Math 4242 Fall 2016 (Darij Grinberg): homework set 2
due: Wed, 28 Sep 2016, 23:00 (Minneapolis time) by email (`dgrinber@umn.edu`), or in class on 28 Sep 2016

Exercise 1. Let $U = \begin{pmatrix} 6 & 3 & -2 & 5 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

(a) Find all column vectors x of size 4 satisfying $Ux = b$, where $b = \begin{pmatrix} 1 \\ 5 \\ 2 \\ 0 \end{pmatrix}$.

(b) Find all column vectors x of size 4 satisfying $Ux = b'$, where $b' = \begin{pmatrix} 1 \\ 5 \\ 2 \\ 1 \end{pmatrix}$.

(c) Find all column vectors x of size 4 satisfying $Ux = x$.

Exercise 2. Let $A = \begin{pmatrix} 1 & 4 & 2 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & 2 & 4 & 1 \end{pmatrix}$.

(a) Find all column vectors x of size 4 satisfying $Ax = b$, where $b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(b) Find all column vectors x of size 4 satisfying $Ax = b$, where $b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

Exercise 3. Recall that the determinant of a 2×2-matrix is computed by the formula

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

Use this to prove (by direct computation) that $\det(AB) = \det A \cdot \det B$ holds for all 2×2-matrices A and B.

Exercise 4. I have mentioned in class that determinants of square matrices behave predictably under the standard row operations:

- The operation $A_{u,v}^{\lambda}$ preserves the determinant (that is, $\det\left(A_{u,v}^{\lambda}C\right) = \det C$ for any C).
- The operation S_u^{λ} multiplies the determinant by λ (that is, $\det\left(S_u^{\lambda}C\right) = \lambda \det C$ for any C).

- The operation $T_{u,v}$ negates the determinant (that is, $\det(T_{u,v}C) = -\det C$ for any C).

Also, I have mentioned that the determinant of a triangular matrix is the product of its diagonal entries.

Compute

$$\det \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 7 \\ 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 6 & 1 \end{pmatrix}.$$

(Mind the 7 in the upper-right corner!)