Comments and Corrections to Nicholas A. Loehr, *Combinatorics*, 2nd edition 2018

GPT-5.6 Sol, edited and proofread by Darij Grinberg

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The following are specific comments and corrections found in chapter-by-chapter proofreading passes through Nicholas A. Loehr's *Combinatorics*, 2nd edition 2018. They are intended especially for course use: the emphasis is on points that could trip up an instructor or students.

I (GPT 5.6 Sol) have separated definite errors from places where the text is essentially correct but would benefit from supplementary explanation.

I (Darij) have proofread these errata, removing spurious errors and streamlining the wording in many of the others. I do not agree with all of GPT's priorities regarding exposition and notations, but I consider them to be legitimate opinions (with a few exceptions that I have removed from this list).

Contents

1 Definite corrections

Page 23, after Definition 1.65.
If $P(A) > 0$, the text says that independence of A and D is equivalent to

$$P(D \mid A) = P(A).$$

This should be

$$P(D \mid A) = P(D).$$

Page 29, Example 1.84(c).
In the proof that the map

$$w_i \longmapsto w_i + i - 1$$

takes weakly increasing words to strictly increasing words, replace

$$1 \leq w_1 + 0 < w_2 + 1 < w_3 + 2 \leq \cdots < w_k + k - 1 \leq n + k - 1$$

by

$$1 \leq w_1 + 0 < w_2 + 1 < w_3 + 2 < \cdots < w_k + k - 1 \leq n + k - 1.$$

Page 30, Theorem 1.88 (Multiset Rule).
The statement needs the hypothesis $n > 0$. As written, for an n-letter alphabet it gives

$$\binom{k+n-1}{k, n-1} = \frac{(k+n-1)!}{k!(n-1)!}.$$

For $n = 0$, the right-hand side is not defined, whereas the empty alphabet has one 0-element multiset and no k-element multisets for $k > 0$.

Page 31, Theorem 1.89(b) (Integer Equation Rule).
The theorem begins with $n > 0$ and $k \geq 0$, and then says that the number of positive integer solutions of $y_1 + \cdots + y_n = k$ is $\binom{k-1}{n-1}$. There are two edge-case problems.

First, when $k = 0$, the displayed binomial coefficient has upper index -1, which is not defined by Definition 1.28. Even if the usual definition of binomial coefficients is applied, the equality is wrong (when $k = 0$, the number of positive integer solutions is 0, but $\binom{k-1}{n-1} = \binom{-1}{n-1} = (-1)^{n-1} \neq 0$).

Second, the proof replaces k by $k - n$ in part (a); this is only legitimate when $k \geq n$. This is only a problem for the proof; the claim itself is true in the case $k < n$, and can be easily proved by inspection (it boils down to $0 = 0$).

The cleanest way to repair these issues is to require $k > 0$ in Theorem 1.89(b), and to add a sentence about the $k < n$ case to the proof.

The corresponding summary on page 39 inherits the same missing qualification.

Page 32, Theorem 1.94 (a), (c).
In part (a), the case $n = 0$ makes the second lower index $n - 1 = -1$, so the displayed multinomial coefficient is not defined under the book's conventions. If one adopts the natural extension that a multinomial coefficient is 0 when a lower entry is negative (see "Page 58" below), this gives the correct answer when $n = 0 < k$, but not when $n = k = 0$: in that case there is one distribution, whereas the extended multinomial coefficient is 0.

Part (c) has a separate issue when $k = 0$, since then its binomial coefficient has top argument -1, and binomial coefficients with negative top argument were not defined.

Page 35, Definition 1.100 and Theorem 1.101 (Catalan/Dyck Path formulas).
For $n \geq 0$, the text includes the formula

$$C_n = \binom{2n}{n, n} - \binom{2n}{n+1, n-1}.$$

Under Definition 1.36, the second multinomial coefficient is not defined when $n = 0$, because one lower entry is -1. The reflection-principle proof of Theorem 1.101 has the same issue: it uses the Lattice Path Rule 1.98, which has only been stated for $a, b \geq 0$.

The simplest fix is to handle $n = 0$ separately ($C_0 = 1$, with one empty Dyck path) and then assume $n \geq 1$ for the difference formula and the reflection proof.

Page 38, Chapter 1 Summary.
The Word Rule says

"There are n^k word of length k using an n-letter alphabet."

Replace "word" by "words."

Page 42, Exercise 1-38.
"For all finte sets S and T" should read "for all finite sets S and T."

Page 44, Exercise 1-49.
The exercise begins:

> "A sequence of four cards is dealt from a 52-card deck (order matters)."

But later parts ask for the probability of a straight and a full house, notions defined in the text for five-card poker hands; in particular a full house cannot occur among four cards.

The exercise almost certainly intends

> "A sequence of five cards is dealt from a 52-card deck (order matters)."

This also makes parts (c)–(f) standard five-card conditional-probability questions.

Page 44, Exercise 1-54.
A "bad poker hand" is described as having "five different non-consecutive values," but the example

$$\{2\heartsuit, 3\heartsuit, 4\heartsuit, 5\heartsuit, 7\diamondsuit\}$$

contains several consecutive values. The intended condition is that the five distinct values *do not form a straight* (equivalently, they are not five consecutive values, with the usual ace convention).

Page 46, Exercise 1-100.
Replace "$e_i = \sum_{k=1}^{\infty} \lfloor n/p_i^k \rfloor$" by "$e_i = \sum_{m=1}^{\infty} \lfloor n/p_i^m \rfloor$", since the letter k is already used for the number of prime factors.

Page 48, Exercise 1-130.
The exercise says that an ordered pair (A, B) of events is "randomly" chosen, but it does not specify a probability distribution on the set of ordered pairs of events. Therefore the requested probabilities are not formally determined.

The natural intended model is to choose (A, B) uniformly from

$$\mathcal{P}(S) \times \mathcal{P}(S),$$

equivalently to choose A and B independently and uniformly from the 2^{25} events of S. This should be stated explicitly.

Page 58, derivation of the commutative Multinomial Theorem after Theorem 2.12.
The displayed formula obtained from the noncommutative expansion is written

$$(z_1 + \cdots + z_s)^n = \sum_{w \in \{1,\ldots,s\}^n} z_{w_1} \cdots z_{w_s}.$$

The last factor should be indexed by the length of the word, not by the size of the alphabet:

$$(z_1 + \cdots + z_s)^n = \sum_{w \in \{1,\ldots,s\}^n} z_{w_1} \cdots z_{w_n}.$$

Page 58, remark after Theorem 2.14.
After proving the lattice-path version of Pascal's identity for $a, b > 0$, the text says that the identity also holds when one (but not both) of a, b is zero. Under Definition 1.36, however, a multinomial coefficient is only defined when all lower entries are nonnegative. Thus, for example, when $a = 0$, the displayed right side contains

$$\binom{b-1}{-1, b},$$

which is not defined by the book's conventions.

One can either omit the boundary-case remark, or explicitly extend multinomial coefficients by the convention that they are 0 when any lower entry is negative. The latter remedy is probably the best one, as the same convention is needed to make Exercise 2-13 and Exercise 12-3 valid in the respective boundary cases.

Page 65, after the multiset recursion in 2.22.
The text says that

$$M(n,k) = \binom{k+n-1}{k, n-1}$$

for all $n, k \geq 0$. This is not literally meaningful when $n = 0$, since the right side then has a lower entry -1 (and the factorial form contains $(-1)!$).

The formula should be stated for

$$n > 0, \qquad k \geq 0.$$

The already stated initial conditions handle the empty alphabet separately:

$$M(0,0) = 1, \qquad M(0,k) = 0 \quad (k > 0).$$

Page 76, Theorem 2.38 (recursion for integer partitions).
The theorem is stated for all positive integers n, k:

$$p(n,k) = p(n-1, k-1) + p(n-k, k).$$

But $p(r,k)$ was only defined for nonnegative r. If $k > n$, the term $p(n-k,k)$ therefore lies outside the stated domain.

A clean repair is either to state the recursion only for

$$n \geq k > 0,$$

using $p(n,k) = 0$ for $k > n$ as a boundary condition, or to extend the convention by setting $p(r,k) = 0$ whenever $r < 0$. The proof given in the text works verbatim with either of the two repairs.

Page 76, Proposition 2.40.
The proposition says that $p(n,k)$ equals the number of integer partitions of n whose first part is k, without specifying a range for n, k. The statement should exclude the empty partition. For example, $p(0,0) = 1$, but the empty partition has no first part.

A clean formulation is: for positive integers n, k, the number of partitions of n into k parts equals the number of partitions of n with first part k.

Page 82, Theorem 2.49 (balls in nonempty boxes).
The theorem does not state a range for a, b. Its formula in part (b),

$$\binom{a-1}{b-1},$$

is not defined under the book's conventions when $a = 0$. The theorem should simply assume that a, b are positive integers, or else the zero-ball/zero-box cases should be stated separately.

Page 84, proof of Theorem 2.55.
In constructing a placement in A_1, the text says to use a placement from C to fill "the bottom $n-1$ rows of Δ_n." But

$$\Delta_n = (n-1, n-2, \ldots, 1)$$

has only $n-1$ rows altogether, and after reserving its top row the remaining board Δ_{n-1} has $n-2$ rows.

Thus "the bottom $n-1$ rows" should be

"the remaining bottom $n-2$ rows"

(or simply "the copy of Δ_{n-1} below the top row"). The counting argument itself is otherwise correct.

Page 86, proof of Theorem 2.58.
Replace "the remaining $n-1$ columns of Δ_n" by "the remaining $n-2$ columns of Δ_n" (since Δ_n itself has just $n-1$ columns, and we are removing the leftmost one). Also, replace "in the first row" by "in the first column".

Page 88, combinatorial proof of Theorem 2.59.
The $n = 0$ case cannot be handwaved away with the "polynomials in x" argument. It is trivial, of course, but that should be said before assuming $x, n \in \mathbb{Z}_{>0}$.

It would also make more sense to have the sum in

$$|A| = \sum_{k=1}^{n} S(n,k)x(x-1)\cdots(x-k+1)$$

start at $k = 1$, not at $k = 0$, since you have explicitly mentioned the set A_0 in the same sentence (and since the theorem itself has "$k = 0$" under the summation sign). If you do this, then the $n = 0$ case need not be treated separately (i.e., you need not assume that $n \in \mathbb{Z}_{>0}$).

Page 89, proof of Theorem 2.62.
Replace "in these rows" by "in these columns" (since the whole proof classifies file-rook placements according to the *columns* that contain rooks).

Page 92, Exercise 2.68.
In the sentence

"The general (complex) solution is $x = s(3+4i)^n + t(3-4i)^n$",

replace "x" by "x_n".

Page 94, Chapter 2 Summary.
There are two index/variable slips in the Multinomial Theorems item.

First, the commutative formula begins with variables $z_1, \ldots, z_s$, but its monomial on the right is printed using $x_1, \ldots, x_s$. It should be

$$(z_1 + z_2 + \cdots + z_s)^n = \sum_{n_1+n_2+\cdots+n_s=n} \binom{n}{n_1, n_2, \ldots, n_s} z_1^{n_1} z_2^{n_2} \cdots z_s^{n_s}.$$

Second, in the noncommutative formula the final factor is printed as Z_{w_s}. It should be

$$Z_{w_n},$$

since $w = w_1 \cdots w_n$ has length n.

Page 96, Exercise 2-13.
The multinomial version of Pascal's identity contains terms with $n_i - 1$. Unless all $n_i > 0$, some of these lower entries are negative, and such multinomial coefficients are not defined by Definition 1.36.

Thus the exercise should either assume all $n_i > 0$, or (better) explicitly adopt the convention that a multinomial coefficient is 0 whenever one of its lower entries is negative.

Page 100, Exercise 2-79(f).
No hypotheses are stated on a, b, n, but the summand contains F_{b-k} for $0 \leq k \leq a$. With the book's Fibonacci numbers indexed only by nonnegative integers, the formula is not even defined when $b < a$.

A natural formulation is to assume

$$a, b, n \in \mathbb{Z}_{\geq 0} \qquad \text{and} \qquad b \geq a.$$

Page 100, Exercise 2-81.
The proposed recursion for $C_{n,k}$, the number of Dyck paths of order n ending with exactly k east steps, cannot hold for $k = n$. Indeed,

$$C_{n,n} = 1$$

(the unique path is $N^n E^n$), whereas the displayed right side has the empty range $1 \leq r \leq n-k = 0$ and hence gives 0.

To fix this, either require $1 \leq k < n$, or extend the sum to start at $r = 0$ rather than at $r = 1$. The latter fix is nicer.

The case $k = 0$ is also problematic: here, the multinomial coefficient has a -1 among its lower arguments. If we understand such multinomial coefficients to be 0 (as above), then the formula holds for $n > 0$, but not for $n = 0$.

Page 101, Exercises 2-92 and 2-93.
In each of the two exercises, replace "the set of" by "the number of".

Page 126, Theorem 3.74.
The theorem allows $d_1, \ldots, d_n \geq 0$ and, when

$$d_1 + \cdots + d_n = 2n - 2,$$

states the formula

$$\binom{n-2}{d_1-1,\ldots,d_n-1} = \frac{(n-2)!}{\prod_j (d_j-1)!}.$$

If some $d_j = 0$, however, the right-hand side contains a negative lower multinomial entry (or $(-1)!$), which is not defined by the book's conventions. The proof later says that the formula is zero "by convention," but this convention has not been introduced.

A clean formulation is: if all $d_j \geq 1$ and $\sum_j d_j = 2n-2$, the displayed multinomial gives the number of trees; in every other case the number is 0.

Page 127, Corollary 3.75.
When summing over degree sequences of trees, the displayed condition is $d_i \geq 0$. For $n \geq 2$, it should be

$$d_i \geq 1.$$

This is also required by the summand, which contains $(d_i - 1)!$, and by the corrected formulation of Theorem 3.74.

Pages 129–130, proof of Theorem 3.80.
In the implication "no odd cycle $\Rightarrow$ no odd closed walk," the proof chooses a shortest odd closed walk and says that, if it is not a cycle, then there must be an interior return to the starting vertex:

$$v_0 = v_i = v_s \qquad (0 < i < s).$$

That need not be true: a non-cycle closed walk may repeat an *intermediate* vertex without revisiting v_0 before the end.

The standard repair is to choose any repeated vertex $v_i = v_j$ with

$$0 \leq i < j \leq s, \qquad (i, j) \neq (0, s).$$

This splits the closed walk into two shorter closed walks of lengths $j - i$ and $s - (j - i)$. Since s is odd, one of these two lengths is odd, contradicting the minimality of s. Hence a shortest odd closed walk has no repeated vertex except its initial/final vertex and is therefore an odd cycle.

Page 137, Corollary 3.99.
Replace "graph" by "simple graph", since χ_G was only defined for simple graphs in Definition 3.92.

Page 141, Theorem 3.110.
The rooted spanning-tree deletion–contraction recursion should specify that the chosen edge z entering the root v_0 is a *non-loop* edge. A loop at v_0 is never used as an edge of a rooted spanning tree (the root loop in Definition 3.109 is part of the rooted-tree structure rather than an edge required from G), and collapsing a loop is not covered by the contraction construction used in the proof.

Equivalently, one may assume throughout this discussion that the digraph is loopless, as is done explicitly in Definition 3.112 for the Matrix–Tree Theorem.

Pages 146–147, Eulerian Tour Rule.
In the passage beginning "Now suppose ... is a cycle in T that is not the 1-cycle at v_0", the appeal to the preceding statement tacitly uses the fact that the last departure from a non-root vertex cannot be a loop. This is clear (after traversing such a loop the tour is still at that vertex and must depart from it again) but should be said. In "Let us show that if e, h are two non-loop edges in T", you are tacitly using the fact that the last departure from a non-root vertex cannot be a loop. This is

clear (after traversing such a loop the tour is still at that vertex and must depart from it again) but should be said.

Later, the sentence

"assume that some edge e in T from x to y is not used"

should likewise read

"assume that some non-loop edge e in T from x to y is not used"

(The loop at the root is the artificial loop in the rooted tree and need not be an edge of the original digraph.)

Page 154, Exercise 3-57.
An edge cover need not exist when G has an isolated vertex, so in that case the phrase "let $ec(G)$ be the size of a minimum edge cover" does not define a finite number. The exercise should assume that G has no isolated vertices (as Exercise 3-59 does), or else adopt an explicit convention such as $ec(G) = \infty$ for the case when G does have isolated vertices.

Page 154, Exercise 3-62.
Replace "graph" by "simple graph". (For non-simple graphs, it is easy to construct counterexamples.)

Page 157, Exercise 3-96.
The complete bipartite graph formula

$$\tau(G) = m^{n-1} n^{m-1}$$

needs $m, n \geq 1$. If one of the two partite sets is empty, the displayed expression can contain a negative exponent and the graph is generally disconnected.

Page 157, Exercise 3-98.
The exercise asks for conditions for an undirected graph to have an Eulerian tour, but Definition 3.117 defines "Eulerian tour" only for digraphs. The intended undirected analogue is standard, but it should be stated: an Eulerian tour of a graph is a walk that visits every vertex and uses every edge exactly once (with "closed" defined as before).

Page 163, Example 4.9.
The bounded-composition formula

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} \binom{m - k(b+1) + n - 1}{n - 1}$$

is written without hypotheses on n, m, b. At a minimum one wants $n \geq 1$ and $m, b \geq 0$. More importantly, when $m - k(b+1) < 0$, the intersection being counted is empty, whereas the displayed binomial coefficient can have a negative upper entry and is not defined by the book's conventions.

A clean version is

$$\sum_{0 \leq k \leq \min(n, \lfloor m/(b+1) \rfloor)} (-1)^k \binom{n}{k} \binom{m - k(b+1) + n - 1}{n - 1}.$$

Page 168, Theorem 4.21 (Involution Theorem).
Replace "sgn(X)" by "sgn(x)" on both sides of the equality.

The same typo occurs in the first displayed sum in the proof; subsequent lines use sgn(x) correctly.

Page 169, Theorem 4.24.
The compact formula

$$\sum_{k=0}^{n}(-1)^k\binom{n}{k}^2 = (-1)^{n/2}\binom{n}{n/2}\chi(n \text{ is even})$$

is not literally well-defined when n is odd under the book's binomial-coefficient conventions, since $n/2$ is then not an integer (and $(-1)^{n/2}$ is likewise not the intended real/integer expression). Multiplying by a truth value does not repair an undefined factor. (Also, the period should not be inside the χ parenthesis.)

The statement should be written piecewise:

$$\sum_{k=0}^{n}(-1)^k\binom{n}{k}^2 = \begin{cases} 0, & \text{if } n \text{ is odd,} \\ (-1)^{n/2}\binom{n}{n/2}, & \text{if } n \text{ is even.} \end{cases}$$

Page 171, Lemma 4.27(b).
Part (b) is stated "for all integers $p, j \geq 0$." When $j = 0$, the summation begins with $k = 0$, so its first binomial coefficient is

$$\binom{k-1}{j-1} = \binom{-1}{-1}.$$

Definition 1.28 does define $\binom{n}{r} = 0$ when $n \geq 0$ and $r < 0$, but it does not define binomial coefficients with negative *upper* index. Thus the displayed left side is still undefined at $j = 0$. (For $k > 0$, the terms with lower index -1 are indeed 0 by Definition 1.28.)

The clean repair is to state part (b) for

$$p \geq 0, \qquad j > 0.$$

Then its right side may simply be written $\chi(p \geq j)$.

Page 174, Example 4.30.
The phrase

"first choosing which of the j inputs will be fixed "

should read

"first choosing which j of the n inputs will be fixed."

Page 174, Definition 4.31.
Replace "distint" by "distinct".

Page 177, Example 4.41.
Replace "prime power" by "prime power or 1", unless you count 1 as a prime power.

Page 181, Theorem 4.58.
The theorem on the Möbius function of a product poset should assume that the factor posets are finite. The book has defined the poset Möbius function only for finite posets, and the proof explicitly uses finite matrices Z_i, M_i, Z, M.

Thus the opening should read, for example:

"Let $(X, \leq)$ be the product of finite posets $(X_i, \leq_i)$, $1 \leq i \leq k$."

(One can extend the theorem to locally finite posets, but that theory has not been introduced here.)

Page 182, Theorem 4.60.
For the same reason, the isomorphism-invariance theorem for Möbius functions should explicitly concern finite posets (or else locally finite posets in a more general treatment). As written, $\mu_{(X,\leq)}$ has not been defined for an arbitrary infinite poset.

Page 182, Example 4.61.
Replace "divisibility poset" by "Boolean poset".

Page 186, Exercise 4-48.
The phrase

"Find the probability that none of the events S_i occurs first by using inclusion-exclusion, and then ..."

is a bit of a crash blossom: it can be read as though "occurs first" were an event. The intended wording is

"Find the probability that none of the events S_i occurs, first by using inclusion-exclusion, and then ..."

Page 187, Exercise 4-53.
The Catalan identity is stated without a range for n:

$$C_n = \sum_{1 \leq k \leq (n+1)/2} (-1)^{k-1} C_{n-k} \binom{n+1-k}{k}.$$

It is false at $n = 0$: the sum is empty, while $C_0 = 1$. The exercise should assume

$$n \geq 1.$$

Page 189, Exercise 4-87.
The exercise says: "Consider an $n \times n$ lower-triangular matrix A such that $A(n, k)$ is the number of Dyck paths ending with exactly k east steps." This specifies only the last row of an $n \times n$ matrix. The intended definition is presumably

$$A(i, k) = \#\{\text{Dyck paths of order } i \text{ ending with exactly } k \text{ east steps}\} \qquad (\text{for all } k \text{ and } i),$$

i.e., $A(i, k) = C_{i,k}$ in the notation of Exercise 2-81.

Page 189, Exercise 4-93(c).
Part (c) is false as written. It claims that every interval $[P, Q]$ in the partition lattice X_n is isomorphic to X_k for some k. In general an interval is a *product* of partition lattices, not a single partition lattice.

For a concrete counterexample, let

$$P = \{\{1\}, \{2\}, \{3\}, \{4\}\}, \qquad Q = \{\{1, 2\}, \{3, 4\}\}.$$

Then $[P, Q]$ has four elements and is isomorphic to $X_2 \times X_2$; it is not isomorphic to any X_k (whose first sizes are $1, 2, 5, \ldots$).

The standard corrected statement is this. For each block B of Q, let r_B be the number of blocks of P contained in B. Then

$$[P, Q] \cong \prod_{B \in Q} X_{r_B}.$$

Consequently part (d) gives

$$\mu_{X_n}(P, Q) = \prod_{B \in Q} (-1)^{r_B - 1}(r_B - 1)! \qquad \text{whenever } P \preceq Q.$$

Page 189, Exercise 4-98.
Replace "n is even" by "m is even".

Indeed, the full symmetric sum

$$\sum_{k=-n}^{n} (-1)^k \binom{2n}{n+k} k^m$$

vanishes for $m < 2n$ by a finite-difference argument. When $m > 0$ is even, the k and $-k$ terms are equal and the $k = 0$ term is zero, so the desired positive-half sum $\sum_{k=1}^{n} (-1)^k \binom{2n}{n+k} k^m$ is zero.

On the other hand, requiring n to be even when m is not is insufficient; for instance, if $n = 2$ and $m = 1$, the sum is -2.

Page 189, Exercise 4-100.
Replace "$n \geq 0$" by "$n \geq 1$". Indeed, at $n = 0$ the sum contains the summand $\binom{2n-2k-1}{n-1} = \binom{-1}{-1}$ which is not defined by the book's conventions.

Page 194, Example 5.8(a).
The example takes $b \in \mathbb{C}$ arbitrary and computes the radius of convergence of $\sum_{n \geq 0} b^n z^n$ using $\lim_{n \to \infty} \left| b^n / b^{n+1} \right| = 1/|b|$. This ratio calculation is not defined when $b = 0$. The conclusion is still correct if the zero case is separated: when $b = 0$, the series is just 1, so its radius of convergence is ∞.

Page 194, Example 5.8(d).
The Ratio Test calculation is said to work "for any $z \in \mathbb{C}$," but when $z = 0$ all the terms c_k are zero and c_{k+1}/c_k is undefined. One should perform the ratio calculation for $z \neq 0$ and note separately that convergence at $z = 0$ is immediate.

Page 196, Example 5.11.
When the ordinary Binomial Theorem is recalled, the displayed formula ends with z^m:

$$(1 + z)^m = \sum_{n=0}^{m} \binom{m}{n} z^m.$$

The exponent should be the summation index:

$$(1 + z)^m = \sum_{n=0}^{m} \binom{m}{n} z^n.$$

Page 209, Example 5.29.
The displayed expansion of $(1-4z)^{1/2}$ contains the term

$$-10^4.$$

It should be

$$-10z^4.$$

Thus the beginning of the expansion is

$$(1-4z)^{1/2} = 1 - 2z - 2z^2 - 4z^3 - 10z^4 - 28z^5 - \cdots .$$

Pages 216–217, derivation in §5.13.
The last displayed equation on page 216 contains the expressions $(n-1)!$ and $(n-2)!$ in certain sums that start at $n=0$. But these expressions are undefined for $n=0$ and for $n=1$. This can be fixed by discarding the $n=0$ addend of the sum back before these expressions appear, and also discarding the $n=1$ addend of the second sum at the point where it still has a $\dfrac{(n-1)c(n-1,k)}{(n-1)!}$ factor. Corresponding changes are also needed in the first paragraph of page 217.

Page 220, Theorem 5.46.
The identity

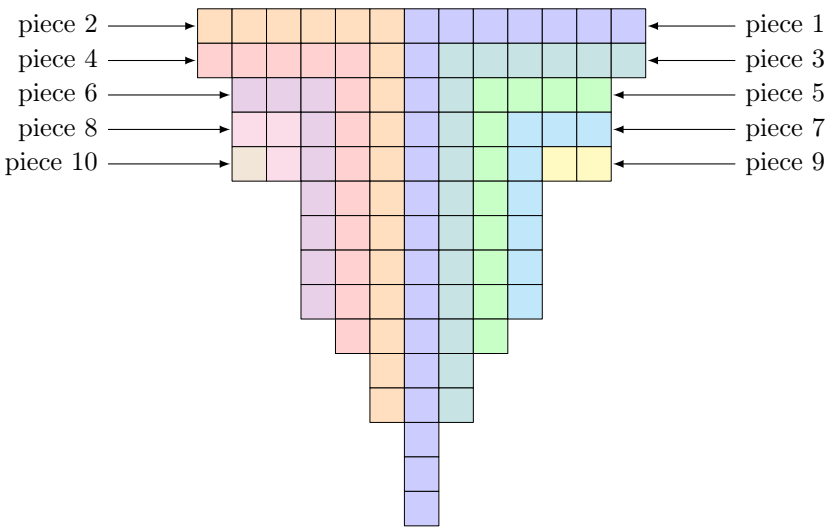
$$\sum_{\mu\in\mathrm{Par}} t^{\ell(\mu)} z^{|\mu|} = \prod_{i=1}^{\infty} \frac{1}{1-tz^i} = \sum_{\mu\in\mathrm{Par}} t^{\mu_1} z^{|\mu|}$$

includes the empty partition. But under the book's definition of an integer partition, the empty partition has no first part, so μ_1 is undefined for this one summand.

The best way to repair this is by explicitly introducing the convention that if μ is a partition of length k, then μ_i means 0 for all $i > k$. (Thus, $\mu_1 = 0$ when μ is the empty partition.) This convention is also introduced in Chapter 9, but it is equally useful outside of symmetric polynomial theory.

Page 222, Figure 5.3.
The intended L-shaped pieces in Sylvester's bijection are not visibly drawn inside the partition diagram in this printing, although the surrounding text and the labels "piece 1," "piece 2," etc. refer to them. A corrected version of the figure is needed if this construction is used in class.

Here is a TikZ replacement figure with the ten L-shaped pieces color-coded:



$$F((13,13,11,11,11,7,7,7,7,5,3,3,1,1,1)) = (21,17,16,13,11,9,8,3,2,1).$$

Page 223, inverse of Sylvester's bijection.
After y_0 has already been defined as the number of cells in column 0, the text says: "For all $i \geq 0$, let y_i be the number of cells in the vertical portion of the $2i$th L-shaped piece." Replace "$i \geq 0$" by "$i \geq 1$" here, since y_0 has already been defined and since there is no 0th L-shaped piece.

Page 230, Exercise 5-10.
The new weight is defined by

$$\mathrm{wt}'(u) = a\ \mathrm{wt}(u) + b \qquad (a, b \in \mathbb{Z}_{\geq 0}).$$

If $a > 0$, the expected answer is $z^b G(z^a)$. But the case $a = 0$ requires separate treatment: if the original weighted set S is infinite, then every object receives the same new weight b, so the weight-b fiber is infinite and (S, wt') is no longer a weighted set in the sense of §5.1.

Thus the exercise should either assume $a > 0$, or explicitly discuss the exceptional case $a = 0$ (which is harmless only when S is finite).

Page 230, Exercise 5-12(b),(d).
Both displayed series begin at $n = 0$, but their $n = 0$ terms are undefined: part (b) contains division by n, and part (d) contains $1/n$. Both sums should begin at $n = 1$.

Page 230, Exercise 5-13(b),(d).
The same endpoint problem occurs here. Part (b) contains $z^n/(n^2 4^n)$, and part (d) contains $(z/n)^n$, while both sums are printed as starting at $n = 0$. They should start at $n = 1$.

Page 236, Exercise 5-109(a).
The statement is false as written. It assumes two sequences of distinct nonzero partitions $\{\mu^i : i \geq 1\}$ and $\{\nu^i : i \geq 1\}$ such that, for every finite $S \subseteq \mathbb{Z}_{>0}$,

$$\sum_{i \in S} |\mu^i| = \sum_{i \in S} |\nu^i|,$$

and asks one to prove that the numbers of partitions avoiding all the μ^i's and all the ν^i's are equal.

For a counterexample, take

$$\mu^1 = (2,2), \qquad \mu^2 = (2,1,1),$$

and

$$\nu^1 = (4), \qquad \nu^2 = (3,1).$$

All four partitions have area 4, so the stated hypothesis holds for every finite subset of $\{1,2\}$. For $n = 6$, however, there are 8 partitions avoiding both μ^1, μ^2, but only 7 partitions avoiding both ν^1, ν^2.

The issue is that an intersection of extension conditions is controlled by the *multiset union* (componentwise maximum of multiplicities), not by the sum of the areas of the individual forbidden partitions. A sufficient corrected hypothesis is that for every finite S,

$$\left|\bigvee_{i\in S} \mu^i\right| = \left|\bigvee_{i\in S} \nu^i\right|,$$

where $\bigvee$ denotes the partition whose multiplicity of each part r is the maximum of the multiplicities of r among the partitions indexed by S. Then inclusion–exclusion gives the desired conclusion.

Page 237, Exercise 5-125.
The divide-and-conquer setup assumes only $a, b, c, d > 0$, but it needs at least

$$b > 1.$$

If $b = 1$, the subproblem size n/b does not decrease and the expression $\log_b a$ appearing in the claimed asymptotic bound is undefined. In the algorithmic interpretation one would normally assume that a and b are positive integers with $b \geq 2$.

Page 239, Definition 6.1.
The notation

$$[\![n]\!] = \{0, 1, \ldots, n-1\}$$

is defined only for positive integers n. Later in the chapter, however, ranking maps for empty combinatorial classes arise naturally (for example, impossible set-partition or anagram parameter choices), and the Bijective Sum Rule also allows finite component sets that may be empty. These situations require a rank set of cardinality 0.

Thus, the above definition of $[\![n]\!]$ should be extended to all **nonnegative** integers n. This boils down to setting

$$[\![0]\!] = \varnothing.$$

Page 242, proof of Lemma 6.4.
Replace "$S \times T$" by "$[\![n]\!] \times [\![m]\!]$".

Page 247, Example 6.15.
The text introduces a three-letter word as

$$w = w_1 w_2 w_2 \in S.$$

This should be

$$w = w_1 w_2 w_3 \in S.$$

Pages 252–253, ranking anagrams.
The text writes

$$C(n; n_1, \ldots, n_k) = \sum_{i=1}^{k} C(n-1; n_1, \ldots, n_i - 1, \ldots, n_k)$$

without any restrictions on $n, n_1, \ldots, n_k$. In truth, this only holds for $n > 0$.

Furthermore, the definition

$$M = (n_1 + \cdots + n_k)!/(n_1! \cdots n_k!)$$

makes sense only if $n_1, n_2, \ldots, n_k$ are nonnegative. If some $n_i < 0$, then M must be defined as 0.

The recursive definition of the ranking function $r_{n_1,\ldots,n_k}$ also requires $n > 0$ (otherwise, there is no "first letter").

Page 256, Section 6.11 (Ranking Trees).
The section opens by saying that there are

$$n^{n-2}$$

rooted trees on $\{1, \ldots, n\}$ rooted at 1, and then encodes them by words of length $n - 2$. This construction requires

$$n \geq 2.$$

For $n = 1$, there is indeed one rooted tree, but $n - 2 = -1$, so neither the word model nor the displayed power should be used literally. The $n = 1$ case should be separated.

Page 257, Rule 6.31 (Successor Sum Rule).
The rule permits some of the sets S_i to be empty and its pseudocode explicitly skips empty components. But it simultaneously assumes that "we already know successor algorithms for each S_i." Under the definition in §6.1, an empty set has no successor algorithm (since a successor algorithm includes subroutines `first` and `last`, but an empty set has no minimum and maximum elements).

The hypothesis should instead say that we can detect which S_i's are empty and that successor algorithms are known for every *nonempty* S_i. The pseudocode then works exactly as intended.

Page 258, Section 6.13.
"rearrangemenets" should read "rearrangements".

Page 259, Figure 6.6 (Anagram Successor Algorithm).
The base case in both `first` and `last` is incorrect. The pseudocode says

```
if (k==0) then return the empty word;
```

where k is the size of the fixed alphabet. But recursive calls decrease the multiplicities $n[i]$, not k. Consequently, when all multiplicities have reached 0 but $k > 0$, the subsequent loop

```
while (n[i]==0)
```

runs past the end of the array.

The base case should be

```
if (all n[i] are 0) then return the empty word;
```

(or equivalently, "if all $n[i] = 0$"). This is a genuine algorithmic bug in the printed pseudocode.

Pages 263–264, Section 6.16 (Successor Algorithms for Dyck Paths).
The text repeatedly applies the subroutines `first`, `last`, and `next` to C_n, although C_n is the *number* of Dyck paths. The underlying set is $DP(n)$. Thus, for example,

$$\texttt{first}(C_n) \quad \text{and} \quad \texttt{last}(C_n)$$

should be

$$\texttt{first}(DP(n)) \quad \text{and} \quad \texttt{last}(DP(n)).$$

Also, after "We use three cases to define $\pi^* = \texttt{next}(\pi, DP(n))$", add "when $\pi \neq \texttt{last}(DP(n))$".

Page 265, Chapter 6 Summary: ranking anagrams.
Again, the expression

$$\frac{(n_1 + \cdots + n_k - 1)!}{n_1! \cdots (n_j - 1)! \cdots n_k!}$$

only makes sense if $n_j \neq 0$, since otherwise this has a $(-1)!$ in the denominator. The sum should run only over

$$j < i \quad \text{with} \quad n_j > 0,$$

or else the text should explicitly use the convention that a fraction with a $(-1)!$ in its denominator must be 0.

Page 272, Exercise 6-123.
The phrase

"Describe a successor algorithm for ranking rooted trees ..."

should read simply

"Describe a successor algorithm for rooted trees ..."

(or "for generating rooted trees"). A successor algorithm does not itself rank the trees.

Page 282, Definition 7.18.
This defines k-cycles only for $k > 1$, but the (arguably trivial) concept of a 1-cycle is used in several places below (explicitly in the proofs of Theorems 7.33, 7.113 and 7.115, for example, but already in Remark 7.19 you are writing "(9)" and treating it as a permutation).

The best remedy is to replace "$k > 1$" by "$k \geq 1$" in Definition 7.18; thus, 1-cycles are put on the same footing as all the other cycles, despite their triviality.

Page 288, Example 7.38.
In "The set $J = \{(1)(2)(3), (1,3), (2,3), (1,3)\}$", replace one of the two "$(1,3)$"s by "$(1,2)$".

Page 299, Definition 7.73.
Replace "*left multiplication by G*" by "*left multiplication by g*", since the map being defined is given by $L_g(x) = g * x$.

Page 311, proof of Theorem 7.118 (Lucas).
In Step 3, the displayed formula at the end starts with

$$\binom{n}{p}.$$

It must start with

$$\binom{n}{k}.$$

Indeed, the next line applies Step 2 using $n = ap + n_0$ and $k = cp + k_0$, and the remainder of the computation proves Lucas's formula for $\binom{n}{k}$.

Page 319, Chapter 7 Summary.
In the discussion of the center of a finite p-group, the summary says

"(Exercise 7-92) groups G of size p^n ... have $|Z(G)| > 1$."

Exercise 7-92 is the Class Equation. The exercise that explicitly asks the reader to deduce the nontriviality of the center of a p-group is **Exercise 7-93**.

Page 322, Exercise 7-45.
The hypercube Q_k is declared to be a *simple graph*, but its edge set is written using ordered pairs:

$$E = \{(v, w) \in V : \ v, w \text{ differ in exactly one position}\}.$$

For a simple undirected graph, the edges should be unordered pairs:

$$E = \big\{\{v, w\} : v, w \in V, \ v, w \text{ differ in exactly one position}\big\}.$$

Page 325, Exercise 7-87(b).
This should assume $n \geq 2$, since the transposition $(1, 2)$ does not exist in S_1.

Page 333, inversion tables after Example 8.22.
The displayed formula defining k_r is missing cardinality bars:

$$k_r = \{(i, j) : 1 \leq i < j \leq n \text{ and } r = w_i > w_j\}.$$

The left side is a number, whereas the right side as printed is a set. It should be

$$k_r = |\{(i, j) : 1 \leq i < j \leq n \text{ and } r = w_i > w_j\}|.$$

This agrees with the sentence immediately following the display, which says that k_r *counts* such inversion pairs.

Page 333, Example 8.23.
Replace "$G(w) = (5, 5, 1, 2, 3, 0, 1, 0)$" by "$G(w) = (5, 5, 1, 2, 3, 1, 0, 0)$". Moreover, replace "since $k_0 = k_1 = 0$" by "since $k_2 = k_1 = 0$".

Page 337, proof of Corollary 8.33.
The induction proof of polynomiality applies the first recursion of Theorem 8.32 with $a = k$ and $b = n - k$ for every $0 \leq k \leq n$. But Theorem 8.32 states that recursion only for

$$a, b > 0.$$

Thus the displayed recursion does not apply when $k = 0$ or $k = n$.

The repair is immediate but should be stated: handle the boundary cases separately using

$$\begin{bmatrix} n \\ 0 \end{bmatrix}_q = \begin{bmatrix} n \\ n \end{bmatrix}_q = 1,$$

and apply the recursion only for $0 < k < n$.

Page 348, Theorem 8.46 heading.
The heading is printed

"Recursion for Dyck Paths Weighted by Area.For all $n \geq 0$, ..."

with a missing space after the period. Replace "Area.For" by "Area. For."

Pages 348–349, Theorem 8.47.
The theorem is stated for all $n \geq 0$, but its middle expression

$$\begin{bmatrix} 2n \\ n, n \end{bmatrix}_q - q \begin{bmatrix} 2n \\ n-1, n+1 \end{bmatrix}_q$$

is not defined at $n = 0$, because the second q-multinomial has the negative lower entry $n - 1 = -1$. The bijection in the proof likewise has target

$$\mathcal{R}(0^{n+1}1^{n-1}),$$

which is meaningful only for $n \geq 1$.

The easiest remedy for the theorem itself is to introduce the reasonable convention that a q-multinomial coefficient is 0 whenever one of its lower entries is negative. As for the proof, the case $n = 0$ is trivial.

Page 351, proof of Theorem 8.48.
In the last paragraph of this proof, replace each "$w(P)$" by "$f(P)$". Indeed, the permutation encoding of a set partition P was denoted by $f(P)$, not by $w(P)$.

Page 355, Exercise 8-31.
Replace "by setting $f(w) = (t_1, \ldots, t_n)$" by "by setting $F(w) = (t_1, \ldots, t_n)$".

Page 357, Exercise 8-66(b).
The displayed recursion for $C_{n,k}(q,t)$ cannot include the case $k = n$. As printed, its right side is then the empty sum

$$\sum_{r=1}^{n-k} (\cdots) = 0,$$

whereas there is one Dyck path beginning with exactly n north steps, namely

$$N^n E^n.$$

For this path,

$$\text{area} = \binom{n}{2}, \qquad \text{bounce} = 0,$$

so

$$C_{n,n}(q,t) = q^{\binom{n}{2}} \neq 0.$$

As in Exercise 2-81, the best fix is to extend the sum to start at $r = 0$ rather than $r = 1$.

The case $k = 0$ is also problematic: here, the q-multinomial coefficient has a -1 among its lower arguments. If we understand such q-multinomial coefficients to be 0 (as above), then the formula holds for $n > 0$, but not for $n = 0$.

Page 357, Exercise 8-67.
The definition of

$$G_n = \{(g_0, g_1, \ldots, g_{n-1}) : g_0 = 0,\ \ldots\}$$

requires $n \geq 1$, since g_0 does not exist when $n = 0$. The exercise should explicitly assume

$$n \geq 1.$$

Pages 365, 385, and 387: scope of $h_\alpha, e_\alpha, p_\alpha$ and Kostka numbers.
There are two related scope inconsistencies in the notation.

First, Definition 9.20 defines

$$e_\alpha, \quad h_\alpha, \quad p_\alpha$$

only when $\alpha = (\alpha_1, \ldots, \alpha_s)$ is a sequence of *positive* integers. Theorems 9.64 and 9.69 later allow α to have zero entries. This is harmless for e_α and h_α, since $e_0 = h_0 = 1$, but it is formally outside Definition 9.20. One should extend the definition of e_α, h_α to finite weak compositions by allowing zero entries (which simply contribute factors $e_0 = h_0 = 1$). It is better *not* to make the analogous extension for p_α, since p_0 has not been defined.

Second, Definition 9.25 defines $K_{\mu,\alpha}$ only for $\mu \in \mathrm{Par}_N$ and $\alpha \in \mathbb{Z}^N_{\geq 0}$. Later statements use Kostka numbers outside this range: for example, Theorem 9.41 writes $K_{\lambda,\lambda}$ for arbitrary partitions, and Theorems 9.64 and 9.69 use an arbitrary finite sequence $\alpha = (\alpha_1, \ldots, \alpha_s)$, with no requirement $s = N$; Theorem 9.69 may also involve $K_{\lambda',\alpha}$ when λ' has more than N parts.

The standard clean convention is to define $K_{\lambda,\alpha}$, independently of an ambient number N of variables, for every partition λ and every finite weak composition α: it is the number of semistandard tableaux of shape λ having α_i copies of the letter i. With this convention the later theorems are well-formed exactly as intended.

Page 388, Definition 9.73.
The definition begins with a list $z_1, \ldots, z_m$, but the displayed monomials are written

$$z_1^{\alpha_1} z_2^{\alpha_2} \cdots z_N^{\alpha_N}, \qquad \alpha \in \mathbb{Z}^N_{\geq 0}.$$

The N's should be m's:

$$z_1^{\alpha_1} z_2^{\alpha_2} \cdots z_m^{\alpha_m}, \qquad \alpha \in \mathbb{Z}^m_{\geq 0}.$$

Page 389, beginning of §9.15.
The recalled definition of p_α is written

$$p_\alpha(x_1, \ldots, x_N) = \prod_{j \geq 1} p_{\alpha_j}(x_1, \ldots, x_N).$$

For the finite sequence $\alpha = (\alpha_1, \ldots, \alpha_s)$, this should be

$$p_\alpha(x_1, \ldots, x_N) = \prod_{j=1}^{s} p_{\alpha_j}(x_1, \ldots, x_N).$$

Otherwise α_j is not defined for $j > s$; padding by zeros would not fix the issue, since p_0 has not been defined.

Page 389, Theorem 9.76.
There are two minor mistakes in the last paragraph of this proof.

First, "$\dfrac{\partial g}{\partial y_k}(g_1, \ldots, g_N)$ is nonzero in $\mathbb{R}[y_1, \ldots, y_N]$" should be "$\dfrac{\partial g}{\partial y_k}(g_1, \ldots, g_N)$ is nonzero in $\mathbb{R}[x_1, \ldots, x_N]$".

Furthermore, the argument that "$\mathbf{Av} = \mathbf{0}$ forces $\mathbf{A}$ to be non-invertible, so that $J = \det(\mathbf{A}) = 0$ by a theorem of linear algebra" does not work over the ring $\mathbb{R}[x_1, \ldots, x_N]$ but only works over its fraction field $\mathbb{R}(x_1, \ldots, x_N)$. Indeed, a square matrix over $\mathbb{R}[x_1, \ldots, x_N]$ can be non-invertible without its determinant being 0 (for instance, the 1×1 matrix $[x_1]$ is not invertible but has nonzero determinant). To repair this, regard $\mathbf{A}$ and $\mathbf{v}$ as a matrix and vector over the fraction field $\mathbb{R}(x_1, \ldots, x_N)$.

Page 393, proof of Theorem 9.86.
In the displayed formula, replace "$\sum_{k=0}^{n}$" by "$\sum_{k=0}^{N}$".

Pages 398 (last paragraph), 417 ("We can define evaluation homomorphisms ..."), and 421 ("Symmetric functions" bullet point in Summary).
The text (and later the Chapter 9 summary) says that a polynomial algebra such as

$$\Lambda_N = \mathbb{R}[p_1, \ldots, p_N] \qquad \text{or} \qquad \Lambda = K[p_1, p_2, \ldots]$$

admits an evaluation homomorphism sending the generators to arbitrary elements of an arbitrary algebra B. If B is noncommutative, this is false unless the chosen images commute pairwise. Thus one should either assume that B is commutative, or require the chosen images of the generators to commute pairwise.

Page 399, Definition 9.94.
The definition

$$\omega(p_j) = (-1)^{j-1} p_j$$

is said to hold "for j between 1 and n." The upper index should be

$$N,$$

not n.

Page 411, proof of Theorem 9.121.
In "where j is a fixed index between 1 and n", replace "n" by "N".

Page 414, Definition 9.130.
A filling of a skew shape is allowed to take arbitrary integer values,

$$T : \mu/\nu \longrightarrow \mathbb{Z},$$

but the text then defines the content monomial of *any* such filling by

$$\mathbf{x}^T = \prod_{(i,j) \in \mu/\nu} x_{T(i,j)}.$$

The variables $x_0, x_{-1}, \ldots$ have not been introduced. The content monomial should be defined only for fillings with values in $\mathbb{Z}_{>0}$ (and, for a polynomial in N variables, for values in $[N]$), just as in Definition 9.6 for straight shapes.

Page 416, Theorem 9.136.
The skew Pieri formulas contain $K_{\lambda/\mu,\alpha}$ while summing over all $\lambda \in \mathrm{Par}_N$. But λ/μ is a skew shape only when $\mu \subseteq \lambda$; no convention assigning zero to the other terms has been stated. One should either restrict the first sum to $\lambda \supseteq \mu$ (and the second accordingly), or explicitly declare the corresponding skew Kostka number to be 0 when the containment fails.

There is also the same Kostka-indexing issue noted above: the theorem allows $\alpha \in \mathbb{Z}^s_{\geq 0}$, whereas the preceding definition of skew Kostka numbers uses $\alpha \in \mathbb{Z}^N_{\geq 0}$. Defining (skew) Kostka numbers for arbitrary finite weak compositions resolves this uniformly.

Page 416, scalar-product identity after Theorem 9.136.
The finite-variable statement

$$\langle s_{\mu/\nu}, f \rangle = \langle s_\mu, s_\nu f \rangle$$

is asserted for a skew shape μ/ν with k cells, $f \in \Lambda^k_N$, and $N \geq k$. This hypothesis is not sufficient for the right-hand side to be defined under Definition 9.122: the right side lies in homogeneous degree

$$|\mu| = |\nu| + k,$$

whereas the Hall scalar product on Λ^d_N was defined there only when $N \geq d$.

A sufficient finite-variable hypothesis is

$$N \geq |\mu|.$$

More naturally, this adjointness identity can be stated in the abstract symmetric-function algebra Λ, where there is no finite-variable degree restriction.

Page 417, §9.28.
After introducing a finite sequence $\alpha = (\alpha_1, \ldots, \alpha_s)$, the abstract symmetric functions are written

$$h_\alpha = \prod_{i \geq 1} h_{\alpha_i}, \qquad e_\alpha = \prod_{i \geq 1} e_{\alpha_i}.$$

The products should run over

$$1 \leq i \leq s,$$

unless one explicitly declares $\alpha_i = 0$ for $i > s$.

Page 420, Chapter 9 Summary.
The displayed generating function for the complete homogeneous symmetric polynomials is incorrectly stated as

$$H_N(t) = \prod_{i=1}^{N} (1 - x_i t)^{-1} = \sum_{k=0}^{N} h_k(x_1, \ldots, x_N) t^k.$$

The "$\sum_{k=0}^{N}$" sign here should be "$\sum_{k=0}^{\infty}$", as in Theorem 9.87.

Page 420, Table 9.2.
The finite-variable monomial expansions in Table 9.2 are written as sums over all partitions of the relevant degree, for example

$$s_\lambda = \sum_{\mu \in \mathrm{Par}(|\lambda|)} K_{\lambda,\mu} m_\mu.$$

But $m_\mu(x_1,\dots,x_N)$ was defined only when $\ell(\mu)\le N$ (Remark 9.24). Unless N is known to be at least the degree, these sums should use

$$\mathrm{Par}_N(|\lambda|)$$

(and similarly $\mathrm{Par}_N(|\mu/\nu|)$ for the skew Schur expansion), or the table should explicitly impose the stable-range hypothesis $N \ge$ degree.

Page 420, Chapter 9 Summary: action of ω.
The summary states, without a degree restriction, that the finite-variable involution

$$\omega : \Lambda_N \longrightarrow \Lambda_N$$

satisfies $\omega(s_\mu) = s_{\mu'}$. Theorem 9.96 proved this only in the stable range

$$|\mu| \le N,$$

and the unrestricted finite-variable statement is false. For example, when $N = 1$, $\Lambda_1 = \mathbb{R}[p_1]$ and ω is the identity, so

$$\omega(s_{(2)}(x_1)) = x_1^2, \qquad s_{(1,1)}(x_1) = 0.$$

The unrestricted identity is valid in the abstract symmetric-function algebra Λ; in Λ_N the stable-range hypothesis should be retained.

Page 421, Chapter 9 Summary, "Symmetric Functions" bullet point.
The summary repeats the claim that one may define an evaluation homomorphism on the abstract symmetric-function algebra by sending each p_n to "any element" of a K-algebra. As on pages 398 and 417, this is correct for a commutative target algebra; for a noncommutative target, the chosen images must commute pairwise.

Page 423, Exercise 9-55.
Replace "x_n" by "x_N".

Page 424, Exercise 9-72.
The displayed recursion is claimed for $k, N \ge 1$, but when $N = 1$ its right-hand side contains symmetric polynomials in the empty list of variables, such as $e_k(x_1,\dots,x_{N-1}) = e_k()$, which have not been defined in the text. One can either state the recursion for $N \ge 2$ and take the $N = 1$ formulas as initial conditions, or extend the definitions by

$$e_0() = h_0() = 1, \qquad \text{and} \qquad e_r() = h_r() = 0 \quad \text{for } r \ne 0.$$

With this extension (which is standard in the literature) the stated recursion is valid also for $N = 1$.

Page 424, Exercise 9-73.
The ranges of n and k should be specified. As written, the formulas can contain negative subscripts when $k > n$, while $k = 0$ leads to symmetric polynomials in zero variables. Without introducing additional conventions, the exercise holds for $1 \le k \le n$. If the boundary cases are desired as well, one can use

$$e_0() = h_0() = 1, \qquad \text{and} \qquad e_r() = h_r() = 0 \quad \text{for } r \ne 0.$$

together with the usual zero conventions for Stirling numbers.

Page 426, Exercise 9-94.
As written, "an algebra isomorphism of Λ_N sending each p_i to $-p_i$" is ambiguous and is false if "each" means every $i \geq 1$. For example, in one variable $p_2 = p_1^2$, so a homomorphism with $p_1 \mapsto -p_1$ must send

$$p_2 = p_1^2 \longmapsto p_1^2 = p_2,$$

not to $-p_2$.

The finite-variable version should say that the isomorphism should send

$$p_i \longmapsto -p_i \qquad \text{for all } 1 \leq i \leq N.$$

With this interpretation, the simple formulas

$$h_n \longmapsto (-1)^n e_n, \qquad e_n \longmapsto (-1)^n h_n$$

hold for $n \leq N$. Alternatively, the exercise can be stated in the abstract symmetric-function algebra Λ, where all p_i are independent generators and the formulas hold for every n.

Page 426, Exercise 9-95.
The exercise asks:

"In the proof of Theorem 9.95(b), where is the assumption $n \leq N$ needed?"

Part (b) has no such variable n or hypothesis. The intended reference is almost certainly **Theorem 9.95(c)**, whose proof explicitly begins "For $1 \leq n \leq N$" and uses part (b) on all partitions of n.

Page 439, proof of Theorem 10.22.
In the second-to-last sentence of the proof, replace "$m_i = |\rho^i|$" by "$m_i = |\nu^i|$.

Page 440, Definition 10.23.
In the last sentence of the definition, replace "$c_k(z) \equiv r \pmod{k}$" by "$c(z) \equiv r \pmod{k}$".

Page 440, construction immediately after Figure 10.1.
In constructing the zeroth 3-quotient, the text says to mark the cells whose associated frontier steps "both have content zero." The construction is based on residue classes modulo 3, so this should read

"both have 3-content zero."

Page 442, Remark 10.26.
In "$t_i \bullet f = \operatorname{sgn}(t_i) f$ for all i between 1 and $N-1$", replace "$N-1$" by "k". Indeed, the t_i are the factors of the given factorization $w = t_1 t_2 \cdots t_k$, so there is no reason why there should be exactly $N-1$ of them.

Page 449, proof of Theorem 10.39.
The proof says that

$$\operatorname{sgn}(v^*) \text{ differs from } \operatorname{sgn}(v) \text{ by } \operatorname{sgn}(R) = (-1)^{\operatorname{spin}(R)},$$

"which is the number of beads that bead i passes over when it moves". Of course, this is not literally true: The sign $(-1)^{\operatorname{spin}(R)}$ is not a number of beads. What the preceding example established is

$$\operatorname{spin}(R) = \#\{\text{beads passed over by bead } i\},$$

and hence the resulting sign change is $(-1)^{\operatorname{spin}(R)}$.

Pages 455–456, Definition 10.48 and Theorem 10.50.
Definition 10.48 allows a weak composition $\alpha \in \mathbb{Z}^s_{\geq 0}$ and requires ν_i/ν_{i-1} to be an α_i-ribbon. But Definition 10.7 defines a ribbon as a nonempty skew shape, so an $\alpha_i = 0$ stage has not been defined. Later Definition 10.63 explicitly speaks of "nonzero rim-hooks," confirming that zero stages are intended.

For rim-hook tableaux it would be natural to adopt the convention that a 0-ribbon is the empty skew shape, contributes no change of partition, and has sign $+1$.

There is a separate but related issue in Theorem 10.50. It is stated for arbitrary $\alpha \in \mathbb{Z}^t_{\geq 0}$, but the polynomial p_α was defined only for sequences of positive integers; in particular p_0 has not been defined. Thus the theorem should either assume

$$\alpha_i > 0 \qquad \text{for all } 1 \leq i \leq t,$$

or explicitly declare that zero entries of α are to be deleted before forming p_α and the corresponding rim-hook tableau:

$$p_\alpha := \prod_{\substack{1 \leq i \leq t; \\ \alpha_i > 0}} p_{\alpha_i} \qquad \text{for all } \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_t) \in \mathbb{Z}^t_{\geq 0}.$$

The latter is the best option for Definition 10.63, since regarding a partition in Par_N as an N-tuple often causes it to contain zeroes.

Page 458, beginning of §10.13.
The set X is defined to consist of pairs (v, T), where v is a justified labeled abacus and T is merely said to be "a semistandard tableau using letters in $\{1, \ldots, N\}$." The shape of T is missing. Since the next displayed generating function is

$$a_{\delta(N)} s_\lambda,$$

the intended definition is

$$T \in \mathrm{SSYT}_N(\lambda)$$

(with semistandardness interpreted relative to the ordering $<_v$).

Page 461, proof of Theorem 10.58.
Replace "take the scalar product of both sides with a given partition λ" by "take the scalar product of both sides with s_λ for a given partition λ".

Pages 465–466, proof of Theorem 10.61.
The proof of the Second Jacobi–Trudi Formula has a genuine finite-variable scope problem. The theorem assumes only

$$\lambda_1 = n \leq N,$$

and then says to obtain the result by applying the finite-variable involution ω to the First Jacobi–Trudi Formula for λ'/μ'.

However, Theorem 10.59 established

$$\omega(s_{\alpha/\beta}) = s_{\alpha'/\beta'}$$

in N variables only under the stronger stable-range hypothesis $N \geq |\alpha|$. Outside that range the corresponding finite-variable assertion is false. For example, in one variable ω is the identity, so

$$\omega(s_{(2)}(x_1)) = x_1^2 \quad \text{whereas} \quad s_{(1,1)}(x_1) = 0.$$

Thus the stated Second Jacobi–Trudi Formula is standard and correct, but the printed proof does not establish it under the stated hypothesis. A clean repair is to apply ω in the *abstract* symmetric-function algebra, where $\omega(s_{\alpha/\beta}) = s_{\alpha'/\beta'}$ holds without a finite-variable restriction, prove the determinant identity there, and then specialize to N variables. Alternatively, the printed argument works after imposing the unnecessarily stronger assumption $N \geq |\lambda|$.

Page 466, beginning of §10.16.
The vector identity

$$\mathbf{e} = \mathbf{K}^{\mathrm{tr}}\omega(\mathbf{s})$$

is potentially incorrect if $\omega(\mathbf{s})$ means applying the finite-variable involution ω entrywise to the vector of all Schur polynomials indexed by Par_N. The identity $\omega(s_\lambda) = s_{\lambda'}$ is only valid when λ has size $\leq N$, whereas this paragraph ranges over all degrees.

The intended relation is better written

$$\mathbf{e} = \mathbf{K}^{\mathrm{tr}}\mathbf{s}^\vee, \qquad \text{where } \mathbf{s}^\vee = (s_{\lambda'} : \lambda \in \mathrm{Par}_N),$$

with the Kostka matrices understood degree by degree. The subsequent explicit inverse formulas remain meaningful with this interpretation.

Page 467, proof of Theorem 10.64.
In the paragraph where you cancel the colliding-beads addends, replace "and switching $e(i)$ and $e(j)$ in the exponent vector" by "and leaving e unchanged". In fact, there is no need to change e in order to preserve the weight, since an object has signed weight

$$\mathrm{sgn}(w) \prod_{r=1}^{N} x_{w(r)}^{N-r+e(r)},$$

which is symmetric in $x_{w(i)}$ and $x_{w(j)}$ if the beads $w(i)$ and $w(j)$ collide (indeed, the beads $w(i)$ and $w(j)$ collide if and only if $N-i+e(i) = N-j+e(j)$). However, if you do switch $e(i)$ and $e(j)$, then the factors $x_{w(i)}^{N-i+e(i)}$ and $x_{w(j)}^{N-j+e(j)}$ become $x_{w(j)}^{N-i+e(j)}$ and $x_{w(i)}^{N-j+e(i)}$, which does not preserve the weight.

Page 469, Theorem 10.66.
The Littlewood–Richardson rule is stated "for all partitions λ, μ, ν," but its right side counts tableaux of skew shape λ/ν. That skew shape has only been defined when

$$\nu \subseteq \lambda.$$

Moreover, the content μ must have the same size as the skew shape:

$$|\mu| = |\lambda| - |\nu|.$$

Thus these hypotheses should either be included explicitly, or the coefficient $c_{\nu,\mu}^{\lambda}$ should first be extended by the convention that it is 0 outside these containment and degree conditions. The latter remedy is better, since it is consistent with later use of $c_{\nu,\mu}^{\lambda}$. (For instance, in Theorem 10.72, you don't presuppose anything about the sizes and containment relations of λ, μ, ν.)

Page 470, Example 10.67.
"parentheis string" should read "parenthesis string."

Page 470, proof of Theorem 10.66.
In the construction of the involution, the sentence "One may verify that the new word is the word of a tableau $T' \in \mathrm{SSYT}_N(\lambda/\nu)$, relative to the ordering $<_{v'}$" hides a nontrivial verification.

Here is the missing argument. The beads $i >_v j$ are adjacent, and v' is obtained from v by interchanging them. Hence $<_{v'}$ differs from $<_v$ only by reversing the order of i and j; all comparisons involving any third letter are unchanged. It therefore suffices to check adjacent row and column comparisons in which both entries belong to $\{i, j\}$.

For the rows, use the parenthesis matching from the proof. Since $i <_{v'} j$, a failure of weak increase in a row of T' involving these two letters would mean that some j occurs in a cell strictly to the left of some i in the same row. In the parenthesis encoding appropriate to (v', T'), the j's are left parentheses and the i's are right parentheses. Thus this row contains a left parenthesis somewhere before a right parenthesis, so the matching contains a matched pair entirely within this row. The construction preserves which parenthesis positions are matched; hence, this matched pair must have already been matched in T. In the original tableau T, the two cells of this matched pair contain an i to the left of a j. But $j <_v i$, contradicting the weak increase of that row of T.

For the columns, suppose two vertically adjacent cells of T' each contain i or j. Let us call these two cells (r, c) and $(r+1, c)$. We must prove that $T'(r, c) = i$ and $T'(r+1, c) = j$.

Clearly, each of $T(r, c)$ and $T(r+1, c)$ must be either i or j, since the transformation of T into T' changes no letters other than i and j. Because T is column-strict and $j <_v i$, we thus have $T(r, c) = j$ and $T(r+1, c) = i$. In the word of T, these entries j and i become a right and a left parenthesis, respectively.

We shall now show that both of these parentheses are matched (though not necessarily to each other). First consider the j. Consider any j in row r that lies in column c or further left. Then, this j has a cell directly below it (since skew rows are intervals, the lower row starts weakly to the left of the upper row, and it extends at least through column c). The entry in this latter cell is strictly larger than j (since T is column-strict) and less than or equal to i (since T is weakly increasing along rows, and there is an i to the right of this entry). Since j and i are adjacent in the alphabet, this entails that this entry is i.

Thus, we have shown that each j in row r that lies in column c or further left must have an i directly below it (in row $r+1$). Similarly, we can show that each i in row $r+1$ that lies in column c or further right must have a j directly above it (in row r). Therefore, rows r and $r+1$ of T look as follows:

		$\cdots$	u_2	u_1	j	j	$\cdots$	j	$*$	$*$	$*$	$*$	$*$
$*$	$*$	$*$	$*$	$*$	i	i	$\cdots$	i	v_1	v_2	$\cdots$		

(1)

where $v_1, v_2, v_3, \ldots$ satisfy $v_\ell >_v i$, whereas $u_1, u_2, u_3, \ldots$ satisfy $u_\ell <_v j$, and where the asterisks denote entries that we are not interested in (they can include i's and j's, but they might also not exist). Note that the columns that have a j in row r and an i in row $r+1$ simultaneously are highlighted in green, whereas the cells of row r that lie to their left as well as the cells of row $r+1$ that lie to their right are highlighted in yellow. In particular, the i in cell $(r+1, c)$ and the j in cell (r, c) are green.

Now, in the reading word $w(T)$, the green and yellow entries form a contiguous block

$$ii \cdots iv_1v_2 \cdots \cdots u_2u_1jj \cdots j.$$

All the i's in this block are left parentheses, and all the j's in this block are right parentheses. Since the intermediate entries $v_1 v_2 \cdots\cdots u_2 u_1$ are not parentheses (because $v_\ell >_v i$ and $u_\ell <_v j$), and since there are equally many i's and j's in this block, we conclude that the left parentheses in this block are matched bijectively with the right parentheses in this block, entirely within the block. In particular, the i in cell $(r+1, c)$ and the j in cell (r, c) are both matched positions (although they need not be matched with each other).

Now, the parenthesis modification in the construction of T' leaves the parenthesis symbols at all matched positions unchanged. When the resulting parenthesis string is decoded for T', however, left parentheses represent j and right parentheses represent i, rather than i and j as they did for T. Thus, any i in a matched position of T becomes a j in T', and vice versa. Hence, the i in cell $(r+1, c)$ of T and the j in cell (r, c) of T become j and i in T', respectively. This is exactly the strict order required by $<_{v'}$ to ensure that the entries are strictly increasing in column c. Comparisons with third letters remain valid because i and j are adjacent. Therefore T' is indeed semistandard relative to $<_{v'}$.

Example 10.67 sketches the row and column checks in a particular case, but the general verification is omitted from the proof itself.

Page 474, Theorem 10.72.
There is a typographical error in the statement: "coefficent" should read "coefficient."

Page 475, Table 10.1.
Two finite-variable formulas in the summary table have lost the hypotheses under which they were proved.

First,

$$\langle s_{\lambda/\mu}, f\rangle = \langle s_\lambda, s_\mu f\rangle$$

is written merely "for $f \in \Lambda_N$." Theorem 10.57 assumes

$$k = |\lambda| - |\mu|, \qquad f \in \Lambda_N^k, \qquad N \geq |\lambda|.$$

The degree condition is also needed for the two scalar products to live in the homogeneous spaces on which the finite-variable Hall scalar product was defined.

Second, the table writes

$$\omega(s_{\lambda/\mu}) = s_{\lambda'/\mu'}$$

without a stable-range restriction. Theorem 10.59 assumes $N \geq |\lambda|$; without such a condition the finite-variable statement is false. The unrestricted formula belongs naturally to the abstract symmetric-function algebra.

Page 479, Exercise 10-46.
The exercise asks the reader to explain why

$$\omega(h_\mu) = e_\mu, \qquad \omega(e_\mu) = h_\mu$$

are special cases of Theorem 10.59, but no finite-variable range is specified. As an unrestricted assertion in Λ_N, this is false: for $N = 1$ and $\mu = (2)$,

$$\omega(h_2) = h_2 = x_1^2, \qquad e_2 = 0.$$

A clean version is to work in the abstract symmetric-function algebra, or to impose a stable-range hypothesis such as $N \geq |\mu|$ before applying Theorem 10.59.

Pages 479–480, Exercise 10-57.
The first sentence asks for $I(v^0, T)$, consistently with the notation v^0 used in the proof of Theorem 10.66 for the standard justified abacus. But the displayed definition immediately below writes

$$v_0 = 5432100\cdots.$$

Replace v_0 by v^0.

Page 483, proof of Theorem 11.7(b).
Replace "$\prod_{n=0}^{\infty} F_n$" by "$\prod_{n=0}^{\infty}(1 + F_n)$". Replace "$\prod_{n=0}^{m} F_n$" by "$\prod_{n=0}^{m}(1 + F_n)$". Replace "of the factors F_n" by "of the factors $1 + F_n$".

Page 484, Theorem 11.10.
There are two small domain/indexing issues in the Infinite Product Rule for Weighted Sets.

First, the hypothesis uses

$$\mathrm{minwt}(S_k - \{o_k\}),$$

but it is possible that $S_k = \{o_k\}$, in which case the minimum of the empty set has not been defined. This should be remedied by adopting the natural convention

$$\mathrm{minwt}(\varnothing) = +\infty.$$

Second, in the weight-additivity hypothesis, replace "u is constructed from $(u_k : k \geq 0)$" by "u is constructed from $(u_k : k \geq 1)$". The same "$k \geq 0$" typo is repeated in the Chapter 11 Summary (page 507).

Page 491, justification of Method 2.70 after Theorem 11.21.
Let k be the order of the recurrence. The text writes

$$\chi(z) = \prod_{i=1}^{k}(z - r_i)^{m_i}$$

and subsequently sums over $i = 1, \ldots, k$. This indexing is wrong when the characteristic polynomial has repeated roots: there need not be k distinct roots.

One should introduce, say, ℓ distinct roots $r_1, \ldots, r_\ell$, with multiplicities $m_1, \ldots, m_\ell$ satisfying

$$m_1 + \cdots + m_\ell = k,$$

and write

$$\chi(z) = \prod_{i=1}^{\ell}(z - r_i)^{m_i}, \qquad P(z) = \prod_{i=1}^{\ell}(1 - r_i z)^{m_i}.$$

All subsequent outer sums in this argument should likewise run from 1 to ℓ.

There is one further tacit step in this application of Theorem 11.18. That theorem allows a polynomial part h in the partial fraction decomposition, and its coefficient formula accordingly contains the extra term $h|_{z^n}$. But the uniqueness proof of Theorem 11.18 identifies h as the quotient obtained when the numerator is divided by the denominator. Thus one has the useful porism

$$\deg f < \deg g \quad \Longrightarrow \quad h = 0.$$

In the present application, $\deg G < k$, while $\deg P = k$ because Method 2.70 assumes that the last recurrence coefficient is nonzero. Hence $h = 0$, which is why the displayed formula for a_n contains only the partial-fraction terms. This should be stated explicitly (or the above porism should be appended to Theorem 11.18).

Page 493, Theorem 11.25(d).
The Formal Power Rule is stated for all $n \geq 0$:

$$(F^n)' = nF^{n-1}F'.$$

At $n = 0$, however, the right side contains F^{-1}, which need not exist. The formula should be stated for

$$n \geq 1,$$

with the case $n = 0$ recorded separately as

$$(F^0)' = 1' = 0.$$

The same boundary issue occurs later in the proof of the Chain Rule when the monomial z^n is treated using $nG^{n-1}G'$ for all $n \geq 0$, and the same Power Rule is repeated without correction in the Chapter 11 Summary.

Alternatively, one could keep a single displayed formula for all $n \geq 0$ by adopting the special convention that the entire expression $nF^{n-1}F'$ is to mean 0 when $n = 0$; this should not be interpreted as making F^{-1} itself defined.

Page 494, proof of Theorem 11.27(b).
Replace "$\exp(-F)$ is the multiplicative inverse of F in $K[[z]]$" by "$\exp(-F)$ is the multiplicative inverse of $\exp(F)$ in $K[[z]]$".

Page 502, Theorem 11.43.
This needs the requirement $n > 0$. Indeed, in the case $n = 0$ (which is explicitly allowed in Definitions 11.41 and 11.42), the one-letter word $w = 1$ satisfies the weight conditions of Theorem 11.43, but is not a list of 0 terms (since that list would be the empty word).

The proof also silently fails at $n = 0$: it prefixes the symbol n to w and claims that nw satisfies the term criterion, but when $n = 0$ its first symbol already has weight -1.

The corresponding characterization in the Chapter 11 (page 509) Summary needs the same qualification.

Page 505, proof of Theorem 11.47.
"which satsifies" should read "which satisfies."

Page 507, Chapter 11 Summary: symmetric function formula for $1/F$.
Part (d) of the "Multiplicative Inverses of Formal Series" bullet points needs the requirement $a_0 = 1$ (not just the $a_0 \neq 0$ requirement of the whole bullet point). Arguably, this may be implicit in "the homomorphism sending e_n to a_n", provided that the n is understood to range over $\mathbb{Z}_{\geq 0}$ (since this implies, in particular, that the homomorphism sends $e_0 = 1$ to a_0, and thus $a_0 = 1$ follows); but a reader might just as well understand n to range over $\mathbb{Z}_{>0}$, and then this requirement is nowhere to be seen.

Page 509, Chapter 11 Summary.
There are two definite typographical errors near the end of the summary.

In the Cycle Lemma item, the number of lists using k_i copies of i for $0 \leq i \leq t$ is printed with the multinomial coefficient

$$\binom{s}{k_0, k_1, \ldots, k_s}.$$

The final lower index should be

$$k_t,$$

not k_s.

In the "Compositional Inverses of Formal Series" item, replace "$F = x/R$" by "$F = z/R$".

Page 509, Exercise 11-13.
There is a small edge case in the instruction to "let (F_{k_n}) be the subsequence of nonzero terms of (F_n)." If (F_n) has only finitely many nonzero terms, there is no infinite subsequence $(F_{k_n} : n \geq 0)$ to which the displayed infinite sum or product can refer. One should either assume that there are infinitely many nonzero terms, or allow (F_{k_n}) to be a finite list and handle that case separately (where convergence is immediate). The same qualification applies to the remark following Theorem 11.7 that one may "always" drop zero summands or factors equal to 1 in order to arrange that all terms are nonzero.

Page 510, Exercise 11-17.
This exercise asks the reader to check convergence of the infinite products at the end of §9.28, but those products live in the rings

$$K[[t, x_1, x_2, \ldots]] \quad \text{and} \quad K[[t, x_1, x_2, \ldots, y_1, y_2, \ldots]],$$

whereas Chapter 11 has only defined convergence for sequences in the one-variable ring $K[[z]]$. In particular, Theorem 11.7 cannot simply be applied using t-adic order: for example, every factor $1 + tx_i$ differs from 1 in t-degree 1.

The intended statement is correct after extending the definition of convergence coefficientwise to these infinite-variable formal power series rings. Then each fixed monomial is affected by only finitely many of the factors. For instance, the coefficient of a given monomial in

$$\prod_{i \geq 1} (1 + tx_i)$$

stabilizes once all variables occurring in that monomial have appeared; the same local-finiteness argument works for the doubly indexed Cauchy product

$$\prod_{i,j \geq 1} (1 - tx_i y_j)^{-1}.$$

Thus the exercise needs a brief extension of the formal convergence framework before it is literally well-posed.

Page 511, Exercise 11-47.
Replace "$g : B \to C$" by "$q : B \to C$".

Page 513, Exercise 11-64(g).
Part (g) is false because the exponent r is itself allowed to be a formal power series. The exercise defines

$$F^r = \exp(r \log F) \qquad (\text{where } F(0) = 1,\ r \in K[[z]]),$$

and then claims

$$\frac{d}{dz} F^r = r F^{r-1} F'.$$

This omits the derivative of r. The correct formula is

$$\boxed{\frac{d}{dz}F^r = F^r\left(r'\log F + r\frac{F'}{F}\right) = F^r r'\log F + rF^{r-1}F'.}$$

The printed formula is valid when r is a constant scalar in K, but not for general $r \in K[[z]]$. For instance, taking $F = 1+z$ and $r = z$ produces a nonzero extra term $(1+z)^z \log(1+z)$.

Page 514, Exercises 11-82 and 11-83.
Both exercises should specify $n \geq 1$. Every term is a nonempty word, so this is the natural range; without it, the case $n = 0$ asks for binary trees with -1 nodes in Exercise 11-82 and for lattice paths ending at $(-1,-1)$ in Exercise 11-83.

Page 514, Exercise 11-86(c).
"a given sequence $F \in V$" should be "a given formal series $F \in V$".

Page 518, proof of Theorem 12.4 (Chung–Feller).
The integer k has already been fixed as the desired number of flaws, but the construction of ϕ_k reuses k as the position of the first step at which the newly constructed path goes below the diagonal:

$$\text{"until we reach a horizontal step } c_k = a_k\text{"}.$$

These two k's need not be equal. Indeed, in the displayed example $k = 6$, while the first switch occurs after transferring the five steps J, K, L, M, N.

The stopping index should be given a new name, say ℓ:

$$c_1 = a_1,\ c_2 = a_2,\ \ldots,\ c_\ell = a_\ell,$$

followed by

$$c_{\ell+1} = b_1,\ c_{\ell+2} = b_2,\ \ldots,\ c_{\ell+m} = b_m, \qquad c_{\ell+m+1} = a_{\ell+1},\ \ldots.$$

The rest of the construction is unchanged.

Page 525, Remark 12.17.
In the discussion of linear probing for hashing, when $h(i)$ is occupied, item i is said to be put in "the earliest position after position i." It should be put in

$$\text{the earliest position after position } h(i).$$

Otherwise the stated correspondence with parking functions is not correct.

Page 531, proof of Theorem 12.30 and Exercise 12-39(b).
The proof of the irreducible-polynomial counting formula begins by taking a finite extension field

$$K \supseteq F, \qquad |K| = q^n,$$

and later uses that K is a splitting field for $x^{q^n} - x$. Existence of such a field (or extension of F) is therefore an input to the printed proof.

This creates a logical dependency problem with Exercise 12-39(b), which asks the reader to *use Theorem 12.30* to prove that a field exists for every prime-power cardinality. If finite-field existence is among the field-theory facts being imported before Theorem 12.30, then Exercise 12-39(b) is circular as a purported proof of that existence theorem.

One should either state explicitly before Theorem 12.30 that the existence of the extension K/F of degree n is being assumed from field theory, and not present Exercise 12-39(b) as an independent proof; or else give a proof of Theorem 12.30 that does not presuppose finite-field existence in the degree being constructed.

There is also a small omitted case inside the proof. The argument for an irreducible g of degree d starts with a nonzero root z. When $g = x$ (the degree-1 irreducible with root 0), this does not apply. That case is immediate and should be separated before assuming $z \neq 0$.

Pages 542–543, Theorems 12.51–12.53.
The Laplace expansion is stated for every $A \in M_n(\mathbb{R})$, but for $n = 1$ the minor $A[1 \mid 1]$ is a 0×0 matrix. Definition 12.39 defines $M_n(\mathbb{R})$ only for positive n, and no determinant of the empty matrix has been defined. Definition 12.52 of the classical adjoint has the same issue for a 1×1 matrix.

The standard clean convention is to allow the unique 0×0 matrix and define its determinant by

$$\det(\varnothing) = 1.$$

With this convention the Laplace formula and the adjoint formula include $n = 1$ exactly as written (and all the other standard properties of determinants are true). Alternatively, one must state the Laplace theorem for $n \geq 2$ and treat the 1×1 cases separately.

Page 551, proof of Lemma 12.72.
There are two boundary/notation problems in the induction.

First, after the initial cell $c_1 = (i_1, j_1)$, the proof says that each possible c_2 is chosen with probability

$$1/(h(c) - 1) = 1/(h(i_1, j_1) - 1).$$

Since $c = (r, s)$ is the *terminal corner*, $h(c) = 1$, so the first denominator is zero. The intended expression is

$$\boxed{1/(h(c_1) - 1) = 1/(h(i_1, j_1) - 1)}.$$

Second, the two possible next cells are written

$$c_2 = (i_1, j_2) \qquad \text{or} \qquad c_2 = (i_2, j_1).$$

But j_2 is undefined when $J = \{j_1\}$, and i_2 is undefined when $I = \{i_1\}$. The intended argument becomes uniform if, after writing

$$I = \{i_1 < \cdots < i_\ell\}, \qquad J = \{j_1 < \cdots < j_m\},$$

one sets

$$i_{\ell+1} = r, \qquad j_{m+1} = s.$$

Then the next cell is (i_1, j_2) or (i_2, j_1), with the terminal row or column used when the corresponding set has only one element.

Additionally, the argument for "we can replace the first factor $1/n$ by $\dfrac{1}{n} \cdot \dfrac{1}{h(c_1) - 1}$" is worth spelling out a bit more: The hook walk is a Markov chain, so the probability of choosing a given sequence of cells $c_3, c_4, \ldots$ after having chosen c_1 and c_2 depends only on c_2 and not on c_1. Thus, the probability of the hook walk being $(c_1, c_2, \ldots, c_k)$ differs from the probability of the hook walk being $(c_2, c_3, \ldots, c_k)$ merely in a single factor, which is the probability of choosing c_2 after c_1 (since the first cell is always chosen with uniform probability $1/n$); and this factor is $\dfrac{1}{h(c_1) - 1}$.

Page 555, proof of Theorem 12.80.
In the $\ell = 1$ case, after sliding w_k leftward, the proof says that "$u_{j-1} \le w_k < u_j$", and applies a Knuth move. This assumes $j > 1$. When $j = 1$, the symbol $u_{j-1} = u_0$ does not exist.

The $j = 1$ case is actually easier: after the preceding slides, one has already reached

$$u_1\, w_k\, u_2 \cdots u_{k-1},$$

which is exactly the reading word of the tableau obtained when w_k bumps u_1 into the second row. Thus one should simply separate $j = 1$ before invoking the displayed inequality.

Pages 555–557, Definition 12.82 and Theorem 12.84.
There is a small but real terminological imprecision in the definition of inc_k and dec_k. A "set of k disjoint increasing subsequences" is literally written as $\{I_1, \ldots, I_k\}$. If the word has length n, then for sufficiently large k no such set exists (and allowing the empty subsequence does not fix the literal set notation, since a set cannot contain several copies of $\varnothing$). Nevertheless inc_k and dec_k are defined for every $k \ge 1$, and Theorem 12.84 is stated for every such k.

A clean repair is to use a k-tuple $(I_1, \ldots, I_k)$ of pairwise disjoint subsequences, allowing empty subsequences; equivalently, maximize over collections of *at most* k pairwise disjoint nonempty subsequences. One should also adopt the usual zero-padding convention

$$\lambda_i = 0 \quad \text{for all } i > \ell(\lambda), \qquad \text{as well as } \lambda'_i = 0 \quad \text{for all } i > \ell(\lambda').$$

With these conventions, the formulas

$$\mathrm{inc}_k(w) = \lambda_1 + \cdots + \lambda_k, \qquad \mathrm{dec}_k(w) = \lambda'_1 + \cdots + \lambda'_k$$

make literal sense for all $k \ge 1$, and the proof is unchanged after padding the chosen families by empty subsequences when necessary.

Page 556, proof of Theorem 12.83.
In the exceptional case of the increasing-subsequence argument, the proof writes quantities such as

$$v_{j_r}, \quad v_{j_{r+1}}, \quad v_{k_s}, \quad v_{k_{s+1}}$$

to compare values immediately before and after the three affected positions. These symbols need not all exist: for example, an increasing subsequence may have no selected position before $i + 1$, or none after $i + 2$.

The construction of the replacement subsequences still works; the missing endpoint inequalities should simply be treated as vacuous. For instance, the inequality "$v_{j_r} \le x$" should be treated as vacuously true if v_{j_r} does not exist (i.e., if $r = 0$).

Page 557, proof of Theorem 12.85.
The proof concludes, "for all $k \ge 1$,"

$$\lambda_k = \mathrm{inc}_k(\mathbf{v}) - \mathrm{inc}_{k-1}(\mathbf{v}) = \mathrm{inc}_k(\mathbf{w}) - \mathrm{inc}_{k-1}(\mathbf{w}) = \mu_k.$$

But Definition 12.82 defines inc_k only for $k \ge 1$, so inc_0 is undefined when $k = 1$.

Add the natural convention

$$\mathrm{inc}_0(\mathbf{u}) = 0 \qquad (\text{and, if desired, } \mathrm{dec}_0(\mathbf{u}) = 0)$$

for every word $\mathbf{u}$, or handle the case $k = 1$ separately.

Pages 559–562, Section 12.14 on Quasisymmetric Polynomials.
In Definition 1.90, compositions of k were only defined for $k > 0$. When working with quasisymmetric functions, it is essentially unavoidable to extend this definition to the case $k = 0$. The definition is precisely the same; the only composition of 0 is, of course, the empty sequence (). Of course, this means that the Composition Rule 1.92 (a) does not hold for $k = 0$ (but this wasn't claimed either).

Likewise, it is worth clarifying that $[k-1] = \{1, 2, \ldots, k-1\}$ is to be understood as the empty set when $k = 0$ (just as it is when $k = 1$). The maps sub and comp for $k = 0$ are the obvious bijections between the 1-element sets $\mathrm{Comp}(0)$ and $\mathcal{P}([0-1]) = \mathcal{P}(\varnothing)$.

In Theorem 12.94, the claim $|\mathrm{Comp}(k)| = 2^{k-1}$ only holds for $k \geq 1$.

Page 561, proof of Theorem 12.97.
There are two small points worth making explicit in the proof.

First, in the sentence following (12.14), replace "the sum of all monomials whose exponent sequences are shifts of $\mathbf{x}^\alpha$" by "the sum of all monomials whose exponent sequences are shifts of α". The shift relation was defined for exponent sequences, whereas $\mathbf{x}^\alpha$ is a monomial.

Second, when the proof replaces the sum over subsets $T \subseteq [k-1]$ by a sum over $\alpha \in \mathrm{Comp}_N(k)$, it silently omits those T for which $\mathrm{comp}(T)$ has more than N parts. This omission is valid: such a T has at least N elements, and thus the corresponding inner sum

$$\sum_{I \in \mathcal{I}:\ T = \mathrm{Asc}(I)} \mathbf{x}_I$$

will be zero (since a weakly increasing sequence

$$I = (i_1 \leq \cdots \leq i_k) \qquad \text{with each } i_j \text{ in } [N]$$

can have at most $N-1$ strict increases, i.e., its ascent set $\mathrm{Asc}(I)$ must have size $\leq N-1$, so it cannot equal the set T which has at least N elements).

Page 563, proof of Theorem 12.99.
In the last paragraph of the proof, the sentence "Since $\mathrm{Des}(U) \subseteq \mathrm{Asc}(I)$, we see (as in the previous paragraph) that every run of equal values in I is used to relabel a horizontal strip of cells in $dg(\lambda)$" hides a non-completely-trivial argument worth spelling out.

Suppose

$$i_a = i_{a+1} = \cdots = i_b$$

is a maximal run of equal values in I. Then none of $a, a+1, \ldots, b-1$ lies in $\mathrm{Asc}(I)$. Since $\mathrm{Des}(U) \subseteq \mathrm{Asc}(I)$, none of these indices is a descent of U. Thus, as the labels $a, a+1, \ldots, b$ increase, their cells move weakly upward through the rows of U. In particular, two of these cells cannot lie in the same column, because entries of a standard tableau strictly increase down a column. Hence the cells form a horizontal strip.

Moreover, consecutive labels in this run occur strictly from left to right: the cell containing $j+1$ is weakly above the cell containing j, and it cannot be weakly to its left, since then it would lie weakly northwest of j while containing the larger entry. Consequently, standardizing $G(U, I)$ from left to right recovers U. This supplies the detail behind "as in the previous paragraph".

Page 564, Theorem 12.105.
The Pfaffian expansion is stated for every positive even N. At $N = 2$, however, $A[[1, 2]]$ is a 0×0 skew-symmetric matrix, while neither SPf_0 nor the Pfaffian of the empty matrix has been defined.

The standard convention

$$\mathrm{SPf}_0 = \{\text{empty permutation}\}, \qquad \mathrm{Pf}(\varnothing) = 1$$

makes the recursion valid at $N = 2$. Otherwise, state Theorem 12.105 for $N \geq 4$ and give $N = 2$ as the base case. The same convention is also needed for the recursive Pfaffian formula repeated in the Chapter 12 Summary on page 578.

Pages 567–568, proof of Theorem 12.112.
The claimed bijection between even-cycle permutations and pairs of perfect matchings is not literally correct with ordinary simple graphs.

For a 2-cycle (a, b), the two directed edges

$$a \longrightarrow b, \qquad b \longrightarrow a$$

become the *same* undirected edge $\{a, b\}$. Thus, after the proof says to view $G(w)$ as an undirected graph, "take all the edges not used in M_1" does not produce a second perfect matching on this component.

The inverse description has the same problem: the ordinary set-theoretic union of two perfect matchings need not be a disjoint union of even cycles when the matchings share an edge. Exercise 12-107(c), where $u = v$, is an explicit instance in which every matching edge is shared.

The repair is to retain the two matching *occurrences* (or colors). Equivalently, form a two-edge-colored multigraph from M_1 and M_2, counting an edge twice when it belongs to both. Every vertex then has one edge of each color, so every component is an alternating even cycle; a doubled edge is a 2-cycle. The orientation is recovered by declaring the M_1-colored edge at the minimum vertex to be the outgoing edge. With this interpretation, the intended bijection and the remainder of the proof work.

Page 574, Lemma 12.118.
Replace "$[U_k(1, b), U_k(2, b), \ldots, U_k(a, b)]^{\mathrm{tr}}$" by "$[U_k(1, b), U_k(2, b), \ldots, U_k(k, b)]^{\mathrm{tr}}$". (This should be a vector with k entries, as it is claimed to be an eigenvector of the $k \times k$ matrix Q_k.)

Page 578, Chapter 12 Summary: description of SPf_N.
The "Pfaffians" bullet point of the summary says that "$w \in \mathrm{SPf}_N$ iff $w \in S_N$, $w_i < w_{i+1}$, and $w_i < w_{i+2}$ for all odd i". This needs a bit of care: The condition $w_i < w_{i+1}$ should be imposed for all odd $i \in [N]$ (or, equivalently, all odd $i \in [N-1]$), whereas the condition $w_i < w_{i+2}$ should only be imposed for all odd $i \in [N-3]$ (since for the final odd index $i = N-1$, the symbol w_{i+2} does not exist).

Page 579, Exercise 12-14.
The displayed product is written over all $k \geq 1$:

$$\prod_{k \geq 1} \binom{a_{k-1} + a_k - 1}{a_{k-1} - 1, a_k}.$$

As written, its tail is undefined under the book's multinomial conventions. Since the sequence $(N-1-\mu_1, N-2-\mu_2, \ldots, 0-\mu_N)$ is finite, we have $a_k = 0$ for all large k; eventually the displayed factor becomes $\binom{-1}{-1,0}$.

The intended product is finite. For example, if

$$K = \max\{k : a_k > 0\},$$

one can replace the product by

$$\prod_{k=1}^{K} \binom{a_{k-1} + a_k - 1}{a_{k-1} - 1, a_k}.$$

These factors are all defined: writing $d_i = N - i - \mu_i$, we have $d_N = 0$ and $d_{i+1} - d_i = \mu_i - \mu_{i+1} - 1 \geq -1$, so if the value $k > 0$ occurs among the d_i, then the value $k - 1$ occurs as well. Hence $a_k > 0$ implies $a_{k-1} > 0$. (Equivalently, one can take the product only over $k \geq 1$ with $a_k > 0$.)

Page 580, Exercises 12-23 and 12-24.
Parking functions were defined in Definition 12.12 only for positive order. However, both recursions on this page use the order-zero quantities: the terms with $m = 1$ and $m = n$ contain p_0 in Exercise 12-23 and $P_0(q)$ in Exercise 12-24. The natural convention is that there is one empty parking function of order 0, so that

$$p_0 = 1 \qquad \text{and} \qquad P_0(q) = 1.$$

This convention should be stated before the two recursions; or, better, Definition 12.12 should be extended to $n = 0$ (no changes to the definition are needed aside of this).

Page 581, Exercise 12-31.
Replace "finite subgroups of the field $\mathbb{C}$" by "finite subgroups of the multiplicative group $\mathbb{C}^\times$". A field is not itself a group until an operation is specified, and the multiplicative interpretation is the one intended by the immediately preceding Theorem 12.29.

Page 582, Exercise 12-57.
In the two displays defining the q-tangent and q-secant numbers, the summation index is printed as k, while the summands use n:

$$\tan_q x = \sum_{k=0}^{\infty} \frac{t_n}{(q;q)_n} x^n; \qquad \sec_q x = \sum_{k=0}^{\infty} \frac{s_n}{(q;q)_n} x^n.$$

Thus n is unbound and k is unused. In both sums, replace the lower limit "$k = 0$" by "$n = 0$" (or, equivalently, replace every n in the summand by k).

Page 585, Exercise 12-98.
Replace "$\mathrm{FQ}_{n,S}$" by "$\mathrm{FQ}_{k,S}$".

Page 585, Exercise 12-99.
The f_y are not well-defined polynomials for an arbitrary surjection $G : \mathbb{Z}_{\geq 0}^N \longrightarrow Y$. Indeed, the exercise defines $f_y \in \mathbb{R}[x_1, \ldots, x_N]$ to be the sum of *all* monomials x^α with $G(\alpha) = y$. A fiber $G^{-1}(y)$ can be infinite, in which case this sum is not a polynomial. For example, if Y is a singleton and G is constant, then f_y would be the sum of every monomial in N variables.

A sufficient correction is to assume

$$|G^{-1}(y)| < \infty \qquad \text{for every } y \in Y.$$

Under this finite-fiber hypothesis, each f_y is a polynomial, the disjoint monomial supports give linear independence, and every G-symmetric polynomial is a finite linear combination of the f_y's. The maps underlying Theorems 9.23 and 12.94 do have finite fibers, so the intended special cases remain valid.

Alternatively, without any such assumptions, the f_y are still well-defined as formal power series in $\mathbb{R}[[x_1, \ldots, x_N]]$. However, they only form a topological basis (not a literal, linear-algebraic basis) of

the topological vector space of all G-symmetric power series; thus, the claim of Exercise 12-99 (b) does not literally hold for them.

Page 591, Appendix: algebra over a commutative ring.
"is a replaced" should be "is replaced".

Page 591, Appendix: definition of a graded algebra.
The appendix defines a graded algebra as

$$A = \bigoplus_{n \geq 0} A_n$$

where the A_n are *subalgebras*. This is incorrect. The A_n should be vector subspaces (or, in a more general base-ring setting, submodules) satisfying $A_n \bullet A_m \subseteq A_{n+m}$. The book's own example of homogeneous polynomials contradicts the printed definition, since homogeneous polynomials of a fixed positive degree are not closed under multiplication and do not contain 1.

Page 591, Appendix.
The formula

$$\Lambda = K[p_m : m \geq 0]$$

should read

$$\Lambda = K[p_m : m \geq 1].$$

Page 592, Appendix.
As on pages 398 and 417, arbitrary elements $c_1, \ldots, c_k$ of an arbitrary algebra B do not necessarily determine "a unique algebra homomorphism $E : F[z_1, \ldots, z_k] \longrightarrow B$". The c_i must commute pairwise, or B should be assumed commutative.

Later on the same page, the phrase

"If E consists of zero alone"

should read

"If $\ker E$ consists of zero alone."

Page 607, Index.
The entry

"connected graphs and OGFs, 499"

should read

"connected graphs and EGFs, 499."

Example 11.33 counts labeled graphs with the Exponential Formula and explicitly uses coefficients $z^n/n!$, hence an exponential generating function.

2 Places where supplementary explanation would help

The following are not necessarily errors, but they are points at which a self-learner or student may reasonably feel that a proof has skipped a nontrivial step, or where the wording leaves a convention implicit.

(DG: Not all of these comments are particularly useful or to-the-point, but I have removed only those that I have found definitely irrelevant. Brace yourself for some pedantism.)

Page 23, Definition 1.66.
The definition of independence of a collection of events says that the product formula holds "for all choices of indices $1 \le i_1 < \cdots < i_k \le n$," but k is not explicitly quantified. The intended statement is: for every $k \in \{1, \ldots, n\}$ and every choice $1 \le i_1 < \cdots < i_k \le n$, we have

$$P(A_{i_1} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \cdots P(A_{i_k}).$$

The meaning is standard, but making the quantifier explicit would be cleaner for beginners.

Page 47, Exercise 1-110(b).
With completely unrestricted yes/no questions, the definition of $K(n)$ actually gives

$$K(n) = \lceil \log_2(n!) \rceil$$

for *every* n: label the $n!$ permutations by distinct binary strings of this length and ask for successive bits (equivalently, ask membership in suitable subsets of the remaining candidates). Thus the restriction "for $n \le 5$" in part (b) is unnecessary. The restriction becomes meaningful in part (c), where only comparison questions "is $w_i < w_j$?" are allowed.

Page 61, proof of Theorem 2.18 (sums of powers).
Equation (2.2) is established only for $n \ge p$. Consequently, setting $p = 2$ proves the sum-of-squares formula only for $n \ge 2$, and setting $p = 3$ proves the sum-of-cubes formula only for $n \ge 3$. The theorem itself is stated for every $n > 0$.

The missing cases are harmless but should be checked separately: $n = 1$ for the sum of squares, and $n = 1, 2$ for the sum of cubes. In the general case, the easiest way to go is to argue that (2.2) holds not only for $n \ge p$ but also for all $n \ge 0$ if you rewrite the left hand side as $\binom{0}{p} + \binom{1}{p} + \cdots + \binom{n}{p}$ (because if $0 \le n < p$, then this equality boils down to $0 = 0$).

Pages 88–89, combinatorial proof of Theorem 2.60.
The theorem asserts a polynomial identity for every real x, but the combinatorial proof explicitly fixes an integer $x \ge 0$ and concludes after proving the identity only for those values. One final sentence should be added: for fixed n, both sides are polynomials in x, so equality for infinitely many nonnegative integers x implies equality for all real x, by Remark 2.7. The case $n = 0$ may be checked separately if desired.

Page 99, Exercise 2-67.
The statement that rhyme schemes using k different rhyme syllables are counted by $S(n, k)$ implicitly treats the rhyme classes as *unlabeled*. If the actual k syllables were regarded as prescribed/distinguishable labels, the count would instead be $k!S(n, k)$.

For students, it would be clearer to say that rhyme schemes are considered up to relabeling of the rhyme symbols, or equivalently that one is partitioning the n lines into k rhyme classes.

Page 125, Theorem 3.71.
The Tree Rule is stated for all $n \ge 1$, whereas the Rooted Tree Rule 3.47 used to deduce it is stated only for $n > 1$. The missing case $n = 1$ is of course immediate: there is exactly one tree on a one-element vertex set. It is worth saying this explicitly, rather than leaving the endpoint case to the formal expression $1^{-1} = 1$.

Page 144, proof of Theorem 3.120.
In the proof of existence of closed Eulerian tours, after constructing a maximal trail W, the text says that, if some edge e is not used by W, then "we can find such an edge that is incident to one of the vertices v_i visited by W". This is true, but not quite immediate in a digraph. One way to justify it is to take a directed path from a visited vertex to the tail of an unused edge and choose the first edge of this path that has not already appeared in W; its initial vertex is already visited. This fills a small logical gap in the maximality argument.

(*Remark:* We could replace "directed path" by "undirected path" in the above argument, and the argument would still work; this shows that Theorem 3.120 holds even if G is assumed to be weakly connected rather than connected.[1])

Page 212, tree bijection g.
The map $g : T \to U$ from full binary trees to ordered trees is announced as a bijection and its recursive construction is given, but the text checks only the weight relation. The inverse map is postponed to Exercise 5-87. If this section is used as the promised bijective proof of the Catalan equinumeracy, it would be better to describe the inverse at once. It has the simple recursion

$$g^{-1}((0)) = (\bullet, \varnothing, \varnothing),$$

and, for an ordered tree $u = (k, u_1, \ldots, u_k)$ with $k \geq 1$,

$$g^{-1}(u) = \big(\bullet, g^{-1}((k-1, u_1, \ldots, u_{k-1})), g^{-1}(u_k)\big).$$

(When $k = 1$, the first argument on the right is the ordered tree (0).) This directly shows that the displayed recursion for g is bijective.

Page 223, proof of Sylvester's bijection.
After solving (5.12) for the integers x_i, y_i, the text says that "we can fit the pieces together to recover the centered diagram of $\mu = G(\nu) \in \mathrm{OddPar}$" and then concludes bijectivity from uniqueness of the solution of (5.12). There is a small geometric verification hidden here: one should check that the reconstructed pieces really do form a centered Ferrers diagram, and hence that $F(G(\nu)) = \nu$.

Here is one way to make this explicit. Number the rows $1, 2, \ldots$ from top to bottom, and number the centered columns by the integers $\ldots, -2, -1, 0, 1, 2, \ldots$. For $i \geq 1$, define the prospective $(2i-1)$st and $2i$th pieces by

$$P_{2i-1} = \{(r, i-1) : i \leq r < i + y_{i-1}\} \sqcup \{(i, c) : i \leq c < i + x_i\},$$

and

$$P_{2i} = \{(i, c) : -i - x_i < c \leq -i\} \sqcup \{(r, -i) : i < r \leq i + y_i\}.$$

Thus P_{2i-1} has a vertical leg of y_{i-1} cells and a horizontal extension of x_i cells, whereas P_{2i} has a horizontal leg of x_i cells and a vertical extension of y_i cells. By (5.12),

$$|P_{2i-1}| = y_{i-1} + x_i = \nu_{2i-1}, \qquad |P_{2i}| = x_i + y_i = \nu_{2i}.$$

The alternating-sum formulas for the x_i and y_i also show that all these numbers are nonnegative: for example,

$$x_i = (\nu_{2i} - \nu_{2i+1}) + (\nu_{2i+2} - \nu_{2i+3}) + \cdots,$$

[1] That said, it is not hard to show (independently of Theorem 3.120) that a weakly connected balanced digraph is automatically strongly connected.

and similarly for y_i.

We claim that

$$D = \bigcup_{j \geq 1} P_j$$

is a centered Ferrers diagram. For a fixed row r and a nonnegative column c, the preceding definitions give

$$(r, c) \in D \quad \Longleftrightarrow \quad \begin{cases} r - c \leq y_c, & 0 \leq c < r, \\ c - r + 1 \leq x_r, & c \geq r. \end{cases} \tag{*}$$

For $c > 0$, the mirror cell $(r, -c)$ satisfies the same criterion. Hence every row is symmetric about column 0.

The rows have no gaps (i.e., each row is a contiguous block of cells). To prove this, we fix a row r. Suppose first that $0 \leq d < c < r$ and that $(r, c) \in D$. Then $r - c \leq y_c$, so $y_c > 0$. Repeated use of

$$y_{j-1} > y_j \qquad \text{whenever } y_{j-1} > 0$$

gives

$$y_d \geq y_c + (c - d) \geq r - d \qquad (\text{since } y_c \geq r - c)\,,$$

and therefore $(r, d) \in D$. Thus the cells of D in row r and columns $0, 1, \ldots, r - 1$, if there are any, form an initial segment $0, 1, \ldots, t$. Together with their mirror cells, they form a contiguous block about column 0; we call this the *central block*. On the other hand, the cells with $c \geq r$ form, by (*) alone, the consecutive block

$$r, r + 1, \ldots, r + x_r - 1.$$

If this block is nonempty, then $x_r > 0$, so that $\nu_{2r} > 0$ (since otherwise, all ν_i with $i \geq 2r$ would be 0, and thus their alternating sum x_r would be 0 as well), therefore $\nu_{2r-1} > \nu_{2r}$ and thus $\nu_{2r-1} - \nu_{2r} > 0$; consequently

$$y_{r-1} - y_r = \nu_{2r-1} - \nu_{2r} > 0,$$

whence $y_{r-1} > 0$, so we have $(r, r - 1) \in D$, and this block joins the central block. By symmetry, the same holds on the left side of column 0.

The rows are weakly decreasing in length. To prove this, it is enough, by symmetry, to show that whenever $(r + 1, c) \in D$ with $c \geq 0$, then $(r, c) \in D$. We prove this case-by-case depending on whether $c < r$ or $c > r$ or $c = r$:

- If $c < r$, this follows immediately from

$$r + 1 - c \leq y_c \quad \Longrightarrow \quad r - c \leq y_c.$$

- If $c > r$, then $c \geq r + 1$, so that $(r + 1, c) \in D$ yields $c - r \leq x_{r+1}$. Since $c - r \geq 1$, this implies $x_{r+1} > 0$, and hence $\nu_{2r+2} > 0$ (otherwise all parts ν_j with $j \geq 2r + 2$ would vanish, forcing $x_{r+1} = 0$). Thus $\nu_{2r+1} > 0$ as well, so the distinctness of the positive parts of ν gives $\nu_{2r} > \nu_{2r+1}$. From the alternating-sum formulas,

$$x_r - x_{r+1} = \nu_{2r} - \nu_{2r+1} > 0.$$

  Therefore $x_r \geq x_{r+1} + 1$; in conjunction with $c - r \leq x_{r+1}$, this yields

$$c - r + 1 \leq x_r,$$

  so that $(r, c) \in D$.

- Finally, if $c = r$, then $(r+1, r) \in D$ means $y_r > 0$. Hence $\nu_{2r+1} > 0$ (otherwise all later parts of ν would vanish and y_r would be 0), so $\nu_{2r} > \nu_{2r+1}$ by distinctness of the positive parts of ν. Using again
$$x_r - x_{r+1} = \nu_{2r} - \nu_{2r+1} > 0$$
and $x_{r+1} \geq 0$, we get $x_r > 0$, and hence $(r, r) \in D$.

Thus every cell of D (except those in the first row) has a cell directly above it. In other words, the rows of D are weakly decreasing in length.

Since $x_i = y_i = 0$ for all sufficiently large i, the set D is finite. We have therefore shown that every nonempty row of D is an interval symmetric about column 0, and that the rows weakly decrease in length. Thus D is the centered Ferrers diagram of a partition $G(\nu)$ into odd parts.

Moreover, the sets P_j are pairwise disjoint: on the right half of the diagram a cell (r, c), $c \geq 0$, belongs to the vertical part of P_{2c+1} when $c < r$, and to the horizontal part of P_{2r-1} when $c \geq r$; the left half is analogous. Consequently the forward cutting procedure defining F removes exactly $P_1, P_2, P_3, \ldots$ in this order. Their sizes are $\nu_1, \nu_2, \nu_3, \ldots$, so
$$F(G(\nu)) = \nu.$$

This is the surjectivity part. Conversely, if $\nu = F(\mu)$, then the x_i, y_i coming from μ solve (5.12); uniqueness of that solution gives $G(F(\mu)) = \mu$, which is the injectivity part. Thus the final sentence of the proof is correct in spirit, but the existence/reassembly argument needed for surjectivity is worth making explicit.

Chapter 5, Theorem 5.43.
The derivation solves a formal-power-series differential equation by dividing by G, taking log, integrating, and exponentiating. The book explicitly postpones formal justification of these operations to Chapter 11 and then independently verifies the resulting answer. In a course, it would be useful to say explicitly that this is a heuristic derivation followed by a rigorous verification.

Page 235, Exercise 5-94(c).
If paths are weighted only by their final horizontal coordinate c, then already the weight-0 fiber contains infinitely many vertical-only paths. Hence this is not a weighted set under the definition in §5.1, and $\mathrm{GF}(S; z)$ is not defined as an ordinary generating function in the chapter's sense. If this is intended as a trick question, it would be useful to signal that possibility; otherwise part (c) needs a different weight or a second variable.

Page 236, Exercise 5-108(a).
In the first case of the proposed involution, one prepends a new part and then decrements all parts by 1. This can create zero parts; for example, $j = 0$ and $\lambda = (1)$ produce zeros. Since the book defines integer partitions using positive parts, the construction should explicitly say to delete any resulting zero parts.

Chapter 6, algorithm conventions.
Several of the pseudocode routines tacitly treat array/list arguments as local copies. For example, `first` and `last` in Figure 6.6 decrement an entry of the multiplicity array before making a recursive call. In an actual implementation with pass-by-reference arrays, one must either copy the array or restore the entry afterward. This is not a mathematical error, but it is worth stating if students are expected to turn the pseudocode into working code.

Chapter 6, poker ranking exercises.
Exercises 6-82 through 6-111 repeatedly ask the student first to "create" ranking, unranking, or successor maps for a class of poker hands and then to compute numerical ranks or successors using the map just created. These numerical answers are not canonical: different perfectly valid choices in the construction can induce different rankings. For grading purposes, it is useful either to specify the intended order of the component choices or to accept any answer consistent with the student's preceding construction.

Page 280, Theorem 7.10.
The Laws of Exponents for arbitrary integer exponents are stated without proof; the text only sketches the idea and sends the complete proof to Exercise 7-10. Since these laws are used repeatedly in the subgroup, homomorphism, coset, and orbit sections, I would regard Exercise 7-10 as part of the main development if the chapter is being used to introduce group theory from scratch. The negative-exponent cases are exactly where students are most likely to make an order-of-factors mistake in a noncommutative group.

Page 295, after Definition 7.57.
The text says that one "readily verifies" that

$$\ker(f) \trianglelefteq G \qquad \text{and} \qquad \text{img}(f) \leq H.$$

For students seeing groups for the first time, the normality assertion is worth proving in the text, since it is used later in the discussion of cosets of a kernel. The missing calculation is short: if $k \in \ker(f)$ and $g \in G$, then

$$f(gkg^{-1}) = f(g)f(k)f(g)^{-1} = f(g)e_H f(g)^{-1} = f(g)f(g)^{-1} = e_H,$$

so $gkg^{-1} \in \ker(f)$.

Page 309, proof of Theorem 7.115.
The centralizer-counting argument is correct, but one bijective step is compressed. After fixing a complete cycle notation for f, the proof says that it is enough to count all complete cycle notations for f (with longer cycles preceding shorter cycles). What is being used is a bijection:

$$g \in C_{S_n}(f) \longleftrightarrow \text{the complete cycle notation obtained by applying } g \text{ to the fixed notation.}$$

Conversely, a second such notation determines g uniquely by matching its entries with the entries of the fixed notation. Making this correspondence explicit removes a possible overcounting concern before the factors $i^{a_i}a_i!$ are introduced.

Page 310, proof of Cauchy's Theorem 7.117.
At the end of the proof, a nonidentity element x is found with $x^p = e$, and the text immediately says that x has order p. One small step is implicit: by the cyclic-group analysis in Example 7.60, the order of x divides every positive exponent m for which $x^m = e$; hence it divides the prime p. Since $x \neq e$, its order is not 1, and therefore it is p, since the prime number p has no positive divisors other than 1 and p.

Page 329, Example 8.7.
The phrase "weight a partition by the length of its largest part" is awkward and potentially confusing: the largest part is an integer, so has no "length"; the statistic intended is simply the *largest part* μ_1 (equivalently, the length of the first row of the Ferrers diagram). This is the statistic equidistributed with the number of parts by conjugation of partitions.

Page 330, The Weight-Shifting Rule 8.12.
The rule allows the shift b to be an arbitrary integer. If $b < 0$, the factor q^b is a Laurent monomial rather than a polynomial, even though the two generating functions themselves are polynomials. Nothing is wrong with the identity, but for complete algebraic precision one should either work temporarily in

$$\mathbb{Z}[q, q^{-1}]$$

or state the rule in the orientation in which the displayed exponent is nonnegative.

Pages 342–343, proof of the q-Binomial Theorem 8.37.
After proving the case $x = 1$, the text obtains the general case by replacing y by y/x and then multiplying by x^n. Since x was introduced as a formal variable, this substitution implicitly passes from a polynomial ring to a Laurent polynomial ring (or a field of rational functions).

A completely rigorous version is to say explicitly that the identity is first interpreted in the ring

$$\mathbb{Z}[q, x, x^{-1}, y]$$

and then multiplied by x^n; the resulting identity lies in $\mathbb{Z}[q, x, y]$. Alternatively, one can avoid division by homogenizing the already proved $x = 1$ identity term by term.

Page 346, Theorem 8.43.
The theorem allows $n_1, \ldots, n_s \geq 0$, so in particular it includes the case $n_1 + \cdots + n_s = 0$, where the anagram class consists only of the empty word. The final sentence should therefore say that the Foata bijection preserves the last letter of each *nonempty* word. (The recursive construction itself already makes exactly this qualification.)

Pages 349–350, inverse in the proof of Theorem 8.47.
The tipping map used for the major-index q-Catalan formula is a central bijection, but the last inverse check (that g and g' are mutually inverse) is compressed into "It can be checked." For course use, it is worth spelling out why the chosen vertex is recovered.

Here is a very rough outline: For the inverse path, choose the last point on the lowest line $y = x - k$ touched by the path. If this point is not the origin, it is entered by an east step; maximality of k forces the next step to be north, and "last point" forces the following step to be north as well (otherwise an east step would return to the same lowest line). This is exactly the local configuration created by the forward tipping operation. The analogous observation in the other direction shows that the two maps really are inverses.

Page 375, Example 9.47.
"Here is a simpler exampler of tableau insertion" should read "Here is a simpler example of tableau insertion."

Pages 382–383, Theorems 9.59 and 9.60.
A containment hypothesis is missing. In Theorem 9.59 one must assume

$$\mathrm{dg}(\mu) \subseteq \mathrm{dg}(\nu)$$

in addition to requiring that $\mathrm{dg}(\nu) - \mathrm{dg}(\mu)$ be the indicated strip. Likewise, the definitions of $H_k(\mu)$ and $V_k(\mu)$ in Theorem 9.60 should include this containment condition.

Without it, ordinary set difference is not enough. For example, take $\mu = (2)$ and $\nu = (1, 1)$. Then

$$\mathrm{dg}(\nu) - \mathrm{dg}(\mu) = \{(2, 1)\},$$

which is both a horizontal and a vertical strip of size 1, although $|\mu| = |\nu| = 2$. Thus ν cannot possibly be obtained from μ by one tableau insertion (which adds one box), and the literal definitions in Theorem 9.60 would even introduce a Schur polynomial of the wrong degree into the Pieri sums. The proofs themselves use the intended stronger condition throughout: the strip consists of boxes *added to* $\mathrm{dg}(\mu)$.

Chapter 9, Theorems 9.52–9.54.
Theorem 9.52(c), asserting that reverse insertion preserves semistandardness, is left as similar to a previous argument. Theorem 9.53, asserting that reverse insertion genuinely reverses insertion, is likewise left to the reader; Theorem 9.54 immediately uses these facts to obtain the insertion bijection. If RSK or the Pieri rules are central to the course, it would be worth proving Theorem 9.53 in class rather than treating it as an optional check. Exercises 9-37 and 9-38 explicitly supply these missing proofs.

Chapter 9, Theorem 9.79.
The power-sum basis theorem is stated without a proof. A short proof is available: Algebraic independence of $p_1, \ldots, p_N$ gives linear independence of the relevant monomials $p_1^{i_1} p_2^{i_2} \cdots p_N^{i_N}$ (with $i_1, i_2, \ldots, i_N \in \mathbb{Z}_{\geq 0}$), while conjugation identifies partitions whose largest part is at most N with partitions having at most N parts, giving the required dimension count.

Chapter 9, Theorem 9.91.
The rather intricate bijection proving the power-sum expansion of h_n defines explicit maps $f : Y \to X$ and $g : X \to Y$, but the assertion that they are two-sided inverses is left to the reader (and then assigned as Exercise 9-85). Since invertibility is the central point of the combinatorial proof, I would regard Exercise 9-85 as part of the proof if this theorem is taught.

Chapter 9, Theorems 9.105–9.106.
The higher-order shadow theorem is proved fully only for the first new level $r = 2$; the general case is summarized by "Iterating this argument." This is believable and standard, but Theorem 9.106 (the symmetry $\mathrm{RSK}(w^{-1}) = (Q(w), P(w))$) immediately relies on all higher-order rows. For a self-contained course proof, it would be useful to formulate the induction from order $r-1$ to order r explicitly.

Chapter 9, Theorems 9.135–9.136.
The symmetry/monomial expansion of skew Schur polynomials and the skew Pieri rules are stated with essentially no proof: the text says that the earlier proofs extend, and Exercises 9-136 and 9-137 ask the reader to supply the details. The assertions are standard, but if skew Schur functions are part of the course rather than optional material, these exercises should be treated as required proof steps.

Chapter 9, Theorem 9.120 and the formal Cauchy identity.
The matrix-RSK proof of the Schur Cauchy identity is naturally an identity of formal power series. The text makes the ambient formal power series ring explicit only at the beginning of the following section. A rigorous reading is coefficientwise: for every fixed monomial, only finitely many matrices/tableaux contribute. Stating this at Theorem 9.120 would prevent the infinite generating-function argument from looking analytic.

Page 412, end of the proof of Theorem 9.121.
The proof obtains an identity in an additional formal variable t and then says "Setting $t = 1$ gives the final formula." Substitution $t \mapsto 1$ is not defined for an arbitrary formal power series in t.

Here it is legitimate degreewise: the coefficient of t^n is homogeneous of bidegree (n,n) in the x- and y-variables, so each fixed x,y-monomial receives a contribution from only one value of n. A sentence to this effect would make the formal justification explicit.

Chapter 10, Theorem 10.5 and formal infinite products.
The Jacobi Triple Product Identity is used as a formal identity involving both positive and negative powers of u. A precise ambient ring is, for example,

$$\mathbb{Z}[u,u^{-1}][[q]],$$

where every coefficient of q^n is a Laurent polynomial in u. The infinite products are meaningful coefficientwise because only finitely many factors can affect a given q-degree. Chapter 11 later develops convergence of formal infinite products, but one sentence here identifying the ambient ring would make the status of the calculation clear.

That said, Exercise 10-6 touches on an even subtler foundational issue. To "deduce Euler's Pentagonal Number Theorem as an algebraic consequence of the Jacobi Triple Product Identity", one wants to make the substitutions

$$q \longmapsto z^3 \qquad \text{and} \qquad u \longmapsto -z^{-2}.$$

There is no substitution homomorphism from $\mathbb{Z}[u,u^{-1}][[q]]$ to $\mathbb{Z}[[z]]$ that does this: in particular, $-z^{-2}$ is not even an element of $\mathbb{Z}[[z]]$. Merely enlarging the target to Laurent series does not solve the problem on the whole source ring either; for example, $\sum_{n\geq 0} u^{2n}q^n$ would be sent formally to $\sum_{n\geq 0} z^{-n}$.

One convenient rigorous repair is to restrict to the following subring R of $\mathbb{Z}[u,u^{-1}][[q]]$. Each series in $\mathbb{Z}[u,u^{-1}][[q]]$ can be written uniquely as

$$\sum_{n\geq 0}\sum_{m\in\mathbb{Z}} c_{n,m}q^n u^m,$$

where for each fixed n only finitely many $c_{n,m}$ are nonzero. Let R consist of those series

$$\sum_{n\geq 0}\sum_{m\in\mathbb{Z}} c_{n,m}q^n u^m$$

whose support satisfies

$$3n-2m\geq 0$$

(that is, which satisfy $c_{n,m}=0$ whenever $3n-2m<0$) and which are locally finite with respect to this quantity: for each $D\geq 0$, only finitely many pairs (n,m) with $3n-2m\leq D$ satisfy $c_{n,m}\neq 0$. Then

$$\phi : R \longrightarrow \mathbb{Z}[[z]], \qquad \phi(q^n u^m) = (-1)^m z^{3n-2m},$$

is a well-defined ring homomorphism. Both sides of the Jacobi Triple Product Identity lie in R. On its left side, the term indexed by $m\in\mathbb{Z}$ has $3n-2m=(3m^2-m)/2\geq 0$; on its right side, the nonconstant terms in the three kinds of factors have respective z-degrees $3n-2$, $3n+2$, and $3n$, which are positive and tend to infinity. Applying ϕ therefore gives

$$\sum_{m\in\mathbb{Z}}(-1)^m z^{(3m^2-m)/2} = \prod_{n\geq 1}(1-z^{3n-2})\prod_{n\geq 0}(1-z^{3n+2})\prod_{n\geq 1}(1-z^{3n}) = \prod_{n\geq 1}(1-z^n),$$

which is Euler's Pentagonal Number Theorem.

The same substitution issue recurs in Exercise 10-8. To apply Theorem 10.5 to the Rogers–Ramanujan product there, the useful specialization is

$$q \longmapsto x^5, \qquad u \longmapsto -x^{-4}.$$

It gives

$$\sum_{m\in\mathbb{Z}} (-1)^m x^{(5m^2-3m)/2} = \prod_{n\geq 1}(1-x^{5n-4}) \prod_{n\geq 0}(1-x^{5n+4}) \prod_{n\geq 1}(1-x^{5n}),$$

and hence

$$\prod_{n\geq 0} \frac{1}{(1-x^{5n+1})(1-x^{5n+4})} = \frac{\prod_{n\geq 1}(1-x^{5n})}{\sum_{m\in\mathbb{Z}}(-1)^m x^{(5m^2-3m)/2}}.$$

Again, the substitution is not defined on all of $\mathbb{Z}[u,u^{-1}][[q]]$; it is justified by restricting to the analogous locally finite subring (now graded by $5n-4m$).

Chapter 10, Theorem 10.16.
The uniqueness proof for k-cores rests on the sentence that the justified k-runner abacus "does not depend on the order in which individual bead moves are made." This is true, but it is the key confluence assertion and deserves a brief justification: on each runner, justification simply packs the same beads into the unique consecutive set of positions having the same eventual justification position. Motions on different runners are independent.

Chapter 10, proof of Theorem 10.22.
In the inverse construction, after replacing each justified runner w^i by $v^i = U(m_i, \nu^i)$, the text defines μ to be the unique partition μ satisfying

$$J(I_k(v^0,\dots,v^{k-1})) = (-1,\mu).$$

The fact that such a μ exists – i.e., that the first coordinate of $J(I_k(v^0,\dots,v^{k-1}))$ is -1 – is tacitly taken for granted here. This fact is true, but is worth justifying. For an abacus w, define its charge by

$$\mathrm{ch}(w) = \#\{j \geq 0 : w_j = 1\} - \#\{j < 0 : w_j = 0\}.$$

If $J(w) = (m,\lambda)$, then $\mathrm{ch}(w) = m+1$: bead moves used in justification do not change this quantity, and an abacus justified at position m has charge $m+1$. Moreover, decimation partitions the positions according to their residue classes, with nonnegative (respectively negative) positions corresponding exactly to nonnegative (respectively negative) runner indices. Hence

$$\mathrm{ch}\big(I_k(a^0,\dots,a^{k-1})\big) = \sum_{i=0}^{k-1} \mathrm{ch}(a^i).$$

Now $J(w^i) = (m_i, 0)$ and $J(v^i) = (m_i, \nu^i)$, so $\mathrm{ch}(v^i) = m_i + 1 = \mathrm{ch}(w^i)$ for every i. Therefore

$$\mathrm{ch}\big(I_k(v^0,\dots,v^{k-1})\big) = \sum_{i=0}^{k-1} \mathrm{ch}(v^i) = \sum_{i=0}^{k-1} \mathrm{ch}(w^i) = \mathrm{ch}\big(I_k(w^0,\dots,w^{k-1})\big) = \mathrm{ch}(U(-1,\rho)) = 0.$$

Thus the first coordinate of $J(I_k(v^0,\dots,v^{k-1}))$ is indeed -1.

Chapter 10, proof of Theorem 10.60.
The Gessel–Viennot cancellation is correct, but the sentence "It can be checked" hides the delicate part of the involution: after switching the two initial path segments, the same chosen earliest intersection point and the same pair of least path indices must again be selected. Exercise 10-51 is particularly instructive here, since it shows that a seemingly natural alternative rule for choosing the intersecting pair need not be an involution. I would make this verification explicit if the proof is used in class.

Chapter 10, later involution arguments.
In the h_k-Pieri proof, in the abacus/tableau involution of §10.13, and especially in the Littlewood–Richardson proof, several substantial checks are compressed into phrases such as "one may check" or "one may verify": preservation of semistandardness, preservation of the first collision, weight preservation, and involutivity. The examples help, but for a course it would be useful to work through at least one of these involutions in full generality rather than relying only on examples.

Chapter 11, Definition 11.4 / Theorem 11.5.
Continuity is defined for maps with codomain $K[[z]]$, but coefficient extraction

$$F \longmapsto [z^n]F$$

has codomain K and is then called continuous. One can regard K as the constant series inside $K[[z]]$, or extend the definition in the evident way.

Chapter 11, Theorems 11.24–11.25.
Two central formal-calculus proofs depend on results deferred to the exercises. The proof of associativity of formal composition in Theorem 11.24(c) uses the uniqueness of the continuous evaluation homomorphism R_G, which is only proved in Exercise 11-48. Likewise, the proof of the Formal Chain Rule in Theorem 11.25(e) concludes that two continuous linear maps agreeing on all monomials agree on all of $K[[z]]$, with this density/continuity argument deferred to Exercise 11-51.

Both arguments are sound once those exercises are supplied, but if Chapter 11 is meant to provide the rigorous foundation missing from Chapter 5, I would either prove these two short lemmas before using them or make Exercises 11-48 and 11-51 explicitly part of the main development.

Chapter 11, Theorem 11.48.
The "weights" used for terms in the combinatorial compositional-inversion formula are

$$r_{w_1} \cdots r_{w_s} z^s,$$

which are elements of $K[[z]]$, rather than the nonnegative-integer weights used in the earlier definition of a weighted set. The intended generalized generating-function argument is sound, but it would help to say explicitly that the Infinite Sum/Product Rules are now being used coefficientwise with algebra-valued weights.

In particular, the sum over all terms is well-defined because there are only finitely many terms of a fixed length; and the class of terms beginning with n has z-degree at least $n+1$, which supplies the local-finiteness needed for the infinite sum over n.

Chapter 11, Exercise 11-72.
This exercise is likely misleading in the context of the Exponential Formula. It asks for "connected simple digraphs" and explicitly points to Definition 3.50. Under that definition, a digraph is connected only when every vertex can reach every other vertex by a directed walk—i.e., the book's "connected" means *strongly connected*.

Strong components do *not* decompose a general digraph as a disjoint union with no edges between components; the book itself notes in Chapter 3 that directed edges may run between distinct strong components. Consequently, the direct analogue of Example 11.33, obtained by taking the logarithm of the EGF for all simple digraphs, counts digraphs whose *underlying undirected graph* is connected, not strongly connected digraphs.

Thus, if Exercise 11-72 is intended as the immediate digraph analogue of Exercise 11-71 using the Exponential Formula, "connected" should be replaced by "weakly connected" (or "having connected underlying undirected graph").

If strong connectivity is genuinely intended, then a recursive enumeration was given by Wright; see Section 2.2 of arXiv:1909.01550v3. The ordinary Exponential Formula does not suffice here. Allowing loops simply multiplies the n-vertex count by 2^n, since the n possible loops may be chosen independently and do not affect strong connectivity.

Thus Exercise 11-72 can be retained with strong connectivity only at the cost of introducing substantial machinery not developed in the text. If it is intended as an immediate application of the Exponential Formula, "connected" should instead be replaced by "weakly connected".

Chapter 12, proof of the Chung–Feller bijection.
After giving a fairly intricate inverse construction ϕ'_k, the proof concludes only that "it can be checked" that ϕ'_k and ϕ_k are two-sided inverses. Since the switching rule is the heart of this proof, I would work through the inverse verification explicitly in class: one needs to show that the reconstructed special point and the successive switching locations are exactly the ones used by the forward map.

Chapter 12, parking-functions-to-trees bijection.
The inverse map from trees to labeled Dyck paths receives a substantial well-definedness argument, but the final assertion that it is the two-sided inverse of the forward map is dismissed as "routine." If this is used as the promised bijective proof of Cayley's tree formula, it is worth spelling out why the parent of each vertex is recovered in both directions.

Chapter 12, Theorems 12.83–12.84.
These two results contain a significant amount of Greene/Schensted-type content, but only the increasing-subsequence half is substantially proved. Theorem 12.83 sends the entire decreasing-subsequence assertion to the reader, and Theorem 12.84 ends with "The proof for $\mathrm{dec}_k(w)$ is similar." Exercise 12-84 explicitly asks the reader to complete both proofs. If the subsequence theorem is part of the course, I would treat that exercise as part of the main text rather than optional practice.

3 Conventions and terminology worth flagging

Fibonacci indexing.
The Introduction uses the sequence $1, 1, 2, 3, 5, \ldots$ with

$$F_0 = F_1 = 1,$$

whereas Example 2.67 later uses the more standard convention

$$F_0 = 0, \qquad F_1 = 1.$$

Students may want an explicit warning that two indexing conventions occur.

Chapter 3: connected digraphs.
For digraphs, the book uses "connected" to mean what is more commonly called *strongly connected*: for every ordered pair u, v, there must be a directed walk from u to v. This is important when comparing Theorem 3.120 with standard Euler-tour criteria, some of which use weak connectedness of the underlying undirected graph.

Chapter 3: rooted trees.
A "rooted tree" in Chapter 3 is a functional digraph whose unique cyclic vertex is the root; in particular, the root carries a loop and all other directed edges point toward it. This differs from the common convention of an undirected tree with a distinguished root, or an arborescence with no root loop.

Chapter 3: Eulerian tours.
Definition 3.117 requires an Eulerian tour to *visit every vertex* as well as use every edge exactly once. Some (most?) graph-theory texts ignore isolated vertices in the definition, so this convention matters when isolated vertices are present.

Chapter 7: left and right cosets.
The terminology in §§7.11 is standard but easy to reverse on a first reading:

$$xH \quad \text{is a left coset,} \qquad Hx \quad \text{is a right coset.}$$

The book derives Hx as an orbit under *left* multiplication by H, and xH as an orbit under a *right* action. It is worth emphasizing that the adjectives "left" and "right" refer to the side on which the subgroup H is written in the coset, not to the side on which the action itself is performed.

Chapter 8: "Stirling permutations".
The term "Stirling permutation of type (n, k)" is defined here for the permutations $f(P)$ obtained by encoding set partitions P. Students consulting other combinatorics sources should be warned that "Stirling permutation" is also widely used for a different standard object (a special permutation of the multiset $\{1, 1, 2, 2, \ldots, n, n\}$). Loehr's definition is unambiguous inside the chapter, but the terminology is not portable without explanation.

Chapter 10: "Pieri Rule for p_k".
The formula called the "Pieri Rule for p_k"

$$s_\lambda p_k = \sum_{\beta/\lambda \text{ a } k\text{-ribbon}} (-1)^{\mathrm{spin}(\beta/\lambda)} s_\beta$$

is more commonly known as the Murnaghan–Nakayama rule. Mentioning this makes it easier for students to recognize the result in other sources.

Chapter 10: the quantities χ^λ_μ.
The signed rim-hook sums denoted χ^λ_μ are not identified in the text with the irreducible character values of the symmetric group. This is not needed for the chapter's proofs, but it is useful context.

Chapter 10: zero parts in rim-hook tableaux.
The chapter allows rim-hook tableau types containing 0 (for example $(8, 7, 6, 1, 0)$), although a 0-ribbon was not explicitly defined. The intended convention appears to be an empty step with sign $+1$.

Chapter 10: Littlewood–Richardson reading convention.
The book reads tableau rows from bottom to top and formulates the lattice condition using suffixes of the reading word. This is equivalent to more standard Littlewood–Richardson conventions after translating conventions, but students comparing with other texts may initially think that a different rule is being used.

Section 9.28: coefficient field.
The presentation of the abstract symmetric-function ring uses

$$\Lambda = K[p_1, p_2, \ldots]$$

over a characteristic-zero field K. Students should not infer that the power sums are polynomial generators over $\mathbb{Z}$ or over an arbitrary coefficient ring.