Cartesian Products and Relations

1. Let $X = \{A, B, C\}$, $Y = \{1, 2\}$, and $Z = \{a, b\}$.
   (a) List the elements in $X \times Y$.

   (b) List the elements in $Y \times X$.

   (c) List the elements in $X \times (Y \times Z)$.

   (d) List the elements in $(X \times Y) \times Z$.

   (e) Is $X \times Y = Y \times X$? Is $X \times (Y \times Z) = (X \times Y) \times Z$?

2. Let $X = [0, 1]$ and $Y = [1, 3]$, written in interval notation. Draw the set $X \times Y$ as a subset of $\mathbb{R}^2$.

3. For the subsets of $\mathbb{R}^2$ drawn on the board, which can be written as $X \times Y$, where X and Y are subsets of $\mathbb{R}$?

4. Consider $X = \{1, 2, 3\}$ and $Y = \{2, 3, 4\}$.
   (a) Which subset R of $X \times Y$ defines the relation $x = y$ for $x \in X$ and $y \in Y$?

   (b) Which subset R of $X \times Y$ defines the relation $x < y$ for $x \in X$ and $y \in Y$?

   (c) Consider $R = \{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 2), (3, 4)\}$. What familiar relation does this define?

5. In a coordinate system, draw the subsets of $\mathbb{R}^2$ that define the following relations:
   (a) $x = y$
   (b) $x < y$
   (c) $x \geq y^2$
   (d) $x \leq \lceil y \rceil$ (this is the ceiling function)