UMTYMP Advanced Topics

Induction

Using induction, prove each of the following statements. You may want to use strong induction for some of these.

1. For every natural number n, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

2. Any whole dollar amount greater than twelve can be formed by a combination of four and five dollar coins.

3. Every natural number $n \geq 2$ can be factored into primes.

4. A set of n elements has 2^n subsets.

5. For every natural number n, $\displaystyle\prod_{i=2}^{n}\left(1-\frac{1}{i^2}\right)=\frac{n+1}{2n}$.

6. Let F_n denote the nth Fibonacci number, so that $F_1 = F_2 = 1$, and $F_{n+2} = F_{n+1} + F_n$. Then $F_n = \dfrac{\phi^n - \psi^n}{\phi - \psi}$, where $\phi = \dfrac{1+\sqrt{5}}{2}$ and $\psi = \dfrac{1-\sqrt{5}}{2}$.