Darij’s list of errata and comments

• Page 232: “counably” → “countably”.

• Page 232: “Frobenious” → “Frobenius”.

• Page 234, (6.1.10): Replace “$Q_{n-2}$” by “$Q_{n-2} (Z)$”.

• Page 235: In the second displayed equality on this page, replace “$\sum_{r+s=m} Z_r \otimes Z_r$” by “$\sum_{r+s=m} Z_r \otimes Z_s$”.

• Page 237, (6.3.9): Add an equality sign after “$\beta \times_{osh} \gamma$”.

• Page 237: It would be better to replace “the RHS of (6.3.8) has a contribution 1 for each pair $(\alpha, \beta \otimes \gamma)$ such that $\alpha$ is a column sum of a matrix in $M_{\beta, \gamma}$” by “the RHS of (6.3.8) has a contribution 1 from each matrix in $M_{\beta, \gamma}$ having column sum $\alpha$”.

• Page 238: On the first line of this page, replace “$[r_1, \ldots, r_m]$” by “$[r_1, \ldots, r_m]$”. Similarly, on the second line of this page, replace “$[s_1, \ldots, s_m]$” by “$[s_1, \ldots, s_m]$”.

• Page 241, Definition 6.5.3: Replace “$\alpha'' = [a_i, a_{i+1}, \ldots, a_m]$” by “$\alpha'' = [a_i, a_{i+1}, \ldots, a_m]$”.

• Page 241, Definition 6.5.3: Replace “A word is Lyndon iff it is” by “A word is Lyndon if it is nonempty and”. (Otherwise, the empty word would be Lyndon, which would break uniqueness of CFL factorization.)

• Page 242, Theorem 6.5.5: The word “nonincreasing” is very confusing here: it suggests (falsely) that each of the $\lambda_1, \lambda_2, \ldots, \lambda_s$ itself is nonincreasing, whereas it is supposed to mean that the sequence $(\lambda_1, \lambda_2, \ldots, \lambda_s)$ is nonincreasing.

• Page 242, proof of Theorem 6.5.5: Replace “for some prefix $\beta$” by “for some nonempty prefix $\beta$”. (Otherwise, $\lambda_1 <_{lex} \beta$ would be false.)

• Page 243, footnote 13: You don’t prove the claim of this footnote, but you later use it in the proof of Lemma 6.7.15.
• Page 243, proof of Theorem 6.5.8, Case A: Replace “The element $a_{i,1}$” by “The element $a_{1,1}$”.

• Page 244, proof of Theorem 6.5.8, Case A2: I think the condition for this case should be “There exists some $i \in \{1, 2, \ldots, \min\{n_1, n_2\}\}$ such that $a_{1,i} > a_{2,i}$ and $(a_{1,j} = a_{2,j}$ for all $j = 1, 2, \ldots, i - 1$)” rather than “[$a_{1,1}, \ldots, a_{1,n_1}] \gg_{\text{lex}} [a_{2,1}, \ldots, a_{2,n_2}]$ and $n_1 \leq n_2$.” Otherwise, for example, the situation when $\alpha_1 = 132$ and $\alpha_2 = 12$ does not fit in any of the Cases A1, A2 and A3.

• Page 244, proof of Theorem 6.5.8, Case A3: Replace “the equal prefixes $[a_{1,1}, \ldots, a_{1,n}] = [a_{2,1}, \ldots, a_{2,n_2}]$” by “the equal prefixes $[a_{1,1}, \ldots, a_{1,n_2}] = [a_{2,1}, \ldots, a_{2,n_2}]$”.

• Page 244, proof of Theorem 6.5.8, Case A3: In “$[b_1, \ldots, b_s] \gg_{\text{lex}} [a_{1,1}, \ldots, a_{1,n}] \gg_{\text{lex}} [a_{2,1}, \ldots, a_{2,n_2}]$”, replace “$a_{1,n}$” by “$a_{1,n_2}$”. But you might want to remove this relation altogether, since I don’t think you ever use $[b_1, \ldots, b_s] \gg_{\text{lex}} [a_{2,1}, \ldots, a_{2,n_2}]$.

• Page 244, proof of Theorem 6.5.8, Case A3: Replace “the CFL factorization of $[\beta_1, \ldots, \beta_s]$” by “the CFL factorization of $[b_1, \ldots, b_s]$”.

• Page 244, proof of Theorem 6.5.8, Case A3: I would replace “$\alpha \gg_{\text{lex}} \beta$” by “$\alpha \geq_{\text{lex}} \beta$” since the latter is easier to prove and just as fine.

• Page 244, proof of Theorem 6.5.8, Case B: In the second displayed equation of Case B, replace “$i_2$” by “$i_1$”.

• Page 244, proof of Theorem 6.5.8, Case B1: I would replace “$\alpha \gg_{\text{lex}} \beta$” by “$\alpha \geq_{\text{lex}} \beta$” since the latter is easier to prove and just as fine.

• Page 244, proof of Theorem 6.5.8, Case B3: I would replace “$\alpha \gg_{\text{lex}} \beta$” by “$\alpha \geq_{\text{lex}} \beta$” since the latter is easier to prove and just as fine.

• Page 244, proof of Theorem 6.5.8, Case B3: I have my doubts about this argument. You write that “relabel the latter $a_{i,j}$ with the equal letters $a_{2,j}$, $j = 1, \ldots, i_1$.” But can’t it happen that, after the relabelling, the letters of $\alpha_2$ will be out of order in $\beta''$, which means $\beta''$ might no longer be a shuffle of $\beta_1, \beta_2, \ldots, \beta_t, \alpha_2, \ldots, \alpha_m$? For an example, let $\alpha = 12141213$, so that $m = 2$, $\alpha_1 = 1214$ and $\alpha_2 = 1213$. Let $\beta$ be the shuffle $a_{2,1}a_{2,2}a_{2,3}a_{1,1}a_{2,4}a_{1,2}a_{1,3}a_{1,4} = 12113214$ of $\alpha_1$
When $i_1 = 3$ and $\alpha_1, a_{i_2}, a_{i_3} = [a_{i_1}, a_{i_2}, a_{i_3}] = 121$. But removing this prefix $[a_{i_1}, a_{i_2}, a_{i_3}]$ from $\beta$ leaves us with $a_{i_1}a_{i_2}a_{i_3}a_{i_4} = 13214$, which cannot be obtained as a shuffle of $[a_{i_1}]$ with $\alpha_2$.

- **Page 245, §6.5.17:** Replace “Shuffle $\otimes \mathbb{Z}/(2)$” by “Shuffle $\otimes \mathbb{Z}/(2)$”.

- **Page 247, proof of Lemma 6.5.32:** Typo: “the the”.

- **Page 248, proof of Construction and Lemma 6.5.33:** I don’t see how you conclude that $\beta \geq_{\text{lex}} \alpha'$. In my opinion, some more arguments are needed here.

- **Page 249:** Remove the opening parenthesis at the beginning of “(from the left $\text{Symm}$ of (6.6.1) to the right $\text{Symm}$” (since no matching closing parenthesis exists).

- **Page 249, footnote 19:** Replace “$h_i$” by “$h_1$”.

- **Page 250:** Replace “a word as in (6.7.2)” by “a word as in (6.7.3)”.

- **Page 251, between (6.7.7) and (6.7.8):** Replace “$\Psi$” by “$\Psi'$”.

- **Page 251, one line below (6.7.8):** Replace “([23], p 49ff), and section 4.9 above.” by “([23], p 49ff, and section 4.9 above).”.

- **Page 252, proof of Lemma 6.7.15:** Replace “$\lambda_n(\alpha)$” by “$\lambda^n(\alpha)$” (throughout the proof, for different values of $n$).

- **Page 253, proof of Theorem 6.7.5:** Replace “$\beta = [b_1, b_2, ..., b_n]$” by “$\beta = [b_1, b_2, ..., b_n]$” (since the letter $n$ will soon be used for something different). In the same vein, replace “$\alpha = \beta_{\text{red}} = [g(\beta)^{-1} b_1, g(\beta)^{-1} b_2, ..., g(\beta)^{-1} b_n]$” by “$\alpha = \beta_{\text{red}} = [g(\beta)^{-1} b_1, g(\beta)^{-1} b_2, ..., g(\beta)^{-1} b_n]$”.

- **Page 253, proof of Theorem 6.7.5:** It would clarify things if you replace “and formula (6.7.18)” by “and the first equation of (6.7.18)” (applied to $r_1$ instead of $n$) we have”.

- **Page 255:** In the first displayed formula on page 255, I believe the summation index “$\text{wt}(\alpha)$” should be “$\text{wt}(\alpha) = \text{j}$”.

- **Page 255, Theorem 6.9.4 (ii):** Replace “$\varphi_*(h(t))$” by “$\varphi_*(Z(t))$”.

- **Page 256, proof of Theorem 6.9.4:** Remove the period after “$\text{NSymmm}$”.

3
• Page 257, (6.9.12): Replace “$v_n(\mathcal{Sym})$” by “$v_n(\mathcal{Sym}_k)$”.

• Page 258: Remove the word “a” in “Now there exist a natural lifts”.

• Page 258, (6.9.15): Replace “$f_n^{Q\mathcal{Symm}}$” by “$f_p^{Q\mathcal{Symm}}$”.

• Page 258, (6.9.15): Replace the equality sign by an “$\equiv$” sign.

• Page 259, (6.9.23): Replace the “$Z$” by a “$Q$”.

• Page 259, footnote 26: Here you conjecture that “The corresponding ideals in $\mathcal{NSymm}$ are most likely the iterated commutator ideals”. If by “the corresponding ideals” you mean the orthogonal spaces $(F_i(Q\mathcal{Symm}))^\perp$ of $Q\mathcal{Symm}$ with respect to the bilinear form, then

\[
\text{I think the conjecture is false. It is true that } \left( F_1(Q\mathcal{Symm}) \right)^\perp = \mathcal{Symm}^\perp = [\mathcal{NSym}, \mathcal{NSym}], \text{ but it is not true that } (F_2(Q\mathcal{Symm}))^\perp = [\mathcal{NSym}, [\mathcal{NSym}, \mathcal{NSym}]]. \text{ For example, } M_{12}^2 = e_1(M_{12})\cdot e_1(M_{12}) (I \text{ write } M_\alpha \text{ for the monomial quasisymmetric function you call } \alpha) \text{ has scalar product } 2 \text{ with } [H_2, [H_3, H_1]], \text{ as the following sage code shows:}
\]

sage: QSym = QuasiSymmetricFunctions(QQ)
sage: M = QSym.M()
sage: NSym = NonCommutativeSymmetricFunctions(QQ)
sage: S = NSym.S()
sage: print (M[1,2]**2).duality_pairing(S[2,3,1]-S[2,1,3]-S[3,1,2]+S[1,3,2])
2

• Page 260, Theorem 6.9.27 (i): Replace “$v$” by “$v_n$”.

• Page 260, Theorem 6.9.27 (i): Replace “[a_1, \ldots, a_m]” by “[a_1, \ldots, a_m]”. Similarly, replace “[n^{-1}a_1, \ldots, n^{-1}a_m]” by “[n^{-1}a_1, \ldots, n^{-1}a_m]”.

• Page 260, Theorem 6.9.27 (iv): Replace the equality sign by an “$\equiv$” sign.