Math 5705, Fall 2018: **Enumerative Combinatorics**
– *Syllabus* –

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##

WARNING! You are reading the syllabus of a class that lies in the past. If you're looking for the current iteration of Math 5705, you are in the wrong place.

##

1. Time & Place

Lectures: MW 14:30–16:25, Vincent Hall 209.

I am planning to do 50 minutes of class + 15 minutes of break + 50 minutes of class. I will include some problem solving on occasion.

The class has a Canvas (https://canvas.umn.edu/courses/75006) with a discussion forum.

Homework will usually be due on **Wednesday** (sometimes weekly, sometimes every other week) **at the beginning of class**. You can also submit your homework electronically via Canvas **provided that the problem set is submitted as 1 single PDF file**. See "Grading" and "Coursework" below for details.

Midterms (3 in total) are like homework, but they count for more, and collaboration is not allowed (see below for details). There will be **no final exam**.

2. Requirements

This is a pure mathematics class and relies heavily on proofs. You have to feel at home reading and writing mathematical proofs. You can catch up on this from:

- Eric Lehman, F. Thomson Leighton, Albert R. Meyer, *Mathematics for Computer Science*,
  https://courses.csail.mit.edu/6.042/spring18/mcs.pdf . (You should know the material from Chapters 1–5, minus the CS parts.)
- Richard Hammack, *Book of Proof*,
  http://www.people.vcu.edu/~rhammack/BookOfProof/
- Martin V. Day, *An Introduction to Proofs and the Mathematical Vernacular*,
  https://www.math.vt.edu/people/day/ProofsBook/IPaMV.pdf .

You also need to understand congruences ($a \equiv b \bmod n$) and summation signs ($\sum$).

3. Texts

3.1. required:

There is no required textbook if you attend class. I won't follow a text directly anyway.

3.2. recommended:

Here are three good texts I know:

- [Loehr] Nicholas A. Loehr, *Bijective Combinatorics*, Chapman & Hall/CRC 2011.
  This is a long book going way beyond what can be done in an undergraduate class. Most relevant to us are Chapters 1, 2, 4, 7, 9, 10 (but we won't use the notion of a group).
- [GKP] Ronald L. Graham, Donald E. Knuth, Oren Patashnik, *Concrete Mathematics*, Second Edition, Addison-Wesley 1994. (Official page, with errata.)
  This is a famous introductory textbook; we shall only use its Chapters 5 and 7.
- [Galvin] David Galvin, *Basic discrete mathematics*.
  These are graduate-level notes. I am currently in charge of correcting and (occasionally) detailing them, so please let me know of any errors and imprecisions you find! We shall follow these notes in a few parts of this course, but their overall structure and focus is different from ours.

Here are four others I've heard people recommend.

- [Aigner] Martin Aigner, *A Course in Enumeration*, Graduate Texts in Mathematics #238, Springer 2007.
- [Bona] Miklós Bóna, *A Walk through Combinatorics*, World Scientific 2011.
- [Bogart] Kenneth P. Bogart, *Combinatorics Through Guided Discovery*, 2003.
- [Brualdi] Richard A. Brualdi, *Introductory Combinatorics*, 5th edition, Prentice-Hall 2010.

You don't have to buy any of these, but I assume any of them would look nice on your shelf. (You can download [Bogart] and [GKP] from the URLs above, and you might be able to get [Aigner] for free from the UMN network. Of course, there are ways to get the other two as well if you know your way around the internet[1].)

[1] Search for "reddit textbooks", for example.

3.3. various:

Combinatorics roughly falls into two parts: the enumerative/algebraic part and the graph-theoretic/extremal part. There is some overlap, but much of the time they feel like two different subjects. This course is about the first part. At `https://math.stackexchange.com/a/1454420/` I have compiled a list of texts for it. Here are just a few:

- [AnFe] Titu Andreescu and Zuming Feng, *A Path to Combinatorics for Undergraduates: Counting Strategies*, Springer 2003.
- [deBr] Nicolaas Govert de Bruijn, J. W. Nienhuys, Ling-Ju Hung, Tom Kloks, *de Bruijn's Combinatorics*, 2012.

  (Informal class notes from de Bruijn, one of the founders of the subject.)
- [Cam] Peter J. Cameron, *Notes on Combinatorics*.

  (Far too terse for an undergrad class, unfortunately.)
- [Ueck] Torsten Ueckerdt and Stefan Walzer, *Lecture Notes Combinatorics*.

  (Also too terse unfortunately.)
- [Wagn] David Wagner, *C&O 330, Introduction to Combinatorial Enumeration*.
- [deMi] Anna de Mier, *Lecture Notes for "Enumerative Combinatorics"*.
- [Daug] Zajj Daugherty, *Math A6800, Fall 2015: Combinatorial Analysis*.
- [detnotes] Darij Grinberg, *Notes on the combinatorial fundamentals of algebra*.

  (Very detailed, but not focussed on enumerative combinatorics. These have occasional overlap with our course, in particularly on the subject of permutations and determinants.)

The Fall 2017 and Spring 2018 sites for Math 4707 also have some useful materials (not a complete set of notes, though), but I won't cover the exact same material (Math 4707 $\neq$ Math 5705).

4. Contact

All material regarding the course (including homework) can be found on my homepage `http://www.cip.ifi.lmu.de/~grinberg/t/18f/`.

The best way to reach me is by email to `dgrinber@umn.edu`.

5. Topics (tentative)

This is still far from finished and decided.

Topics marked with an * **may** be excluded. Topics marked with an ** probably **will** be excluded.

1. Introduction.

   a) Main goals: counting; proving polynomial identities; discrete structures and maps between them.

   b) Domino tilings, following Chapter 0 of [Ueck]. Include the connection with Fibonacci numbers (counting $2 \times n$ tilings) and discuss what constitutes a closed form (how to we compute with ϕ and with cosines).

   c) Formulas for $1^k + 2^k + \cdots + n^k$. (Basic examples.)

   d) Factorials and binomial coefficients. (Up to Vandermonde convolution? Stress the usefulness for $1^k + 2^k + \cdots + n^k$.)

   e) Derangements. (First contact with the OEIS. No proofs at this point.)

   f) Fibonacci numbers count lacunar subsets (same as domino tilings).

   g) * More Fibonacci numbers: identities, Zeckendorf representation, Zeckendorf family identities, Fibonacci multiplication, Fibonomial coefficients, etc., with no proofs at this point.

   h) Basic properties of the $\sum$ sign.

2. Binomial coefficients.

   a) Basic identities. (Follow Chapter 5 of [GKP] up to Section 5.3.)

   b) Counting maps (all injective, surjective).

   c) Properties of $\operatorname{sur}(m, n)$ (number of surjective maps, aka $n!$ times Stirling numbers of the 2nd kind).

   d) Proof of $1^m + 2^m + \cdots + k^m$ formula.

   e) * Stirling numbers of the 2nd kind.

   f) Vandermonde convolution and its mutates (using upper negation).

   g) Bijective proofs.

   h) Polynomial functions and the “Zariski density trick”.

   i) Principle of inclusion and exclusion.

   j) Derangements: now with proof.

   k) More examples of inclusion-exclusion: $\operatorname{sur}(m, n)$, perhaps Euler totient function.

   l) ** Finite differences.

m) * Multinomial coefficients.

n) ** Generating functions (demo).

3. The twelvefold way.

    a) Follow Section 1.5 of [Ueck].

    b) Equivalence classes. (Assume these known in 5705.)

    c) Two ways to count multisets (bijection and induction).

    d) * Application of multisets to lacunar subsets.

4. Permutations. (Follow [detnotes] whenever possible.)

    a) Inversions and sign.

    b) Cycle decomposition.

    c) * Lehmer code.

    d) Determinants.

    e) Stirling numbers of the 1st kind.

    f) Eulerian numbers.

    g) * Pattern avoidance: length-3 classes.

    h) ** Weak Bruhat order.

5. Endofunctions (= maps from a set to itself).

    a) Eventual image of f (aka $f^n(S) = \operatorname{Rec} f$).

    b) Tortoise and hare algorithm. Apply this to define f^ω.

    c) Various examples (use exercises from previous years).

    d) Cayley's n^{n-2} theorem. (Proof: Aigner/Ziegler, fourth proof. Possibly use Putnam 2013 B5 solutions.)

    e) * Connection to trees (briefly).

    f) * Other endofunction counts.

    g) ** various tree theorems in terms of endofunctions (e.g., Laplacian kernel?).

6. Lattice paths and Catalan combinatorics.

    a) Lattice paths counted by $\binom{n+m}{n}$.

    b) Dyck paths.

    c) The number of n-Dyck paths is the Catalan number C_n.

   d) * Various proofs of the number of Dyck paths: coplactic operation; cycling; gen.fun.; what else? Best done at the generality of the ballot problem.
   e) * Some identities.
   f) ** Dyck paths as binary trees.
   g) ** Dyck paths as triangulations.

7. Pólya enumeration for the cyclic group.
   a) The μ and ϕ functions from number theory.
   b) Counting necklaces and aperiodic necklaces.
   c) Fermat's little theorem.
   d) * Counting lacunar necklaces (no two adjacent beads).
   e) ** Dirichlet series.

8. Generating functions. (Partly follow Oct/Nov notes of Fall 2017 classes.)
   a) Some examples of their use (without rigor).
   b) Formal linear combinations.
   c) Rigorous definition of power series (and polynomials!) and outline of proofs.
   d) Some applications.
   e) Solving linear recurrences (and occasionally others) in surds or in binomial coefficient sums.
   f) ** Bernoulli numbers and the ultimate $1^k + 2^k + \cdots + n^k$ formula.
   g) * $\mathbb{Q}[S_n]$.
   h) * Dynkin idempotent.
   i) * Young-Jucys-Murphy elements.
   j) * Solomon's Mackey formula for descent classes?
   k) * Some applications (card-shuffling probabilities?).

9. Partitions.
   a) Basics.
   b) Young diagrams and conjugation.
   c) Durfee squares.
   d) Glaisher bijection.
   e) * Euler's pentagonal number theorem.
   f) * Jacobi's triple product formula.

g) * Basic tableau counting (§1.3 in Leeuwen:RSK).
h) * Hook-length formula. (sketch proofs?)
i) * Cores of partitions (proof using β-set/abacus).

10. Combinatorics on words.

a) Lyndon words and Chen-Fox-Lyndon factorization.
b) ** Gessel-Reutenauer bijection.
c) ** Apply Gessel-Reutenauer to permutation enumeration?
d) ** de Bruijn cycles via Lyndon words.
e) ** Christoffel words (and Stern-Brocot tree?).

6. Schedule (tentative)

week	**material**	**due**
Sep 5		
Sep 10, 12		
Sep 17, 19		hw1
Sep 24, 26		hw2
Oct 1, 3		
Oct 8, 10		hw3
Oct 15, 17		
Oct 22, 24		MT1
Oct 29, 31		hw4
Nov 5, 7		
Nov 12, 14		MT2
Nov 19, 21		
Nov 26, 28		hw5
Dec 3, 5		
Dec 10, 12		MT3

7. Grading

The grade will be computed based on three take-home **midterms** (totalling to **60%** of the final grade, each giving 20% of the final grade) and about 5 **homework sets** (totalling to **40%** of the final grade, but the lowest score will be dropped).

Points will be deducted if your proofs are ambiguously worded or otherwise hard to understand. Writing readable arguments is part of mathematics; you can learn this from the references in the "Requirements" section above and you can practice it on math.stackexchange.

8. Coursework

Collaboration on homework is allowed, as long as:

- you **write** up the solutions autonomously and in your own words (in particular, this means that you have to **understand** them), and
- you **list the names of your collaborators** (there will be no penalties for collaboration, so you don't lose anything doing this!).

On the midterms, you have to **work alone** (you can **read** whatever you want, but you must **not contact** anyone about the midterm problems[2]; in particular, you must **not ask** them on the internet).

Homework and midterms should be submitted either in person during class, or by email to `dgrinber@umn.edu` . (Note: "dgrinber", not "dgrinberg"!)

If you handwrite your solutions:

- Make sure that your writing is legible.
- If you submit your solutions by email, make sure that your submission is **1 single PDF file** for a given homework set (not many 1-page JPGs!). Double-check that your scans are readable and aren't missing any relevant text near the margins.

If you type up your solutions:

- Again, make sure that your submission is **1 single PDF file** for a given homework set.
- Double-check that your text doesn't go over the margins (something that often happens when using LaTeX). If something is not on the page, we cannot grade it...

[2]It is OK to contact **me** with questions.

Calculators and computer algebra systems may be used, but are not necessary (and you are responsible for any resulting errors). For emails, I suggest using "[Math 5705] Homework set #n submission" (n = the number of the problem set) as the subject line.

Late homework or late midterms are **not accepted** in any situation; if you are not finished, submit whatever you have before the deadline. If you want to update your submission, you can do so (before the deadline!) by sending me an email that includes the whole updated submission (not just the parts you want changed).

See also the following university policies:

- https://policy.umn.edu/education/gradingtranscripts
- https://policy.umn.edu/research/academicmisconduct