Math 4707 & Math 4990, Fall 2017: **Introduction to Combinatorics**
– *Syllabus* –
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##

WARNING! You are reading the syllabus of a class that lies in the past. If you're looking for the current iteration of Math 4707 or Math 4990, you are in the wrong place.

##

Time & Place

4707: Lectures: MW 14:30–16:25, Amundson Hall 120.

4990: Lectures: Tue 16:00–18:00, Vincent Hall 211.

I am planning to do 50 minutes of class + 15 minutes of break + 50 minutes of class. I might include some problem solving on occasion.

Note that both classes have Moodles with discussion forums.

Homework will usually be due

- on **Wednesday** sometimes weekly, sometimes every other week (4707);
- on **Tuesday** weekly (4990),

both times **at the beginning of class**. You can also submit your homework electronically via moodle **provided that the problem set is submitted as 1 single PDF file**. See "Grading" and "Coursework" below for details.

Midterms (3 in total) in 4707 are like homework, but they count for more, and collaboration is not allowed (see below for details). There will be **no final exam**. In 4990, there are neither midterms nor finals.

Requirements

This is a pure mathematics class and focuses heavily on proofs. You have to feel at home reading and writing mathematical proofs. You can catch up on this from :

- Eric Lehman, F. Thomson Leighton, Albert R. Meyer, *Mathematics for Computer Science*, `https://courses.csail.mit.edu/6.042/fall17/mcs.pdf` . (You should know the material from Chapters 1–5, minus the CS parts.)
- Richard Hammack, *Book of Proof*, `http://www.people.vcu.edu/~rhammack/BookOfProof/`
- Martin V. Day, *An Introduction to Proofs and the Mathematical Vernacular*, `https://www.math.vt.edu/people/day/ProofsBook/IPaMV.pdf` .

It is helpful to understand congruences ($a \equiv b \mod n$) and summation signs ($\sum$).

Texts
recommended:
Here are three texts I know:

- [Loehr] Nicholas A. Loehr, *Bijective Combinatorics*, Chapman & Hall/CRC 2011.
  This is a long book going way beyond what can be done in an undergraduate class; but it comes the closest to covering what I am planning to cover. Most relevant to us are Chapters 1, 2, 4, 7, 9 (but we won't use the notion of a group).

- [GKP] Ronald L. Graham, Donald E. Knuth, Oren Patashnik, *Concrete Mathematics*, Second Edition, Addison-Wesley 1994. (Official page, with errata.)
  This is a famous introductory textbook that you can start reading before we even start; we shall only use its Chapters 5 and 7.

- [Galvin] David Galvin, *Basic discrete mathematics*.
  These are graduate-level notes. I am currently in charge of correcting and (occasionally) detailing them, so please let me know of any errors and imprecisions you find! We shall follow these notes in a few parts of this course, but their overall structure and focus is different from ours.

Here are four others I've heard people recommend.

- [Aigner] Martin Aigner, *A Course in Enumeration*, Graduate Texts in Mathematics #238, Springer 2007.

- [Bona] Miklós Bóna, *A Walk through Combinatorics*, World Scientific 2011.

- [Bogart] Kenneth P. Bogart, *Combinatorics Through Guided Discovery*, 2003.

- [Brualdi] Richard A. Brualdi, *Introductory Combinatorics*, 5th edition, Prentice-Hall 2010.

You don't have to buy any of these, but I assume any of them would look nice on your shelf. (You can download [Bogart] and [GKP] from the URLs above, and you might be able to get [Aigner] for free from the UMN network. Of course, there are ways to get the other two as well if you know your way around the internet[1].)
various:
Combinatorics roughly falls into two parts: the enumerative/algebraic part and the graph-theoretic/extremal part. There is some overlap, but much of the

[1]Search for "reddit textbooks", for example.

time they feel like two different subjects. Most of this course (70%?) will be on the first part. Either part has lots of books written about it; let me just link two collections of references:

- On enumerative/algebraic combinatorics, I have compiled a list at `https://math.stackexchange.com/a/1454420/`.
- On graph theory, there is a list in the Preface of my Notes on Graph Theory. (The notes are unfinished, but the list is quite long.)

A book that is commonly used for 4707 at UMN is:

- [LPV] L. Lovász, J. Pelikán, and K. Vesztergombi, *Discrete Mathematics: elementary and beyond*, Springer 2003.

It gives a nice overview of several concepts, although it skimps on the details. Here are some more:

- [AnFe] Titu Andreescu and Zuming Feng, *A Path to Combinatorics for Undergraduates: Counting Strategies*, Springer 2003.
- [deBr] Nicolaas Govert de Bruijn, J. W. Nienhuys, Ling-Ju Hung, Tom Kloks, *de Bruijn's Combinatorics*, 2012.

  (Informal class notes from de Bruijn, one of the founders of the subject.)
- [Cam] Peter J. Cameron, *Notes on Combinatorics*.

  (Far too terse for an undergrad class, unfortunately.)
- [Ueck] Torsten Ueckerdt and Stefan Walzer, *Lecture Notes Combinatorics*.

  (Also too terse unfortunately.)
- [detnotes] Darij Grinberg, *Notes on the combinatorial fundamentals of algebra*.

  (Very detailed, but not focussed on enumerative combinatorics. These have occasional overlap with our course, in particularly on the subject of permutations and determinants.)

I give some more references in `http://www.cip.ifi.lmu.de/~grinberg/t/17f/cafe.pdf` .

Contact

All material regarding the course (including homework) can be found on my homepage `http://www.cip.ifi.lmu.de/~grinberg/t/17f/`.

The best way to reach me is by email to `dgrinber@umn.edu`.

Topics (tentative)

This is still far from finished and decided.

Topics marked with an * **may** be excluded. Topics marked with an ** probably **will** be excluded.

1. Introduction.

   a) Domino tilings, following Chapter 0 of [Ueck]. Include the connection with Fibonacci numbers (counting $2 \times n$ tilings) and discuss what constitutes a closed form (how to we compute with ϕ and with cosines).

   b) Formulas for $1^k + 2^k + \cdots + n^k$. (Basic examples.)

   c) Factorials and binomial coefficients. (Up to Vandermonde convolution? Stress the usefulness for $1^k + 2^k + \cdots + n^k$.)

   d) Derangements. (First contact with the OEIS. No proof at this point.)

   e) * More Fibonacci numbers: identities, Zeckendorf representation, Fibonacci multiplication, etc.

2. The twelvefold way.

   a) Follow Section 1.5 of [Ueck], introducing basic principles (Sections 1.1–1.4) as they become important.

   b) Equivalence classes.

   c) Two ways to count multisets (bijection and induction).

   d) Fibonacci numbers count lacunar subsets (same as domino tilings).

3. Binomial coefficients.

   a) Follow Chapter 5 of [GKP] up to Section 5.3.

   b) Bijective proofs.

   c) Polynomial functions and the "Zariski density trick".

   d) Upper negation; mutating Vandermonde.

   e) Principle of inclusion and exclusion.

   f) Derangements: now with proof. (More examples of inclusion-exclusion?)

   g) Finite differences.

   h) Multinomial coefficients.

   i) Stirling numbers of the 2nd kind.

4. Generating functions.

   a) Some examples of their use (without rigor).

   b) Rigorous definition of power series (and polynomials!) and outline of proofs.

   c) Some applications.

   d) ** Bernoulli numbers and ultimate $1^k + 2^k + \cdots + n^k$ formula.

   e) ** Lagrange inversion.

   f) ** Umbral calculus (examples).

5. Permutations.
   a) Inversions and sign.
   b) Determinants.
   c) Cycle decomposition.
   d) * Lehmer code.
   e) * Weak Bruhat order.
   f) ** Strong Bruhat order.

6. Partitions.
   a) Basics.
   b) Young diagrams and conjugation.
   c) Durfee squares.
   d) Glaisher bijection.
   e) * Euler's pentagonal number theorem.
   f) ** Jacobi's triple product formula.
   g) * Hook-length formula (just state).
   h) * Cores of partitions. (Proofs unlikely.)

7. Graphs (see also Math 5707).
   a) Definitions.
   b) Degrees and their basic properties.
   c) * Dominating sets.
   d) Eulerian walks/circuits.
   e) * Hamiltonian paths/cycles.
   f) Trees.
   g) Cayley's n^{n-2} theorem. (Inductive proof using Pascal for multinomial coefficients. Follow [Galvin].)
   h) * Prüfer code. (Again, follow [Galvin].)
   i) * The chromatic polynomial. (Cf. Exercise 4 in 5707 Midterm 2.)

8. Digraphs (see also Math 5707).
   a) Definitions.
   b) ** Tournaments.

c) ** The matrix-tree theorem and the BEST theorem. (Proofs very unlikely.)
d) Max-flow-min-cut. (See 5707, specifically the handwritten notes and references.)
e) Applications of max-flow-min-cut: Hall and Menger.
f) * Some more matching theory.
g) * Non-bipartite matching (no proofs).
h) ** Pfaffians (overview).

9. Catalan combinatorics.
   a) Dyck paths.
   b) The number of n-Dyck paths is the Catalan number C_n.
   c) * Various proofs of the number of Dyck paths: coplactic operation; cycling; gen.fun.; what else? Best at the generality of the ballot problem.
   d) Some identities.
   e) * Dyck paths as binary trees.
   f) * Dyck paths as triangulations.

10. Pólya enumeration for the cyclic group.
    a) The μ and ϕ functions from number theory.
    b) Counting necklaces and aperiodic necklaces.
    c) Fermat's little theorem (homework).
    d) * Counting Fibonacci necklaces (no two adjacent beads).
    e) ** Dirichlet series.

11. Combinatorics on words.
    a) Lyndon words and Chen-Fox-Lyndon factorization.
    b) * Fine-Wilf theorem.
    c) ** Gessel-Reutenauer bijection (w/o proof I guess).
    d) ** de Bruijn cycles via Lyndon words.
    e) ** Christoffel words (and Stern-Brocot tree?).

12. ??? (material to be added whenever it fits)
    a) Stable marriage theorem (Gale-Shapley).
    b) Discrete continuity (30 shoes).

Grading

The grade will be computed based on three take-home midterms (totalling to 60% of the final grade, each giving 20% of the final grade) and about 6 homework sets (totalling to 40% of the final grade, but the lowest score will be dropped).

Points will be deducted if your proofs are ambiguously worded or otherwise hard to understand. Writing readable arguments is part of mathematics; you can learn this from the references in the "Requirements" section above and you can practice it on math.stackexchange.

Coursework

Collaboration on homework is allowed, as long as:

- you **write** up the solutions autonomously and in your own words (in particular, this means that you have to **understand** them), and
- you **list the names of your collaborators** (there will be no penalties for collaboration, so you don't lose anything doing this!).

On the midterms, you have to **work alone** (you can **read** whatever you want, but you must **not contact** anyone about the midterm problems[2]; in particular, you must **not ask** them on the internet).

Homework and midterms should be submitted either in person during class, or by email to `dgrinber@umn.edu` . (Note: "dgrinber", not "dgrinberg"!)

If you handwrite your solutions:

- Make sure that your writing is legible.
- If you submit your solutions by email, make sure that your submission is **1 single PDF file** for a given homework set (not many 1-page JPGs!). Double-check that your scans are readable and aren't missing any relevant text near the margins.

If you type up your solutions:

- Again, make sure that your submission is **1 single PDF file** for a given homework set.
- Double-check that your text doesn't go over the margins (something that often happens when using LaTeX). If something is not on the page, we cannot grade it...

Calculators and computer algebra systems may be used, but are not necessary (and you are responsible for any errors they make, or you make at using them). For emails, I suggest using "[Math m] Homework set #n submission" (n = the number of the problem set; $m \in \{4707, 4990\}$) as the subject line.

[2]It is OK to contact **me** with questions.

Late homework or late midterms are **not accepted** in any situation; if you are not finished, submit whatever you have before the deadline. If you want to update your submission, you can do so (before the deadline!) by sending me an email that includes the whole updated submission (not just the parts you want changed).

See also the following university policies:

- https://policy.umn.edu/education/gradingtranscripts
- https://policy.umn.edu/research/academicmisconduct