Equivalence Classes and Partitions

1. For each of the equivalence relations you found on the previous work sheet, describe and/or draw the equivalence classes.

2. List all partitions of the set $\{1, 2, 3, 4\}$.

3. For each of the following partitions of $\mathbb{Z}$, describe the corresponding equivalence relation.

   (a) $\{..., \{-1\}, \{0\}, \{1\}, \{2\}, \{3\}, ...\}$

   (b) $\{\{..., -1, 0, 1, 2, 3, 4, 5, ...\}\}$

   (c) $\{\{..., -1, 1, 3, 5, 7, 9, ...\}, \{-2, 0, 2, 4, 6, 8, ...\}\}$

   (d) $\{\{0\}, \{-1, 1\}, \{-2, 2\}, \{-3, 3\}, \{-4, 4\}, ...\}$

4. In this problem, you will prove that equivalence relations and partitions are "essentially the same."

   (a) Suppose we are given an equivalence relation $\sim$ on a set X. Show that the equivalence classes form a partition of X. That is, show:

   - Every element $x \in X$ is in some equivalence class.
   - For any two equivalence classes E_x and E_y, either $E_x = E_y$ or $E_x \cap E_y = \emptyset$.

   (b) Suppose we are given a partition $\mathcal{P}$ of a set X. Consider the relation defined by:

   $$x \sim y \Leftrightarrow x, y \text{ are in the same set in } \mathcal{P}$$

   Show that $\sim$ is an equivalence relation on X. That is, show:

   - For all $x \in X$, $x \sim x$.
   - If $x \sim y$, then $y \sim x$.
   - If $x \sim y$ and $y \sim z$, then $x \sim z$.