On Bidigare's proof of Solomon's theorem
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`http://www.ma.rhul.ac.uk/~uvah099/Maths/Bidigare4.pdf`
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Errata and addenda by Darij Grinberg

1. Errata

- **page 1:** As usual, I think it's worth explaining that your functions (or permutations, at least) act on the right of their values and are multiplied accordingly (so that $\alpha\beta$ sends i to $(i\alpha)\,\beta$).
- **page 1:** Also, I think "natural number" should be defined (to warn the reader that 0 doesn't count as a natural number).
- **page 1:** Also, the Young subgroup S_p should be defined.
- **page 1 and later:** You occasionally use "Ξ_p" as a synonym for "Ξ^p". (Probably, a search for "`\Xi_`" will quickly locate all the instances of this.)
- **page 1:** In the definition of $\operatorname{Des}(g)$, replace "$xg < (x+1)\,g$" by "$xg > (x+1)\,g$" (or does gravity, too, work the other way round in Britain?).
- **page 1:** "Given compositions p, q and r of $\mathbf{N}$ such that p has k parts and q has ℓ parts" $\to$ "Given compositions $p = (p_1, p_2, \ldots, p_k)$, $q = (q_1, q_2, \ldots, q_\ell)$ and r of $n \in \mathbf{N}$". (Two things corrected here: "of $\mathbf{N}$" became "of $n \in \mathbf{N}$", and the notations p_i and q_j have been defined explicitly since you refer to them later.)
- **page 1, Theorem 1:** "If p, q and r" $\to$ "If p and q".
- **page 1, §2:** "the sets $P_1, \ldots, P_n$ are disjoint" $\to$ "the sets $P_1, \ldots, P_k$ are disjoint".
- **page 1, §2:** In the first displayed equation of §2, add a comma after "$P_1 g$" in "$(P_1 g \ldots, P_k g)$".
- **page 1, §2:** It might be worth saying a few words about why this product $\wedge$ is associative. To me, this becomes really clear when I identify each set composition $P = (P_1, \ldots, P_k)$ of n with a total pre-order on the set $\{1, 2, \ldots, n\}$ (namely, the pre-order under which two elements i and j satisfy $i \leq j$ if and only if $i \in P_u$ and $j \in P_v$ for some $u \leq v$), and then the product $\wedge$ becomes a "lexicographic order" product (i.e., two elements i and j of $\{1, 2, \ldots, n\}$ satisfy $i \leq j$ in $P \wedge Q$ if and only if they satisfy $i \leq j$ in P and (if $i \sim j$ in P, then $i \leq j$ in Q)). Thus, $P \wedge Q$ means "order the elements according to P, and use Q to break ties".

- **page 2, basic property (2):** Here, "$\{1, \ldots, n\}$" should be replaced by "$(\{1, \ldots, n\})$".
- **page 2, basic property (3):** This statement relies on a somewhat unusual concept of "refinement", as you explain a few paragraphs below; with the normal concept of refinement for compositions, it is false[1].

  Let me, however, suggest to replace **(3)** by the weaker claim that "If $Q \in \Pi_n$ has type (1^n), then $P \wedge Q$ has type (1^n) for any $P \in \Pi_n$.". This is all you need in the following, and it has the advantage of being obviously true.
- **page 2, basic property (C):** "By (3) above" $\to$ "By (3) and (4) above" (at least if you follow my suggestion in nerfing (3)).
- **page 2, basic property (D):** "If q has ℓ parts" $\to$ "If $q = (q_1, \ldots, q_\ell)$".
- **page 2, basic property (D):** In the displayed equation that defines T^q, replace "$\{1 \ldots q_1\}$" by "$\{1, \ldots, q_1\}, \{q_1 + 1, \ldots, q_1 + q_2\}$" (I've added missing commas and also added a second set to make the construction clearer).
- **page 3, proof of Proposition 3:** "Let p be a composition with k parts" $\to$ "Let $p = (p_1, \ldots, p_k)$ be a composition".
- **page 3, proof of Proposition 3:** Remove the "where q has ℓ parts" (you never use ℓ).
- **page 3, proof of Proposition 3:** "for $1 \leq i < k$" $\to$ "for $1 \leq i \leq k$".
- **page 3, proof of Proposition 3:** Replace "T_q" by "T^q". This, too, appears several times, so it's worth searching for it.
- **page 3:** You write "is a subalgebra of $\mathbf{Z}S_n$ isomorphic to $(\mathbf{Z}\Pi_n)^{S_n}$". You seem to be going a tad too fast here; the isomorphism only follows once you realize that the Ξ^p are linearly independent, which follows from the distinctness of their "leading terms" with respect to some order on the permutations; but this isn't really so obvious that it isn't worth further mention, if you ask me.
- **page 3:** "say that $T \in \Pi_n$ is *increasing*" $\to$ "say that $T = (T_1, \ldots, T_\ell) \in \Pi_n$ is *increasing*".
- **page 3:** "for $1 \leq i < i' \leq \ell$" $\to$ "for $1 \leq i < j \leq \ell$" (or rename j as i' later).

[1]For example, the type of

$$(\{1,2,3\}, \{4\}) \wedge (\{1,4\}, \{2,3\}) = (\{1\}, \{2,3\}, \{4\})$$

is $(1,2,1)$, which is a refinement of $(3,1)$ but not a refinement of $(2,2)$ in the usual sense of this word.

- **page 3, proof of Proposition 4:** "Suppose that p has k parts, q has ℓ parts and that r has m parts" $\rightarrow$ "Suppose that $p = (p_1, \ldots, p_k)$ and $q = (q_1, \ldots, q_\ell)$". (You don't need m.)