus, by Proposition 4.6.9, There is a homomorphism".

43. **Page 118, Exercise 4.6.13 (6):** *Missing hypothesis.* The formulas use orders as finite integers, although the book permits $o(x) = \infty$. In particular, an expression such as $x^{o(f(x))}$ need not make sense. Add that the elements under discussion have finite order (for example, assume the groups are finite). Under this hypothesis, if $m \mid o(x)$, then

$$o(x) = m \cdot o(x^m),$$

and the subsequent deductions are valid.

44. **Page 123, Example 4.7.8:** The conjugation homomorphism has codomain $\mathrm{Aut}(\mathbf{A}_n)$:

$$\Gamma\colon \mathbf{S}_n \longrightarrow \mathrm{Aut}(\mathbf{A}_n),$$

not $\Gamma\colon \mathbf{S}_n \to \mathbf{A}_n$.

45. **Page 126, Remarks 4.7.19:** In "The extensions $f\colon G \to K$ and $f'\colon G \to K$ are equivalent ...", replace "$f'\colon G \to K$" by "$f'\colon G' \to K$".

46. **Page 130, Exercise 4.7.27 (3):** Replace "$H \subset \mathbf{A}_n$ of Problem 1" by "$H \subset \mathbf{A}_4$ of Problem 1".

47. **Page 130, Exercise 4.7.27 (8):** Replace "Show that f is isomorphic to either D_{16} or Q_{16}" by "Show that G is isomorphic to either D_{16} or Q_{16}".

48. **Page 133, Exercise 4.7.27 (19) (d):** The expression

$$\frac{m^k - 1}{m - 1}$$

is undefined when the action is trivial, i.e., when $m = 1$. Use throughout the polynomial expression

$$1 + m + \cdots + m^{k-1},$$

which equals k when $m = 1$.

49. **Page 133, Exercise 4.7.27 (19) (g):** The assertion "If n is prime, then $H = \mathbf{Z}_n$" is false without a condition on the action. Let

$$S = 1 + m + \cdots + m^{k-1}.$$

Part (d) shows that H consists of those b^i for which $n \mid iS$. Thus, when n is prime, $H = \mathbf{Z}_n$ if and only if $n \mid S$; otherwise H is trivial. A nontrivial action ($m \not\equiv 1 \pmod{n}$) implies $n \mid S$, but a trivial action need not. For example, with $n = 5$, $k = 3$, and the trivial action,

$$G = \mathbf{Z}_5 \times \mathbf{Z}_3 \cong \mathbf{Z}_{15}.$$

The subgroup $\mathbf{Z}_5$ is characteristic, as required by the standing hypothesis of the exercise, but the subgroup H from part (d) is trivial.

50. **Page 135, chapter introduction:** In the assertion that the full converse of Lagrange's Theorem holds for nilpotent groups, replace "A nilpotent group G" by "A *finite* nilpotent group G". The result proved later as Corollary 5.7.10 has this finiteness hypothesis.

51. **Page 136, proof of Corollary 5.1.2:** Replace "A second application of Cauchy's Theorem given an element $a \in G$ of order 2" by "A second application of Cauchy's Theorem gives an element $a \in G$ of order 2".

52. **Page 137, proof of Proposition 5.2.2:** In both appearances of "$[G : \mathcal{C}_G(g)]$", replace "g" by "x".

53. **Page 139, proof of Proposition 5.2.7:** Replace

$$\alpha(a^j) = \overline{m^j}$$

by

$$\alpha(f(a^j)) = \overline{m^j}$$

(or say directly that conjugation by a^j acts as the j-th power of conjugation by a), since α has domain $\mathbb{Z}_p$, not G. Also, the result that $K(p, 2)$ is cyclic of order p generated by $1 + p$ is Corollary 4.5.19, not Corollary 7.3.15.

54. **Page 140, proof of Lemma 5.2.9:** The representative range in

$$\mathbf{Z}_p \times \mathbf{Z}_p = \{a^i b^j \mid 0 \leq i, j \leq p\}$$

should be $0 \leq i, j < p$ (equivalently, $0 \leq i, j \leq p - 1$).

55. **Page 145, proof of Corollary 5.3.18:** Replace the citation to Corollary 5.3.16 by a citation to Corollary 5.3.13. Once the 2-Sylow subgroup H has been shown to be normal, Corollary 5.3.13 gives

$$G \cong H \rtimes_\alpha \mathbf{Z}_3.$$

Corollary 5.3.16 cannot be applied here: neither choice of the two primes in $12 = 2^2 \cdot 3$ satisfies its hypothesis $q^s < p$.

56. **Page 149, proof of Lemma 5.4.5:** Taken literally, the printed argument proves only

$$f([G, G]) \subseteq [G, G].$$

Apply the same argument to the automorphism f^{-1} to obtain

$$f^{-1}([G, G]) \subseteq [G, G],$$

or equivalently $[G, G] \subseteq f([G, G])$. These two inclusions give the equality required for $[G, G]$ to be characteristic.

57. **Page 151, first paragraph of Section 5.5:** *Wording.* The reference to Galois theory should say that there is no *general* formula by radicals solving every polynomial of degree at least 5. There are, of course, many individual polynomials of such degrees that are solvable by radicals.

58. **Page 151, proof of Lemma 5.5.4:** Replace "the result follows by induction on k" by "the result follows by induction on i", in agreement with the index quantified in the statement.

59. **Page 152, paragraph before Definition 5.5.8:** In the statement that G_{i+1} need not be normal in G, replace "if $i > 1$" by "if $i \geq 1$". For $i = 1$, the subgroup G_2 is required to be normal in G_1, but need not be normal in $G = G_0$.

60. **Page 152, proof of Proposition 5.5.9:** In the forward direction, replace "the quotient group G_i/G_{i+1}" by "the quotient group $G^{(i)}/G^{(i+1)}$". The displayed subnormal series at this point is the derived series $G^{(k)} \lhd \cdots \lhd G^{(0)}$, not the separately indexed series $G_k \lhd \cdots \lhd G_0$ used in the converse.

61. **Page 153, paragraph after the Hall Converse:** Replace "every order n dividing G" by "every order n dividing $|G|$".

62. **Page 154, proof of Lemma 5.6.5:** In "let H_p be the p-torsion subgroup of G," replace G by H. The next sentence correctly treats H_p as a characteristic subgroup of H.

63. **Pages 154–155, proof of Theorem 5.6.2:** *Missing boundary cases.* At the beginning of the induction, handle $n = 1$ and $k = 1$ separately: for $n = 1$, the desired Hall subgroup is e; for $k = 1$, it is G. This also supplies the case $|G| = 1$.

    The subsequent second branch implicitly requires $n, k > 1$. Without this reduction, the inference $k = p^r$ from $k \mid p^r$ and $(n, k) = 1$ fails when $k = 1$, while when $n = 1$ the quotient G/P may be trivial and therefore has no nontrivial minimal normal subgroup $\overline{Q}/P$ to choose.

64. **Page 155, proof of Theorem 5.6.2:** Remove the comma after "Then Lemma 5.6.5".

65. **Page 159, paragraph before Corollary 5.8.2:** The map

$$\iota_n \colon \mathrm{Gl}_n(A) \longrightarrow \mathrm{Aut}(A^n)$$

    is an injective *group* homomorphism, not an injective ring homomorphism; neither displayed object is a ring in this context.

66. **Page 160, Proposition 5.8.4:** It is worth reminding the reader that the groups A^0 and $\mathrm{Gl}_0(A)$ are trivial. Without this convention, the asserted right-hand side would not be defined when $n = 1$.

67. **Page 160, proof of Proposition 5.8.4:** *Proof error over general rings.* The conclusion is correct, but the argument using linear dependence ("as otherwise the first $n-1$ rows of the full matrix would be linearly dependent") is false in this generality: Already a one-row matrix (2) over $\mathbb{Z}$ is noninvertible without its row being linearly dependent. Instead, the easiest way to argue that $\begin{pmatrix} M & 0 \\ X & 1 \end{pmatrix} \in \mathrm{Gl}_n(A)$ implies $M \in \mathrm{Gl}_{n-1}(A)$ is using the determinantal identity

$$\det \begin{pmatrix} M & 0 \\ X & 1 \end{pmatrix} = \det M.$$

68. **Page 165, Definition 6.1.6:** Replace the composition rule

$$(f \times g) \circ (f' \times g') = (f \circ f') \times (g \circ g')$$

by

$$(f' \times g') \circ (f \times g) = (f' \circ f) \times (g' \circ g).$$

69. **Page 166, Exercise 6.1.10 (2):** *Missing exception.* Not every injection in $\mathfrak{Sets}$ has a left inverse. If $X = \varnothing$ and $Y \neq \varnothing$, the injection $X \to Y$ has no left inverse because there is no function $Y \to \varnothing$. Add the hypothesis $X \neq \varnothing$ (or separately exclude this case).

    The assertion concerning surjections is the Axiom of Choice adopted in Chapter 1.

70. **Page 169, Exercise 6.2.6 (10) (b):** Replace "$\iota_Y(X) = X \times Y$" by "$\iota_Y(X) = (X, Y)$".

71. **Page 171, Exercise 6.3.2 (6):** *Substantive false assertion.* The empty set has a unique G-action, and there is exactly one G-map from it to every G-set. Hence the empty G-set is an initial object in $G\text{–}\mathfrak{sets}$ for every group G, including every nontrivial group. The requested assertion is therefore false and should be deleted (or replaced by the assertion just stated).

72. **Page 181, proof of Proposition 6.4.5:** Insert the missing right parenthesis in the equality

$$h((g(a), -f(a)) = f'g(a) - g'f(a) = 0.$$

    Thus, the left-hand side should be $h((g(a), -f(a)))$.

73. **Page 183, first paragraph of Section 6.5:** Binary products and coproducts give products and coproducts of any *nonempty* finite family by induction. The empty cases require a terminal object and an initial object, respectively, and do not follow merely from the existence of binary products or coproducts.

74. **Page 186, proof of Proposition 6.5.10:** Replace "$f\colon \bigoplus_{i\in I} \longrightarrow B$" by "$f\colon \bigoplus_{i\in I} A_i \longrightarrow B$".

75. **Page 188, proof of Lemma 6.6.3:** Replace "with A and object of C" by "with A an object of C".

76. **Page 188, paragraph before Proposition 6.6.4:** The parenthetical statement that one-point sets are the only sets for which free groups are abelian overlooks the empty set: the free group and the free abelian group on $\varnothing$ are both trivial. Replace "one-point sets" by "sets of cardinality at most one".

77. **Page 188, Proposition 6.6.5 and the following paragraph:** The free-object structure maps were denoted by η_X and η_Y, not by ι_X and ι_Y. Accordingly, replace "$F(f) \circ \iota_X = \iota_Y \circ f$" by "$F(f) \circ \eta_X = \eta_Y \circ f$". In the following paragraph, likewise replace "its image under ι_X" by "its image under η_X".

78. **Page 189, Exercise 6.6.6 (5):** Replace "G is generated" by "A is generated".

79. **Page 190, Definition 6.7.1:** In "induced by $\{x_1, \ldots, x_n\} \subseteq G$", replace "$n$" by "$k$".

80. **Page 192, Proposition 6.8.3:** In the displayed description

$$G = \{(g_i \mid i \geq 0) \mid h_i(g_i) = g_{i-1} \text{ for all } i \geq 0\},$$

replace the final "$i \geq 0$" by "$i \geq 1$". The morphism h_0 and the term g_{-1} are not defined.

81. **Page 194, Exercise 6.8.8 (7):** Insert "is" in "the image of ι_i the subgroup", so that it reads "the image of ι_i is the subgroup".

82. **Page 196, Definition 6.9.3 and Proposition 6.9.5:** *Nonstandard terminology.* The definition of a skeleton given here is highly nonstandard. The standard definition would additionally require that D have no two distinct isomorphic objects. Thus, for example, any category C is a skeleton of itself according to Definition 6.9.3, but not according to the standard definition unless it has no two distinct isomorphic objects.

83. **Page 204, Definitions 7.1.10:** "for some integer n" should be "for some positive integer n" or "for some nonnegative integer n".

84. **Page 211, paragraph before Exercises 7.1.36:** The sentence "$\mathbb{C}$ and $\mathbb{H}$ are the only division rings that are algebras over $\mathbb{R}$ in such a way that the induced real vector space structure is finite dimensional" omits $\mathbb{R}$ itself. Frobenius' theorem says that the three finite-dimensional associative real division algebras are

$$\mathbb{R}, \qquad \mathbb{C}, \qquad \mathbb{H}.$$

85. **Page 218, Definitions 7.2.16:** Replace "$k \geq 1$" by "$k \geq 0$" three times in this definition. (The sums should be allowed to be empty, since ideals must contain 0 even when S is empty.)

86. **Page 219, Definitions 7.2.21:** Again, replace "$k \geq 1$" by "$k \geq 0$" three times.

87. **Page 223, proof of Proposition 7.3.3:** "And $\overline{g}_b(1) = \iota(1) = 1$" should be "And $\overline{g}_b(1) = g(1) = 1$".

88. **Page 226, Exercises 7.3.16 (6):** The assertion that the units of $A[X]$ are precisely the constant units is false for a general ring. For instance, $1 + 2X \in (\mathbf{Z}/4)[X]$ is a unit (in fact, $(1+2X)^2 = 1$ in $(\mathbf{Z}/4)[X]$) but not constant.

    What is true is that if A is commutative, then the units of $A[X]$ are precisely those polynomials whose constant term is a unit and whose all remaining coefficients are nilpotent. See, e.g., `https://math.stackexchange.com/questions/19132/characterizing-units-in-polynomial-rings` for a proof of this fact. Thus, if A is an integral domain (or, more generally, if the only nilpotent element of A is 0), then the claim of the exercise does hold.

89. **Page 227, Exercise 7.3.16 (20):** Require $n > 1$ here (otherwise, the denominator $\zeta_n^k - 1$ will be zero).

90. **Page 236, Definition 7.4.6:** This does not define the degree of the zero polynomial. The standard convention is to define the degree of the zero polynomial to be $-\infty$ (which is not an actual number but a symbol that is understood to behave according to the rules $\min\{-\infty, k\} = -\infty$ and $(-\infty) + k = -\infty$ for all $k \in \mathbf{Z} \cup \{-\infty\}$, so that the standard properties of degrees still hold for the zero polynomial).

91. **Page 237, proof of Proposition 7.4.7:** The claim that "h must have total degree less than or equal to that of f" is slightly underexplained. The reason why it holds is the following: Consider the polynomial rings $A[X_1, \ldots, X_n]$ and $A[X_1, \ldots, X_{n-1}]$ as graded A-algebras with their usual grading by total degree (i.e., each variable X_i is homogeneous of degree 1). Their subrings $A[X_1, \ldots, X_n]^{\mathbf{S}_n}$ and $A[X_1, \ldots, X_{n-1}]^{\mathbf{S}_{n-1}}$ inherit this grading from them. Consider the polynomial rings $A[Y_1, \ldots, Y_n]$ and $A[Y_1, \ldots, Y_{n-1}]$ as graded A-algebras as well, but with the grading now defined by letting each indeterminate Y_i be homogeneous of degree i. This is called "weighted degree". Thus, $A[Y_1, \ldots, Y_{n-1}]$ is a graded subalgebra of $A[Y_1, \ldots, Y_n]$. Moreover, the A-algebra homomorphisms $\alpha_n : A[Y_1, \ldots, Y_n] \to A[X_1, \ldots, X_n]^{\mathbf{S}_n}$ and $\alpha_{n-1} : A[Y_1, \ldots, Y_{n-1}] \to A[X_1, \ldots, X_{n-1}]^{\mathbf{S}_{n-1}}$ are graded (since $s_i(X_1, \ldots, X_n)$ and $s_i(X_1, \ldots, X_{n-1})$ are homogeneous of the same degree – namely, i – as the variable Y_i).

Now, the equality

$$\pi(f) = g(s_1(X_1,\ldots,X_{n-1}),\ldots,s_{n-1}(X_1,\ldots,X_{n-1})) = \alpha_{n-1}(g)$$

shows that the total degree of $\pi(f)$ equals the weighted degree of g (since the homomorphism α_{n-1} is graded). Hence, the weighted degree of g is the total degree of $\pi(f)$, which is clearly less than or equal to that of f. Since the homomorphism $\alpha_n : A[Y_1,\ldots,Y_n] \to A[X_1,\ldots,X_n]^{\mathbf{S}_n}$ is also graded, we thus conclude that the total degree of $\alpha_n(g)$ is less than or equal to that of f. But let us now recall that

$$h = h(X_1,\ldots,X_n) = \underbrace{f(X_1,\ldots,X_n)}_{=f} - \underbrace{g(s_1(X_1,\ldots,X_n),\ldots,s_{n-1}(X_1,\ldots,X_n))}_{=\alpha_n(g)}$$
$$= f - \alpha_n(g).$$

Hence, h is a difference of two polynomials whose total degree is less than or equal to that of f. Therefore, h itself must also have total degree less than or equal to that of f.

92. **Page 238, Definition 7.4.12:** This definition (which is arguably somewhat nonstandard) is only well-defined if a_0 is a unit of A (since a_0 appears in denominators).

93. **Page 239, Exercise 7.4.14 (3):** Again, a must be assumed to be a unit of A (in order for the fraction to be well-defined).

94. **Page 239, monoid rings:** The canonical map $M \to A[M]$ is injective if the ring A is nontrivial, but is constant if A is trivial. Thus, strictly speaking, it cannot be regarded as an inclusion (as you do in Lemma 7.5.3), and you cannot really *identify* each $m \in M$ with the corresponding element of $A[M]$ (as you do in Definitions 7.5.1). This is a very pedantic point, since its injectivity is not actually used in practice and the identification only serves as a convenient abuse of notation. (One almost never derives the equality of two elements of M from the equality of the respective elements of $A[M]$; and when one does, one usually has the option of choosing $A = \mathbb{Z}$ or $A = \mathbb{Q}$ or $A = \mathbb{C}$.)

95. **Page 242, Exercises 7.5.10 (3) and (4):** These exercises require A to be nontrivial, since the zero ring has no zero-divisors.

96. **Page 247:** "every commutative ring has a maximal ideal" should be "every nontrivial commutative ring has a maximal ideal". The zero ring has no proper ideals.

97. **Page 248, proof of Corollary 7.6.14:** The union argument applies to a nonempty chain (i.e., to the case $S \neq \varnothing$). For the empty chain, use any element of the already nonempty poset X, for example the original ideal $\mathfrak{a}$, as an upper bound.

98. **Page 254, Lemma 7.7.19:** "a two-sided ideal of M" should be "of A".

99. **Page 254, Lemma 7.7.21:** "$1 \leq i \leq n$" should be "$1 \leq i \leq k$".

100. **Page 255, Definitions 7.7.25:** Again, replace "$k \geq 1$" by "$k \geq 0$".

101. **Page 256, Exercise 7.7.27 (3):** "Corollary 7.7.26" $\to$ "Proposition 7.7.26".

102. **Page 256, Exercise 7.7.27 (9):** "Let m be" $\to$ "Let M be".

103. **Page 256, Exercise 7.7.27 (10):** the displayed matrix should be labelled x, not a.

104. **Page 258, proof of Lemma 7.7.30:** "from A^n to N" should be "from A^n to M".

105. **Page 259, footnote [12]:** Strictly speaking, "In the case of a basis, the elements $x_1, \ldots, x_n$ must all be distinct" is only true if A is nontrivial. But as a justification for an abuse of notation, this is fine, and the case of a trivial ring A is trivial anyway.

106. **Page 261, Lemma 7.7.38:** This is only true if A is nontrivial.

107. **Page 261, Lemma 7.7.40:** "Thus, $m_1, \ldots, m_k$ is a basis" $\to$ "Thus, $m_1, \ldots, m_n$ is a basis".

108. **Page 262, Corollary 7.7.41:** "some integer n" $\to$ "some nonnegative integer n".

109. **Page 262, after Definition 7.7.44:** "Thus, $M/g\left(M'\right)$" should be "Thus, $M/f\left(M'\right)$".

110. **Page 265, proof of Proposition 7.7.51:** Replace "$\pi\left(m_i\right)$" by "$g\left(m_i\right)$" in the first sentence of the proof.

111. **Page 266, Exercise 7.7.52 (3):** This requires $x \neq 0$.

112. **Page 271, proof of Lemma 7.8.7:** Replace "$g\left(N_k\right) \subset f\left(N_{k+l}\right)$" by "$g\left(N_k\right) \subset g\left(N_{k+l}\right)$".

113. **Page 272, proof of the Hilbert Basis Theorem (Theorem 7.8.14):** Strictly speaking, the set $\mathfrak{b}$ is not an ideal, since it does not contain 0. Moreover, if f and g are two polynomials of equal degrees $m = n$ whose leading coefficients add to zero, then the leading coefficient of $f + X^{m-n}g = f + g$ is not $a + b = 0$ but something else.

    Both of these issues can be fixed by redefining $\mathfrak{b}$ as follows:

    $$\mathfrak{b} = \{0\} \cup \{\text{leading coefficients of all nonzero polynomials in } \mathfrak{a}\}.$$

    In the second paragraph of the proof, then, we need to assume that all x_i are nonzero (since a zero generator is redundant).

114. **Page 275, Theorem 7.9.8:** *Missing hypothesis.* Require A to be a nonzero commutative ring. For the zero ring, $A^m = A^n = 0$ for every m, n, so the conclusion $m = n$ is false. The proof's choice of a maximal ideal also requires $A \neq 0$.

115. **Page 276, proof of Proposition 7.9.11:** Replace "$1 \leq i \leq k$" by "$1 \leq i \leq l$".

116. **Page 279:** On the first two lines of this page, replace both "n"s by "r"s.

117. **Page 284, Exercise 7.10.23 (3):** Replace "$\Phi_{n,n}$" by "$\Phi_{n+k,n+k}$".

118. **Page 284, Definition 7.11.1:** The set S should be required to be nonempty here. Otherwise, $S^{-1}A$ is an empty set and thus cannot be a ring.

119. **Page 290, proof of Proposition 7.11.16:** Replace "Thus, $S^{-1}f$ is onto" by "Thus, $\ker\left(S^{-1}g\right) \subseteq \operatorname{im}\left(S^{-1}f\right)$".

120. **Page 306, Exercise 8.1.37 (14) (d):** The element z should be required to be a nonzero nonunit.

121. **Page 307, Exercise 8.1.37 (16) (e):** This is false when $m = n$. For example, the element $2 + 2\zeta_3$ is prime in $\mathbf{Z}[\zeta_3]$ (since it equals $-2\zeta_3^2$ and thus is associate to 2), but $\varphi(2 + 2\zeta_3) = (2 + 2\zeta_3)\overline{(2 + 2\zeta_3)} = 4$ is not prime in $\mathbf{Z}$.

    This mistake might be limited to the $m = n$ case, but it needs to be fixed nevertheless.

122. **Page 315, proof of Lemma 8.3.4:** Replace "$A[X_1, \ldots, X_{n+1}]$" by "$K[X_1, \ldots, X_{n+1}]$".

123. **Page 316, proof of Lemma 8.3.8:** Replace "whose total degree is k less than that of f" by "whose total degree $\deg f_k$ satisfies $\deg f_k \leq \deg f - k < \deg f$".

124. **Page 316, proof of Proposition 8.3.9:** "Lemma 8.3.5" should be "Corollary 8.3.5".

125. **Pages 318–319, Lemmas 8.4.6 and 8.4.8 and proof of Proposition 8.4.9:** Nonzero constant polynomials do not have roots. Every occurrence of "every polynomial has a root" should say "every polynomial of positive degree has a root". In particular, the f_i in Lemma 8.4.6 must have positive degree.

126. **Page 320, proof of Gauss's Lemma 8.5.4:** Replace "in $\mathbf{Q}(X)$" by "in $\mathbf{Q}[X]$".

127. **Page 321, Lemma 8.5.6:** After "Let $f(X) \in \mathbf{Q}[X]$" add "be nonzero".

128. **Page 325, Exercise 8.6.8 (2):** Require $\mathbf{k}$ to be a perfect field of characteristic $p > 0$. As written, p is undefined, and "perfect field" also includes characteristic-zero fields.

129. **Page 333, proof of Proposition 8.9.6:** The zero module needs a separate base case: if $M = 0$, take $n = 0$. The claim that a one-generated module $M = Am$ has $\mathrm{Ann}\,(m) = 0$ fails when $M = 0$.

130. **Page 333, proof of Proposition 8.9.6:** After "Let $m_1, \ldots, m_n$ be a generating set for M.", add "We WLOG assume that all m_i are nonzero.". This is used when you apply Lemma 8.9.5 to $m = m_1$.

131. **Page 333, proof of Proposition 8.9.6:** Replace "A/Am_1" by "M/Am_1".

132. **Page 334, proof of Proposition 8.9.6:** The claim "Since M is torsion-free, $q \neq 0$" is unnecessary, and (strictly speaking) fails when $m = 0$.

    Later, "generate m" should read "generate M".

133. **Page 335, Lemma 8.9.8:** There is an exceptional case when $a = x = 0$. Then $c = 0$, and the equation $a = bc$ does not determine b; choosing $b = 0$, for example, gives the wrong annihilator. Either assume $a \neq 0$, or add: "If $a = x = 0$, take $b = 1$".

134. **Page 335, Lemma 8.9.11:** Require $a_1, a_2, \ldots, a_k \neq 0$, in order for the weights to be defined.

135. **Page 336, proof of Corollary 8.9.13:** The indices are inconsistent: The "k" in "$a = p_1^{r_1} \cdots p_k^{r_k}$" is not the same number as the "k" in Corollary 8.9.13. Also an "n" and an "l" appear without definition in the last paragraph of the proof.

136. **Page 339, proof of Theorem 8.9.17:** After writing $a_i = pa_i'$, some a_i' may be units, so the displayed decomposition

$$A/\left(a_1'\right) \oplus \cdots \oplus A/\left(a_k'\right)$$

    may contain zero summands $A/(1)$. The same holds for the decomposition

$$A/\left(b_1'\right) \oplus \cdots \oplus A/\left(b_k'\right).$$

    These zero summands must be deleted before invoking the induction hypothesis, since Theorem 8.9.17 requires a_k and b_l to be non-units. Since $k = l$ has already been proved, the number of deleted zero summands can then be recovered.

137. **Page 340, Theorem 8.9.20:** The summands should be required to be nonzero. Otherwise one can add arbitrary zero cyclic modules, which are p-torsion for every p, and the decomposition is not literally unique.

138. **Page 340, Exercise 8.9.21 (1):** Replace "no element of $\mathbf{Z}$ annihilates all of $\mathbf{Q}/\mathbf{Z}$" by "no nonzero element…" Of course, 0 annihilates every module.

139. **Page 344, proof of Proposition 9.1.8:** "since since" $\to$ "since".

140. **Page 344, proof of Lemma 9.1.9:** Replace "for $n, m \geq 0$" by "for $n, m \geq 1$". Otherwise $n + m - 1$ can be negative.

141. **Page 345, Exercise 9.1.17 (2):** This needs an additional requirement that A is not a field. Indeed, a field is a P.I.D with Jacobson radical 0 but only one prime ideal.

142. **Page 347, Examples 9.2.6 (3):** Replace "a nonzero ideal (a)" by "a nonzero proper ideal (a)". Primary ideals must be proper by definition.

143. **Page 351, Lemma 9.3.1:** "where A Noetherian" $\to$ "where A is Noetherian".

144. **Page 352, proof of Corollary 9.3.3:** In the final two sentences, take α to be an arbitrary *nonzero* element of $A/\mathfrak{p}$. Then $K[\alpha]$ is a field, so α is invertible. It follows that every *nonzero* element of $A/\mathfrak{p}$ is invertible and hence that $A/\mathfrak{p}$ is a field. (As printed, the assertion that α is invertible fails for $\alpha = 0$.)

145. **Page 353, Definitions 9.3.7:** Replace "$\mathfrak{m}_{a_1,\ldots,a_n} \in \mathrm{Max}\,(A)$" by "$\mathfrak{m}_{a_1,\ldots,a_n} \in \mathrm{Max}\,(K[X_1, \ldots, X_n])$".

146. **Page 355, paragraph after proof of Lemma 9.3.12:** Merely as topological spaces, neither $\mathrm{Spec}(A)$ nor $\mathrm{Max}(A)$ is a scheme. The spectrum $\mathrm{Spec}(A)$ equipped with its structure sheaf is the affine scheme associated to A; $\mathrm{Max}(A)$ is generally not a scheme. The sentence should refer here simply to spectra and their Zariski closed subsets, reserving "affine scheme" for the locally ringed space

$$(\mathrm{Spec}(A), \mathcal{O}_{\mathrm{Spec}(A)}).$$

147. **Page 356, Corollary 9.3.16:** Require A to be commutative; otherwise $\mathrm{Spec}\,(A)$ has not been defined in the book's sense.

148. **Page 356, Examples 9.3.17 (1):** Require the PID A not to be a field. For a field, $\mathrm{Spec}\,(A) = \mathrm{Max}\,(A) = \{(0)\}$, so the claimed description of the proper closed subsets fails.

149. **Page 356, Remarks 9.3.18:** For functions $f_i \colon \mathbf{R}^m \to \mathbf{R}$, the definition of $f \colon \mathbf{R}^m \to \mathbf{R}^k$ should say "for $x \in \mathbf{R}^m$", not "for $x \in \mathbf{R}^n$". Later, if $f \colon \mathbf{C}^n \to \mathbf{C}^k$ has component functions f_i, the index range is $i = 1, \ldots, k$, not $i = 1, \ldots, n$.

150. **Page 356, Remarks 9.3.18, second paragraph:** The statement "Every smooth manifold is diffeomorphic ... to a manifold obtained by the above procedure" is false. For instance, non-orientable manifolds cannot be obtained in this way. A safe replacement is: "Every smooth manifold is *locally* of this form, and many embedded manifolds arise globally as regular level sets".

151. **Page 360, proof of Corollary 9.3.31:** "representing f_i for $i = 1, \ldots, n$" should be "representing f_i for $i = 1, \ldots, m$".

152. **Page 361:** "whose objects and are" $\to$ "whose objects are".

153. **Page 365, Exercise 9.3.39 (5):** To be maximally unambiguous, replace "the upper half plane in $\mathbf{C}$" by "the closed upper half plane in $\mathbf{C}$". Indeed, the maximal ideals $(X - a)$ for $a \in \mathbf{R}$ correspond to points on the real axis. Under this identification, the map

$$\mathrm{Max}(\mathbf{C}[X]) \longrightarrow \mathrm{Max}(\mathbf{R}[X])$$

identifies each $z \in \mathbf{C}$ with its complex conjugate.

154. **Page 366, Proposition 9.4.3:** Replace "$\overline{f} : M \otimes_A N$" by "$\overline{f} : M \otimes_A N \to G$".

155. **Page 367, Lemma 9.4.6:** Replace "left and right" by "right and left" (both times).

156. **Page 368, Proposition 9.4.7:** Replace "$\iota : M \times N$" by "$\iota : M \times N \to M \otimes_A N$".

157. **Page 369, Proposition 9.4.11:** "B-bilinear in M" should be "B-linear in M".

158. **Page 376, proof of Corollary 9.4.26:** "every maximal ideal of M" should be "every maximal ideal $\mathfrak{m}$ of A".

159. **Page 381, paragraph after proof of Proposition 9.5.14:** The claim that "the commutative ring A maps to the field $A/\mathfrak{m}$ if $\mathfrak{m}$ is a maximal ideal of A" holds only if the ring A is nonzero (i.e., nontrivial).

160. **Page 381, Proposition 9.5.15:** Replace "commutative ring" by "nonzero commutative ring". Over a zero ring A, all modules are isomorphic, so the existence of a surjective homomorphism $A^m \to A^n$ does not imply anything about m and n.

161. **Page 382, Lemma 9.5.17:** Add "nonzero" before "commutative ring", since a zero ring has no prime ideals.

162. **Page 383, Theorem 9.5.20:** *Missing hypothesis.* Require A to be a nonzero commutative ring. Over the zero ring there is an injection

$$A^m = 0 \longrightarrow 0 = A^n$$

for arbitrary m, n, so $m \leq n$ need not hold. The proof's reduction to $A/\mathfrak{p}$ also presupposes the existence of a prime ideal.

163. **Page 383, Exercise 9.5.21 (5):** "a left A-module M" should be "a left A-module N".

164. **Page 384, Proposition 9.6.1:** The "only if" part of the assertion "$B \otimes_A C$ is commutative if and only if both B and C are commutative" is false, because the canonical maps

$$B \longrightarrow B \otimes_A C, \qquad C \longrightarrow B \otimes_A C$$

need not be injective. For example, take

$$A = \mathbb{Z}, \qquad B = M_2(\mathbf{Z}/2\mathbf{Z}), \qquad C = \mathbf{Z}/3\mathbf{Z}.$$

Then B is noncommutative but $B \otimes_{\mathbb{Z}} C = 0$, which is commutative.

The forward implication (i.e., "only if" part) is valid if both canonical maps are injective. Without additional hypotheses, retain only the "if" part of the proposition (i.e., the implication that commutativity of B and C implies commutativity of $B \otimes_A C$). (Fortunately, this is the one implication that is commonly used.)

165. **Page 388, proof of Proposition 9.7.5:** The displayed equation

$$\psi : \mathrm{Hom}_A\left(M_1 \otimes_B M_2, M_3\right) \to \mathrm{Hom}_A\left(M_1, \mathrm{Hom}_A\left(M_2, M_3\right)\right)$$

should be

$$\psi : \mathrm{Hom}_A\left(M_1 \otimes_B M_2, M_3\right) \to \mathrm{Hom}_B\left(M_1, \mathrm{Hom}_A\left(M_2, M_3\right)\right)$$

instead.

166. **Page 391, paragraph after proof of Proposition 9.7.12:** The claim that "Infinitely generated free modules are not reflexive" is false as written. Replace it by "Infinitely generated free modules are not always reflexive". For instance, the infinitely generated free $\mathbb{Z}$-module $\mathbb{Z}^{(\mathbb{N})}$ is reflexive by Specker's theorem.

167. **Page 397, proof of Proposition 9.8.16:** Require P to be projective in the first sentence.

168. **Page 408, Examples 9.10.8 (2):** "gives an isomorph of the following" is garbled; probably "gives an example of the following".

169. **Page 408, paragraph after Lemma 9.10.10:** Replace "the elements $m_1 \otimes m_2 - m_1 \otimes m_1$" by "the elements $m_1 \otimes m_2 - m_2 \otimes m_1$".

170. **Page 418, Proposition 10.1.9 (3):** Replace

$$\mathrm{tr}(f \oplus g) = \mathrm{tr}(f) + \mathrm{tr}(g)$$

by

$$\mathrm{tr}(f_1 \oplus f_2) = \mathrm{tr}(f_1) + \mathrm{tr}(f_2).$$

The maps introduced in the hypothesis are f_1 and f_2, not f and g.

171. **Page 419, proof of Proposition 10.2.3:** In the first displayed calculation, replace $\Delta(x_1, \ldots, x_j)$ by $\Delta(x_1, \ldots, x_n)$.

172. **Page 423, Remarks 10.2.14:** Near the bottom of the page, "the natural map from the tensor algebra $T_A(M)$" should be "the natural map from the tensor algebra $T_A(N)$".

173. **Page 424, proof of Proposition 10.3.1:** The phrase "the nonzero summands above are in one-to-one correspondence with elements of $\mathbf{S}_n$" is not to be taken fully literally: even a summand indexed by a permutation can vanish. Replace "the nonzero summands" by "the summands not forced to vanish by the preceding argument".

174. **Page 426, Proposition 10.3.9:** In the displayed short exact sequence of multiplicatively written groups, replace the two endpoint 0's by 1's:

$$1 \longrightarrow \mathrm{Sl}_n(A) \xrightarrow{\subset} \mathrm{Gl}_n(A) \xrightarrow{\det} A^\times \longrightarrow 1$$

(since the groups are multiplicative).

175. **Page 426, paragraph after Definition 10.3.10:** "a choice of basis for n" should be "a choice of basis for N".

176. **Page 428, Exercise 10.3.12 (10) (d):** Replace "of degree $n-1$" by "of degree at most $n-1$". For example, if all the y_i are zero, the unique interpolating polynomial subject to the corrected bound is the zero polynomial, which does not have degree $n-1$.

177. **Page 430, paragraph preceding Lemma 10.4.5:** Replace "$XI_n - (M' \oplus M'')$ is the block sum of $XI_n - M'$ and $XI_n - M''$" by "$XI_{n+k} - (M' \oplus M'')$ is the block sum of $XI_n - M'$ and $XI_k - M''$".

178. **Page 432, Exercise 10.4.11 (4):** "Let $\mathfrak{a}$ and $\mathfrak{b}$ ideals" should be "Let $\mathfrak{a}$ and $\mathfrak{b}$ be ideals".

179. **Page 433, Lemma 10.5.2:** Insert "a" in "$a \in K$ is root of the characteristic polynomial", so that it reads "$a \in K$ is a root of the characteristic polynomial".

180. **Page 436, second paragraph:** Replace "every torsion module over $K[X]$" by "every finitely generated torsion module over $K[X]$". (The module under discussion is indeed finitely generated, since its underlying K-vector space is finite-dimensional.)

181. **Page 437, proof of Proposition 10.6.7:** In the first sentence and the following display, replace "$XI_n - C(f)$" by "$XI_m - C(f)$". (The companion matrix $C(f)$ is $m \times m$, not $n \times n$.)

182. **Page 440, Exercise 10.6.14 (7):** Require f_1 and f_2 to be *distinct* monic irreducible polynomials.

183. **Page 441, Definition 10.7.2:** "Jordan block with eigenvalue m" should be "Jordan block with eigenvalue a".

184. **Page 442, proof of Lemma 10.7.6:** In "$\mathrm{ch}_M(X) = \mathrm{ch}_{C((X-a)^k)}(X) = (X-a)^k$", replace both occurrences of "k" by "m". The matrix $M = J(a,m)$ is similar to $C((X-a)^m)$ (by Lemma 10.7.3) and has characteristic polynomial $(X-a)^m$.

185. **Page 442, proof of Lemma 10.7.6:** Insert "is" in "if $b \neq a$, then $bI_m - M$ invertible".

186. **Page 443, Exercise 10.7.8 (4):** Insert "is" in "such that $\rho(a)$ not diagonalizable if $a \neq 0$".

187. **Page 446, proof of Proposition 10.8.9:** "the first $k-1$ entries of the i-th row are empty" should be "the first $k-1$ entries of the i-th row are zero".

188. **Page 446, proof of Proposition 10.8.9:** The final sentence, "The result now follows by induction on n", names the wrong induction variable. Replace it by, for example, "Continuing this induction through $k = n$ proves the result".

189. **Page 446, Exercise 10.8.13 (2):** Insert "has" before "order 3".

190. **Page 448, Lemma 10.9.6:** As on page 426, replace the endpoint 0's in this short exact sequence of multiplicatively written groups by 1's:

$$1 \longrightarrow \mathrm{Sl}(A) \xrightarrow{\subset} \mathrm{Gl}(A) \xrightarrow{\det} A^\times \longrightarrow 1.$$

191. **Page 450, sixth paragraph:** "the subfields of the algebraic closure of L" should be "the subfields of L".

192. **Page 454, Example 11.2.2 (3):** "The complex numbers **C** is a splitting field" is a somewhat ungrammatical shorthand for "The field **C** of complex numbers is a splitting field".

193. **Page 456, proof of Proposition 11.2.9, last paragraph:** "the minimal polynomials of the α_i over **K**" should be "the minimal polynomials of the α_i over **k**".

194. **Page 457, Corollary 11.2.11:** Require the collection $\{\mathbf{K}_i \mid i \in I\}$ to be nonempty. For the empty collection, the intersection inside the ambient field **K** is **K**, which has not been assumed normal over **k**.

195. **Page 457, Proposition 11.2.13:** Insert "of" in "the collection of all roots in $\mathbf{K}$ the polynomials $f_1, \ldots, f_n$".

196. **Page 457, Corollary 11.2.14:** Insert "be" after "let $\mathbf{K}_1$".

197. **Page 458, proof of Proposition 11.3.1:** The formal derivative of $X^{q^n} - X$ is -1, not 1. The conclusion that it has no repeated roots is unchanged.

198. **Page 459, Exercise 11.3.5 (1):** Require $n > 1$. As written, the smallest answer is trivially 2: in $\mathbf{Z}_2$, take $\alpha = 0$. Even requiring $\alpha \neq 0$ would only change the answer to 3, since $1 \in \mathbf{Z}_3$ generates the prime field but is not a generator of $\mathbf{Z}_3^\times$. With the intended requirement $n > 1$, the smallest answer is 9: an element of order 4 in $\mathbf{F}_9^\times$ generates $\mathbf{F}_9$ over $\mathbf{F}_3$ but is not primitive in the multiplicative group of order 8.

199. **Page 460, paragraph before Proposition 11.4.2:** "a better ideal" should be "a better idea".

200. **Page 463, proof of Proposition 11.4.12:** Near the end of the proof, "for $i = 1, \ldots, n$" should be "for $i = 1, \ldots, k$", in accordance with $\mathbf{K} = \mathbf{k}(\alpha_1, \ldots, \alpha_k)$.

201. **Page 464, proof of Proposition 11.4.14:** Replace both occurrences of "$\mathbf{k}(\alpha_1, \ldots, \alpha_n)$" by "$\mathbf{k}(\alpha_0, \ldots, \alpha_{n-1})$". (Recall that $\alpha_n = 1$; the printed field omits the possibly nontrivial coefficient α_0.)

202. **Page 464, proof of Corollary 11.4.15:** "a finite extension $\mathbf{k}(\alpha_{i_1}, \ldots, \alpha_{i_k})$ of $\mathbf{K}$" should end with "of $\mathbf{k}$".

203. **Page 464, Corollary 11.4.16:** Replace "any collection of separable elements of $\mathbf{k}[X]$" by "any collection of separable polynomials in $\mathbf{k}[X]$".

204. **Page 465, Exercise 11.4.20 (2):** *Missing finiteness hypothesis.* Require $\mathbf{K}$ to be a *finite* extension of $\mathbf{k}$, at least in parts (b) and (c). Otherwise the minimal polynomial in part (b) need not exist, and the separability and inseparability degrees in part (c), as defined in this section, need not be defined. For example, the printed hypotheses allow $\mathbf{K} = \mathbf{k}(t)$.

205. **Page 465, paragraph before Lemma 11.5.2:** "the minimal polynomial of α over $\mathbf{K}$" should be "the minimal polynomial of α over $\mathbf{k}$".

206. **Page 466, Lemmas 11.5.5 and 11.5.7:** In each statement, "an intermediate field between $\mathbf{K}$ and $\mathbf{k}$" should be "an intermediate field between $\mathbf{k}$ and $\mathbf{K}$".

207. **Page 467, proof of Lemma 11.5.10:** Replace "Let $\alpha_1, \ldots, \alpha_m$ be a basis for $\mathbf{K}$ as a vector space over $\mathbf{k}$" by "Let $\alpha_1, \ldots, \alpha_m$ be m elements of $\mathbf{K}$ that are linearly independent over $\mathbf{k}$", since the existence of a finite basis is not a-priori guaranteed by the conditions of the lemma.

208. **Page 470, Remarks 11.5.15:** The reduction of $X^3 + bX^2 + cX + d$ by substituting $X - b/3$ requires $\operatorname{char} \mathbf{k} \neq 3$. Add this hypothesis to the paragraph; in characteristic 3, the printed substitution is not defined (and translation does not in general remove the quadratic term).

209. **Page 471, proof of Theorem 11.5.17:** Before invoking the Fundamental Theorem of Galois Theory, state that $\mathbf{K}/\mathbf{k}$ is finite by Lemma 11.5.10. This supplies a hypothesis needed for the cited subgroup–subfield correspondence.

210. **Page 472, proof of Proposition 11.5.19:** "adjoining the roots of $g(X)$ to $\mathbf{k}$" should be "adjoining the coefficients of $g(X)$ to $\mathbf{k}$".

211. **Page 475, proof of Corollary 11.6.3:** The 2-Sylow subgroup G_2 of $G = \mathrm{Gal}(\mathbf{L}/\mathbf{R})$ acts on $\mathbf{L}$, not necessarily on the intermediate field $\mathbf{K}$. Replace

$$\mathbf{K}_1 = \mathbf{K}^{G_2}$$

by

$$\mathbf{K}_1 = \mathbf{L}^{G_2}.$$

The following statements should then read

$$G_2 = \mathrm{Gal}(\mathbf{L}/\mathbf{K}_1), \qquad [\mathbf{L} : \mathbf{K}_1] = |G_2|, \qquad [\mathbf{K}_1 : \mathbf{R}] = [G : G_2].$$

With these replacements, the argument proves the desired contradiction.

212. **Page 476, Lemma 11.7.2:** "elments" should be "elements".

213. **Page 477, paragraph before Corollary 11.7.4:** "the splitting field of $X^n - 1$ over ζ" should be "the splitting field of $X^n - 1$ over $\mathbf{k}$".

214. **Page 477, Proposition 11.7.7:** "obtained by adjoining a primitive n-th root of unity to $\mathbf{K}$" should end with "to $\mathbf{k}$".

215. **Page 480, proof of Lemma 11.8.1:** In the first sentence, let $\mathbf{L}$ be an algebraic closure of $\mathbf{k}$, not of $\mathbf{K}$; the proof is constructing the splitting field $\mathbf{K}$, which is then set equal to $\mathbf{k}(\alpha, \zeta)$.

216. **Page 481, paragraph after Proposition 11.8.2:** "a principal n-th root of unity" should be "a primitive n-th root of unity".

217. **Page 481, proof of Proposition 11.8.3:** "divides is order" should be "divides its order".

218. **Page 484, Exercise 11.8.6 (11):** Replace "$n = p_1, \ldots, p_k$" by "$n = p_1 \cdots p_k$".

219. **Page 486, proof of Proposition 11.9.2:** In the last sentence, "a nontrivial dependence relation between $f_2(x), \ldots, f_n(x)$" should be "a nontrivial dependence relation between the functions $f_2, \ldots, f_n$".

220. **Page 486, Proposition 11.9.4:** Require G to be finite. Both sums over G in the proof, as well as the application of Corollary 11.9.3, require this hypothesis.

221. **Page 487, proof of Proposition 11.9.5, first sentence:** "any element $a \in \mathbf{K}$" should be "any element $a \in \mathbf{k}$".

    Also, the "by induction" argument in the second paragraph could use some more explanation:

    We argue by induction on n. Put

    $$d = [\mathbf{K} : \mathbf{k}] = |\mathrm{Gal}(\mathbf{K}/\mathbf{k})|.$$

    Since $\mathrm{Gal}(\mathbf{K}/\mathbf{k})$ is cyclic and has exponent n, we have $d \mid n$. If $d < n$, then the inductive hypothesis, applied with d in place of n, shows that $\mathbf{K}$ is the splitting field of $X^d - a$ for some $a \in \mathbf{k}$ (since $\mathrm{Gal}(\mathbf{K}/\mathbf{k})$ is cyclic and thus has exponent $|\mathrm{Gal}(\mathbf{K}/\mathbf{k})| = d$). Writing $n = dk$, we see from the first paragraph that $X^d - a$ and $X^n - a^k$ have the same splitting field over $\mathbf{k}$. Thus, we can WLOG assume that $d = n$.

222. **Page 492, proof of Corollary 11.10.5:** (This was reported by GPT-5.6; I have not thoroughly verified it.)

    Proof gap. The printed induction does not justify that $B_1 \subseteq B$, yet uses this to compare the two indices and to choose $b_{s+1} \in B \setminus B_1$. A shorter argument avoids this issue. Since $B/(\mathbf{k}^\times)^r$ is finite, choose $b_1, \ldots, b_s \in B$ whose residue classes generate it, and let $\mathbf{K}_0$ be the splitting field of

    $$\{X^r - b_i \mid 1 \leq i \leq s\}.$$

    For any $b \in B$, write

    $$b = a^r b_1^{e_1} \cdots b_s^{e_s}$$

    with $a \in \mathbf{k}^\times$ and $e_i \in \mathbf{Z}$. If $\gamma_i^r = b_i$ in $\mathbf{K}_0$, then $a\gamma_1^{e_1} \cdots \gamma_s^{e_s}$ is an r-th root of b. Since $\boldsymbol{\mu}_r \subseteq \mathbf{k}$, all the roots of $X^r - b$ lie in $\mathbf{K}_0$. Thus $\mathbf{K}_0$ is already the splitting field of $\{X^r - b \mid b \in B\}$.

223. **Page 493, first sentence of Section 11.11:** Delete one occurrence of "finding" in "the issue of finding finding roots of polynomials".

224. **Pages 493–494, Example 11.11.1 (Cardano's formula):** *Substantive degenerate case.* The instruction to choose β uniquely so that $3\alpha\beta = -p$, and the later expression involving $p/(3\alpha)$, require $\alpha \neq 0$. When $p = 0$, one of the two choices of the square root can make $\alpha^3 = 0$, so the printed formula is undefined. Handle $p = 0$ separately: the depressed cubic is

    $$X^3 + q,$$

whose roots are the three cube roots of $-q$ (with the evident multiplicity convention when $q = 0$). Alternatively, when $q \neq 0$, choose the sign of the square root so that $\alpha^3 = -q$ and take $\beta = 0$. For $p \neq 0$, the printed linked choice of α and β is valid.

225. **Page 493, Example 11.11.1:** Delete one occurrence of "in" in "As shown in in Problem 5 of Exercises 7.4.14".

226. **Page 494, first paragraph:** Delete the second "that" in "We leave it to the reader to verify that in this degenerate case, that the procedure ...".

227. **Page 497, proof of Theorem 11.11.9:** In the first sentence of the proof, "$\mathbf{Q} \subset \mathbf{K}$" should be "$\mathbf{Q} \subset \mathbf{k}$".

228. **Page 497, Exercise 11.11.10 (1):** *Missing hypothesis.* Require f to be irreducible over $\mathbf{Q}$. Without irreducibility the claim is false: for $p = 5$, the polynomial $(X^3 - X)(X^2 + 1)$ has exactly $3 = p - 2$ real roots, but its splitting field is $\mathbf{Q}(i)$ and its Galois group is not $\mathbf{S}_5$.

229. **Page 499, fifth paragraph:** "the i-th elementary function s_i in the variables $X_1, \ldots, X_1$" should be "the i-th elementary symmetric function S_i in the variables $X_1, \ldots, X_n$".

230. **Page 500, proof of Proposition 11.12.3:** "gives in embedding" should be "gives an embedding".

231. **Page 502, opening paragraph of Section 11.14:** "Write $\mu_\alpha : K \to K$" should be "Write $\mu_\alpha : \mathbf{K} \to \mathbf{K}$". In the following sentence, insert "a" in "Note that μ_α is a transformation of vector spaces".

232. **Page 504, Exercise 11.14.5 (1):** "with respect to any **k**-basis of **k**" should be "with respect to any **k**-basis of **K**".

233. **Page 508, Example 12.1.2 (2):** "Its nonzero ideals are $K \times 0$ and $0 \times L$" should be "Its proper nonzero ideals are $K \times 0$ and $0 \times L$". The full ring $K \times L$ is also a nonzero ideal.

234. **Page 511, paragraph before Theorem 12.1.8:** Capitalize "compact lie group" as "compact Lie group".

235. **Page 514, proof of Proposition 12.2.6:** Delete one occurrence of "and" in "By Lemmas 12.2.3 and and 12.2.5".

236. **Page 514, proof of Corollary 12.2.8:** "Then D is the direct sum of the ideals $\mathfrak{a}_i$" should be "Then $M_n(D)$ is the direct sum of the ideals $\mathfrak{a}_i$".

237. **Page 516, paragraph before Proposition 12.2.14:** "a hands-on argument in instructive" should be "a hands-on argument is instructive".

238. **Page 516, proof of Proposition 12.2.14:** In the sentence beginning "Since $\mathfrak{b}$ is generated…", both occurrences of "$\mathfrak{b}$" should be "$\mathfrak{b}_i$".

239. **Page 517, proof of Proposition 12.2.14:** In the directness argument, replace "$\mathfrak{a}_1\mathfrak{a} = 0$" by "$\mathfrak{a}_i\mathfrak{a} = 0$". Indeed, $\mathfrak{b}_1 + \cdots + \mathfrak{b}_{i-1}$ is a sum of simple submodules of A non-isomorphic to $\mathfrak{a}_i$, so Corollary 12.2.13 gives $\mathfrak{a}_i(\mathfrak{b}_1 + \cdots + \mathfrak{b}_{i-1}) = 0$. Since $\mathfrak{a} \subset \mathfrak{b}_1 + \cdots + \mathfrak{b}_{i-1}$, this entails $\mathfrak{a}_i\mathfrak{a} = 0$. This contradicts the preceding equality $\mathfrak{a}_i\mathfrak{a} = \mathfrak{a}$ when $\mathfrak{a} \neq 0$.

240. **Page 524, proof of Theorem 12.3.10:** The first sentence should cite Corollary 12.2.21 for the equivalence of left and right simplicity. Theorem 12.2.19 gives the equivalence between being simple and being a matrix ring over a division ring (conditions 4 and 5 here).

241. **Pages 526–529, Exercise 12.3.14 (2), Corollaries 12.4.2 and 12.4.4, and Proposition 12.4.8:** *Zero-ring exceptions.* These statements should require A to be nonzero, in keeping with Definitions 12.1.1 and 12.1.4. If A is the zero ring, then $A/\mathfrak{R}(A) = 0$, and every unitary A-module is the zero module and hence projective, so the zero ring has global left homological dimension 0 (and thus at most 1). However, the book's definitions do not call the zero ring Jacobson semisimple, left semisimple, or left hereditary.

242. **Page 526, Exercise 12.3.14 (4):** Insert "isomorphism classes of" before "those $K[G]$-modules". A choice of basis turns a module structure on K^n into a homomorphism $G \to \mathrm{Gl}_n(K)$; changing the basis conjugates the homomorphism. Thus, conjugacy classes correspond to isomorphism classes of n-dimensional modules, not to the individual modules.

243. **Page 528, Lemma 12.4.7:** On the second exact sequence, replace the label "f" on the arrow $Q \to M$ by "g". The proof and its pullback diagram use g for this map.

244. **Page 529, proof of Lemma 12.4.7:** In the second split short exact sequence, replace "$P \times_M A$" by "$P \times_M Q$".

245. **Page 529, proof of Proposition 12.4.8:** In the last short exact sequence, replace "$A\mathfrak{a}$" by "$A/\mathfrak{a}$", thus making it

$$0 \longrightarrow \mathfrak{a} \longrightarrow A \longrightarrow A/\mathfrak{a} \longrightarrow 0.$$

246. **Page 530, proof of Proposition 12.5.1:** "every ideal is a principal" should be "every ideal is principal".

247. **Page 532, proof of Proposition 12.6.6:** After "$a_1b_1 + \cdots + a_kb_k = a$", the range "$i = 1, \ldots, n$" should be "$i = 1, \ldots, k$".

248. **Page 533, proof of Theorem 12.6.8:** In the implication (2)⇒(1), the proof treats only $\mathfrak{a} \neq 0$. The omitted case $\mathfrak{a} = 0$ is easy (just take $\mathfrak{c} = 0$).

249. **Page 533, proof of Theorem 12.6.8:** "every maximal ideal of $\mathfrak{a}$ is invertible" should be "every maximal ideal of A is invertible".

250. **Page 533, proof of Theorem 12.6.8:** In "$\mathfrak{m}_i$ is a factor if the principal ideal (a)", replace "if" by "of".

251. **Page 534, paragraph after Proposition 12.6.10:** Insert "are" in "if $\mathfrak{a}$ and $\mathfrak{b}$ nonzero ideals of a Dedekind domain".

252. **Page 535, proof of Lemma 12.6.14:** The proof silently assumes that the ideals to which Lemma 12.6.4 is applied are nonzero. Add that if either $\mathfrak{a} = \mathfrak{c} = 0$ or $\mathfrak{b} = \mathfrak{d} = 0$, the conclusion is immediate; otherwise the printed proof applies.

253. **Page 536, Proposition 12.6.20:** Require $\mathfrak{a}$ and $\mathfrak{b}$ to be nonzero ideals. For example, if $\mathfrak{a} = 0$, the asserted isomorphism would give $\mathfrak{b} \cong A$ for every ideal $\mathfrak{b}$, which is false for a nonprincipal ideal in a Dedekind domain. The printed proof also uses the invertibility of $\mathfrak{a}$.

254. **Page 537, proof of Theorem 12.6.21:** "an k-dimensional one" should be "a k-dimensional one".

255. **Page 538, Exercise 12.6.23 (2) (a):** Replace "an A-submodule $\mathfrak{i}$ of A" by "an A-submodule $\mathfrak{i}$ of K", and require the element $a \in A$ satisfying $a\mathfrak{i} \subset A$ to be nonzero.

256. **Page 538, Exercise 12.6.23 (2) (c):** The groups in the displayed exact sequence are written multiplicatively, so replace the two endpoint 0's by 1's:

$$1 \longrightarrow A^\times \longrightarrow K^\times \longrightarrow \mathfrak{I}(A) \xrightarrow{p} \mathrm{Cl}(A) \longrightarrow 1.$$

257. **Page 540, proof of Corollary 12.7.6:** "a monic polynomial over B which has c is a root" should be "a monic polynomial over B which has c as a root". Also, all three occurrences of "$b_1, \ldots, b_n$" should be "$b_0, \ldots, b_{k-1}$".

258. **Page 540, Proposition 12.7.7:** "there is an $a \in A$" should be "there is a nonzero $a \in A$".

259. **Page 542, Lemma 12.7.13:** Delete the comma in "for a prime number, p".

260. **Page 543, proof of Lemma 12.7.15:** In the noninjective case, "$g(bt)$ gives an element of A that lies in the kernel of η" should say "$g\,(bt)$ gives an element of B that lies in the kernel of $\eta : B \to S^{-1}B$".

261. **Page 547, proof of Theorem 12.7.30:** The converse argument immediately assumes that A has Krull dimension 1, whereas the theorem allows dimension 0. Add first that if A is a field, then it is a Dedekind domain. Otherwise, a domain of Krull dimension at most 1 that is not a field has Krull dimension exactly 1, and the printed argument applies.