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A Lecture on Fitts' Law

2nd Edition

June 5, 2023

Preface

This is a lecture specifically addressing the HCI (Human Computer Interaction) community. It seems necessary as there is an unmanageable amount of publications on Fitts' Law, and many of them contribute more to confusion than clarification. A reasonable chapter in HCI textbooks seems to be missing. The confusion on Fitts' Law is a breeding ground for more strange theories, and consequently, the number of questionable publications grows. The situation may worsen with the recent rise of a new field called Computational Interaction. People doing research in this field are invited to read this text.

In the recent years, terms like 'alternative facts' have become popular, and we are in danger of losing the truth. So-called 'filter bubbles' and 'echo chambers' are a phenomenon of social media platforms and a topic of HCI research. The HCI community should have a look at itself. Filter bubbles in the sciences cause severe damage to scientific truth. This lecture is an attempt to prevent this problem.

The first edition of this lecture on Fitts' Law was written in 2013 in the hope of triggering a discussion within the HCI community. However, this did not happen and the HCI community continues to refer to Fitts' Law but uses MacKenzie's formula and calls it the Shannon formulation. Most likely, Shannon would not be amused. Fitts presented his theory in 1954, and the HCI community produces new papers on Fitts' Law up to this day. This raises the question of how long it will take until Fitts' Law is understood and further research is no longer necessary. It seems that Fitts' Law will be an eternal research topic for HCI.

The second edition was completely re-edited and restructured and now also contains a section on MacKenzie's theory. The aim of this lecture is to convince the HCI community to use Fitts' formula for Fitts' Law, even if this means that the HCI community does not have their own and putatively better MacKenzie formula. MacKenzie based his theory on the imperfections of Fitts' formula, but there is nothing wrong with Fitts' formula, and consequently, there is no need for an alternative formulation. Additionally, this lecture hopes to prevent the HCI community from trifling with information theory and other hard sciences. Therefore, i t would be a good idea to ban Fitts' Law research from HCI.

The HCI community claims to be scientific and tries to support this claim with impressive formulas and advanced statistics with all kinds of post-hoc corrections. It seems that, for parts of the HCI community, the more difficult and complicated a topic is presented to be, such that few people understand it, the more scientific the topic must be. In contrast, this lecture tries to be as simple as possible, so that hopefully everybody is able to understand it with only simple mathematics and common sense.

The one piece of good news since the release of the first edition is that parts of the HCI community now agree that saccadic eye movements are not ruled by Fitts' Law. For psychologists, this was clear since the beginning of eye movement research and every textbook states that eye movements are ballistic, which is the opposite of what Fitts' Law suggests. For the HCI community, eye movements are now only Fitts' Law-like.

The main focus of this lecture lies in explaining Fitts' theory and related topics. However, as it is impossible to ignore the existing literature on Fitts' Law, criticism is unavoidable. Therefore, it should be emphasized here that this lecture is the author's private work and should not be associated with his affiliation.

Munich, June 2023

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Chapter 1
Fitts' Law Basics

Abstract This chapter introduces Fitts' research and the Index of Difficulty. It presents four approaches to deriving Fitts' formula and discusses the differences between a model and reality and the limits of Fitts' Law. Additionally, this chapter explains how (and how not) to evaluate measured data of a Fitts' Law pointing experiment.

1.1 Introduction

Fitts published his paper '*The information capacity of the human motor system in controlling the amplitude of movement*' [10] in 1954. Fitts' research allows one to predict the time a human needs to point at a target of a given size at a given distance. For the HCI community, this law has some significance because it applies to pointing with a mouse device, and pointing is very important for operating a graphical user interface.

Fitts' Law states that it takes more time to hit a target if the target is further away and also if the target is smaller. Both statements can be proven using common sense. Fitts' Law also states that the target acquisition time increases drastically if the target gets tiny. Fitts' Law states that an infinitely small target is impossible to hit because it would take an infinitely long time, which also aligns with common sense.

For people designing graphical user interfaces, especially those who have only a design background and no solid mathematics education, this understanding of Fitts' Law is nearly enough to do their work well. Perhaps some concept of the steering law (see Section 2.2) – which states that we can steer a car more quickly on a wide and straight street than on a narrow and curved road – would be a good addition to the knowledge of a designer, as this law also holds for steering a mouse pointer through a cascading menu. Finally, it would be good to know that there are limits to Fitts' Law (see Section 1.8).

Besides Fitts' original formula, other formulas claim to be better, especially in their predictive power. The time predicted from these formulas is only valid for a mean time over many pointing actions. However, the pointing performance has a high variance, not only between different persons but also within the pointing actions of a single person. The completion time for the same pointing task differs by ±20% (see Figure 1.22) within a person and much more within a group. Therefore, the question of which formula is the best is unimportant for practitioners, as the results from different formulas typically differ little.

Consequently, using other Fitts' Law formulas do not affect practical design issues, and the discussion which is the correct formula is a little bit academic. However, if Fitts' Law is the scientific claim of the HCI community, the question of the correct formula becomes very important. It gets even more important when building new theories on top of questionable formulas.

1.2 Fitts' Research

Fitts' research question was: What is the limiting factor for the speed of controlled body movements? He had two possible hypotheses:

- H1: The speed of controlled movements is limited by the muscle force.
- H2: The speed of controlled movements is limited by the information processing capacity of the human nervous system.

Fitts published his research in 1954 [10]. At that time, the concept of measurable information was relatively new. Shannon published his work on information theory in 1948 [24]. At that time, very few computers existed, all of which filled big rooms. The invention of the computer mouse dates back to the late sixties. Consequently, Fitts could not do a mouse click experiment, as typically done nowadays, to demonstrate Fitts' Law. Instead, he used a mechanical setup.

To answer his question, Fitts designed three simple tasks, the 'Reciprocal Tapping' (see Figure 1.1), the 'Disc Transfer' (see Figure 1.2), and the 'Pin Transfer' (see Figure 1.3).

To test the first hypothesis, Fitts varied the stylus weight in the reciprocal tapping experiment:

'Two metal-tipped styluses were used. One weighed 1 oz. and was about the size of a pencil. The other weighed 1 Ib. and was slightly larger.' [10]

However, Fitts did not find significant differences in the performance times of his study participants.

For the second hypothesis, Fitts needed to quantify the information amount in the task. For this, he used an analogy to mechanical waves and the knowledge from information theory. This analogy is legitimate as information theory is also valid for mechanical waves, for example, acoustic waves. The mathematics for mechanical and electrical waves are the same, and the reader may be more familiar with electrical waves and the question of how much information we can transfer with voltage. With a voltage U and a noise signal (or inaccuracy) of ΔU it is possible to distinguish $U/\Delta U$ discrete voltage levels, which can encode $log_2(U/\Delta U)$ bits. This is well-known from computer science lectures on data transmission on the physical layer of the network layer model (see Figure 1.4).

Therefore, Fitts introduced the Index of Difficulty ID as

$$ID = log_2(\frac{2A}{W}) \tag{1.1}$$

which is the number of bits needed to fulfill the pointing task. A is amplitude, which refers here to the distance from an initial position to the center of the target. W is the width of the target. The binary logarithm gives the number of binary digits

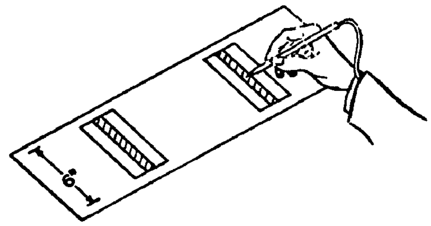

Figure 1. Reciprocal tapping apparatus. The task was to hit the center plate in each group alternately without touching either side (error) plate.

Fig. 1.1 Experimental setup of Fitts' 'Reciprocal Tapping'. Taken from Fitts' publication [10]

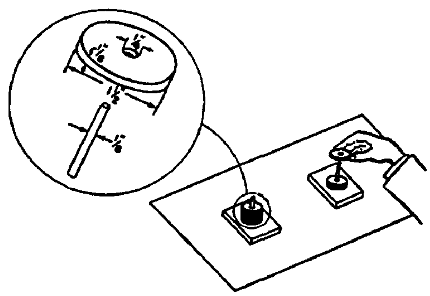

Figure 2. Disc transfer apparatus. The task was to transfer eight washers one at a time from the right to the left pin. The inset gives the dimensions for the W_S = 1/8 in. condition.

Fig. 1.2 Experimental setup of Fitts' 'Disc Transfer'. Taken from Fitts' publication [10]

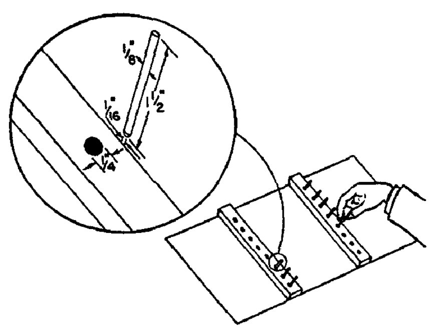

Figure 3. Pin transfer apparatus. The task was to transfer eight pins one at a time from one set of holes to the other. The inset gives the dimensions of pins and holes for the W_S = 1/8 in. condition.

Fig. 1.3 Experimental setup of Fitts' 'Pin Transfer'. Taken from Fitts' publication [10] and therefore has the unit bit. This factor 2 is the starting point for MacKenzie's confusion discussed in 3.1.

Fitts introduced the factor of 2 with the words:

'The use of 2A rather than A is indicated by both logical and practical considerations. Its use insures that the index will be greater than zero for all practical situations and has the effect of adding one bit $(-log_2 1/2)$ *per response to the difficulty index. The use of 2A makes the index correspond rationally to the number of successive fractionations required to specify the tolerance range out of a total range extending from the point of initiation of a movement to a point equidistant on the opposite side of the target.'* [10].

Perhaps Fitts did not express himself in the best way. This passage sounds a little bit like passing the explanation he got from an expert. Within the analogy to information theory, he mapped the noise amplitude to the target width. However, the target width corresponds to the amplitude from peak to peak. Consequently, he also took the peek-to-peek value for the movement amplitude, which is 2A (see Figure 1.5).

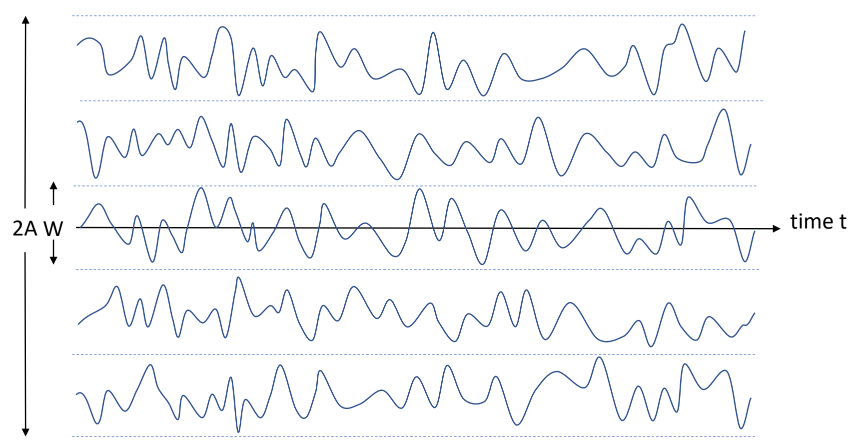

Fig. 1.4 The number of distinguishable levels (here 5) with the presence of noise as an illustration of Fitts' formula.

Fitts' explanation is correct. Finally, the question is whether the radius or the diameter should be in the formula. Choosing the diameter creates a factor of 2. Fitts mentioned that a factor of 2 ensures positive IDs. Fitts' formula produces negative IDs if $A < W/2$. However, if the target center is closer than $W/2$ it means that the pointer is already inside the target. In this case, the entry to the target happened in the past, meaning negative time. Fitts expressed this with the words *'the index will*

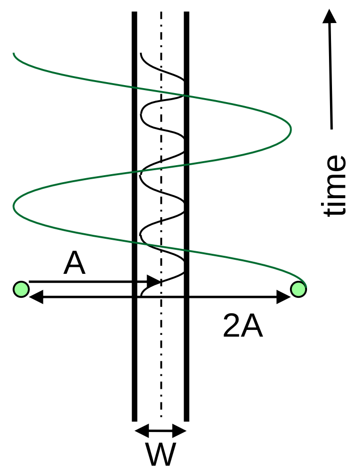

Fig. 1.5 Amplitude and peek-to-peek value. This figure illustrates Fitts' explanation for the '2' in his formula.

be greater than zero for all practical situations', which means a starting position outside the target. If the starting position is exactly at the target edge, the distance to the target center is $W/2$, and therefore the ID is zero. This means the goal is reached and there are no bits to transfer. Everything is alright with Fitts' definition of the ID. Figure 1.6 shows the values for the ID over distance to the target center.

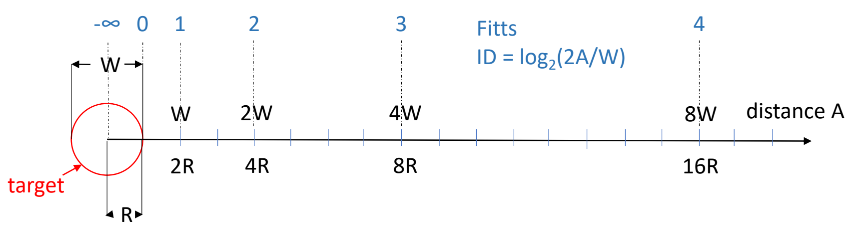

Fig. 1.6 Index of Difficulty over pointer distance A to a target of given size W. If the pointer is at the target edge the task is fulfilled and the ID is zero. Every doubling of the pointer distance to the target center adds one bit to the ID.

Although it is trivial and exactly the same, it may cause less confusion to define the ID as

$$ID = log_2(\frac{A}{\frac{W}{2}}) = log_2(\frac{A}{R}) \quad (1.2)$$

where R is the radius (or half the width) of the target.

Assuming a constant time for the human nervous system to process one bit, typically named the b-constant, it is possible to calculate the time T necessary to steer the stylus into the target by multiplying the ID with the b-constant.

$$T = b \cdot ID = b \cdot log_2(\frac{2A}{W}) = b \cdot log_2(\frac{A}{R}) \tag{1.3}$$

Fitts called $1/b$ the performance index. This value is the number of bits transferred per second. Fitts' experimental setup did not involve reaction times. He used metal-tipped styluses that opened an electrical circuit at the moment of lifting and closed it at contact with the target. In experiments where reaction time is involved, typically a mouse click task, the reaction time a must be added. Then Fitts' Law has the following popular form:

$$T = a + b \cdot ID = a + b \cdot log_2(\frac{2A}{W}) = a + b \cdot log_2(\frac{A}{R}) \tag{1.4}$$

However, the a is never mentioned in Fitts' paper.

The result of Fitts' experiments was that the speed of the movement is not limited by the muscle force; the subjects showed the same performance independent of the weight of the stylus. Instead, the measured times agreed, although not perfectly, with the hypothesis of information processing (see Figure 1.21).

The definition of the ID is more plausible with the plotter analogy presented in the next section (Section 1.4), but such devices were invented just at the time when Fitts wrote his paper.

There are two more derivations of Fitts' Law based on a control-feedback loop presented in Sections 1.5 and 1.6, where no bits or noise are involved. Even if Fitts' analogy with signal and noise amplitudes (not power!) may be debatable, the three other approaches (plotter accuracy, discrete step, and continuous approach) produce the same formula.

1.3 Critique of Fitts' Theory

At the time when Fitts wrote his paper, information theory was extremely popular and authors with all levels of understanding wrote papers using the vocabulary and concepts of information theory. In 1958 Elias, a member of MIT and president of the information theory society wrote an editorial titled 'Two Famous Papers' [9] where he recommends that other disciplines should stop writing papers using the framework and the vocabulary of information theory. In the case of Fitts' theory, it seems to be legitimate to use the framework of information theory. The derivation of the Index of Difficulty from the ratio of the amplitudes of signal and noise leads to a correct definition. Fitts' Law gives valuable guidelines for the design of graphical user interfaces operated with a mouse.

However, the way Fitts phrased his argument with information theory is very debatable. It was an unlucky choice to refer to Shannon's Theorem 17 to explain a limitation by bandwidth. Unfortunately, Fitts continued his argumentation with Shannon's Theorem 17 in a publication in 1964 [11] and prepared the ground for MacKenzie's theory which is the topic of chapter 3.1.

Shannon's Theorem 17 deals with information transmission capacity on a channel in the presence of noise, but for Fitts' research, the limitation is in the processing capacity. Mentioning Shannon's Theorem 17 as proof for a bit transfer limitation is unnecessary as there is nothing infinite in the physical world.

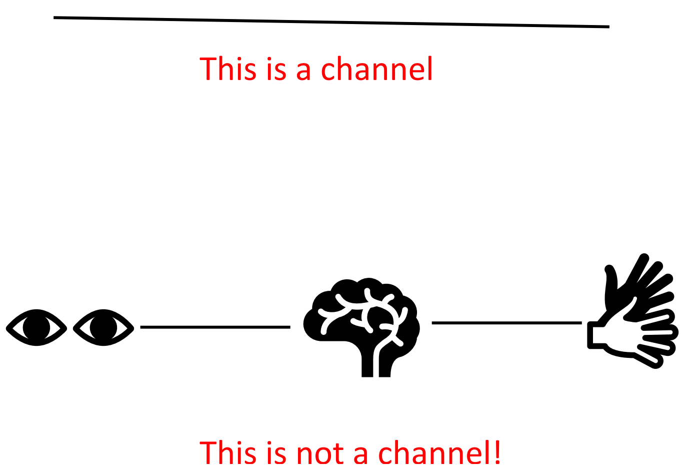

Fig. 1.7 Fitts' feedback loop contains a brain with computing power and memory and therefore is not a information-theoretic channel. Shannon's Theorems, however, only apply to channels.

Shannon defined:

'The channel is merely the medium used to transmit the signal from transmitter to receiver. It may be a pair of wires, a coaxial cable, a band of radio frequencies, a beam of light, etc.' [24]

In contrast, Fitts wrote about:

'the performance capacity of the human motor system plus its associated visual and proprioceptive feedback mechanisms' [10].

Figure 1.7 illustrates that the feedback loop of the visual and motor system is not a channel. According to Shannon's definition, a channel is merely the medium. Fitts' feedback loop, however, contains a brain with computing power and memory. As the feedback loop is not a channel Shannon's Theorems do not apply. Fitts' argumentation with Shannon's Theorem 17 sounds scientific, but finally reveals that he was not familiar with Information Theory. Unfortunately, MacKenzie built a theory based on Shannon's Theorem 17 (see 3.1) which became very popular in the HCI community although the theory has no foundation.

Shannon's Theorem 17 is not applicable to Fitts' Law.

1.4 Information in Accuracy and the Index of Difficulty

Everybody who knows how a digital plotter works understand the relationship between information and accuracy. If we send n bits to a one-dimensional plotter, we can address 2^n positions. From this, it is possible to calculate the Index of Difficulty (ID).

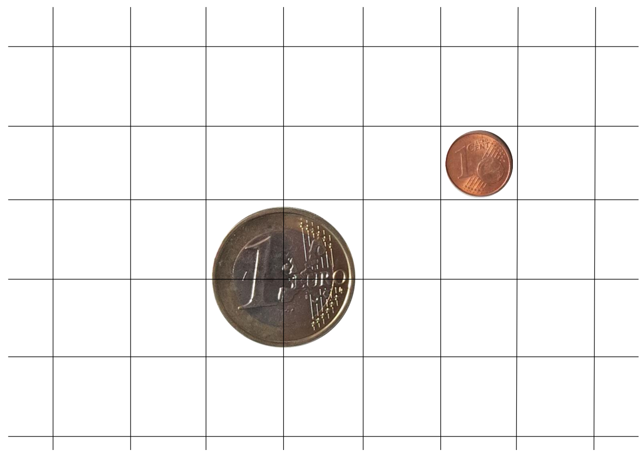

Fig. 1.8 With three bits the plotter can actuate eight positions and hit the big coin. To hit the small coin, it needs one bit more. There is a relation between the size of the coin and the bits needed to hit it.

Let us assume the plotter pen is at position 0 and the target center of a target with width W is at the other end at position A. With every bit sent, the plotter pen approaches position A with half of the step size of the previous step. After n bits, the distance to position A is $A/2^n$. The plotter pen arrives inside the target when the distance to position A is smaller than $W/2$.

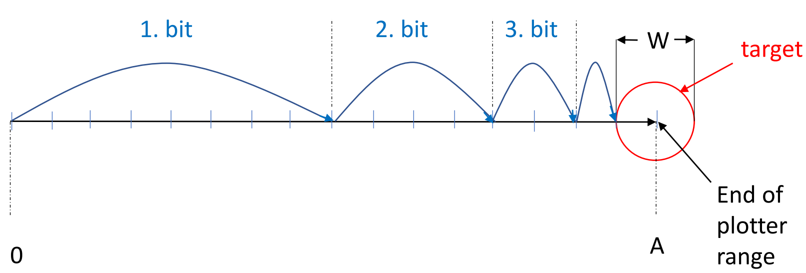

Fig. 1.9 Hitting the target with a plotter. The pen is at the zero position and the target center is at the end of the plotter range.

$$\frac{A}{2^n} \leq \frac{W}{2} \tag{1.5}$$

Dividing by A and inverting leads to:

$$2^n \geq \frac{2A}{W} \tag{1.6}$$

As we want to calculate the value when the pointer reaches the target edge, we can drop the inequality and keep only the equal sign. Taking the logarithm, which yields the unit bit, leads to the definition of the ID, which is the number of bits needed to reach the target:

$$n = log_2(\frac{2A}{W}) = ID \tag{1.7}$$

This derivation of the ID gives a nice, understandable definition:

> If the pointer moves at every step to the middle of the remaining distance to the target center, the ID is the number of steps to reach the target.

Annotation: For a real plotter, Fitts' hypothesis 1 is valid. The transfer of bits to a plotter needs milliseconds or even less, while the positioning of the pen takes fractions of seconds. It is the strength of the motor that limits the speed of a plotter. However, we can add a camera and image processing software with the ability to detect the pen and target position. If this software sends bits to the plotter to position the pen inside the target, the situation is similar to what the human nervous system does, and the limiting factor might be the speed of the image processing.

1.5 The Discrete Step Model

The discrete step model[1] is another derivation of Fitts' Law which does not need the concept of bits. The model starts with a step-wise movement of the pointer towards the target. Every step consists of aiming at the target, moving the pointer to the target, and estimating how close the pointer is to the target. The derivation uses two basic assumptions:

- The distance to the target after each step is proportional to the distance at the beginning of the step.
- Every step takes the same amount of time.

The first assumption reflects the scalability of nature – twice the distance means twice the error. The second assumption reflects a constant information processing power.

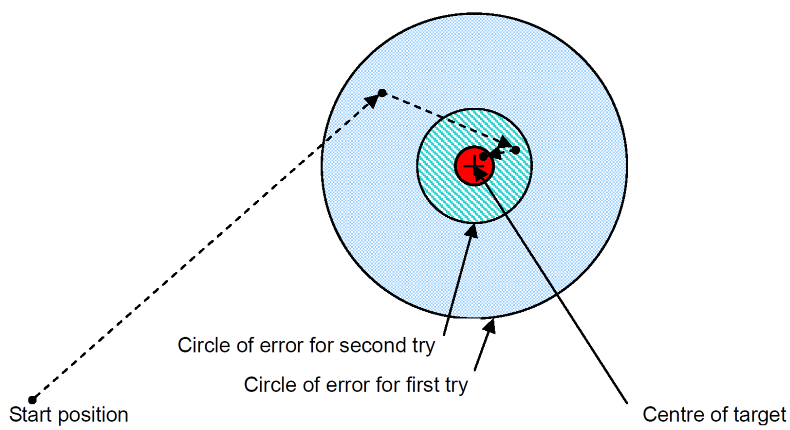

Fig. 1.10 Discrete Step Model: In each step, the pointer moves into the next circle of error.

Figure 1.10 shows discrete steps of sensing and movement. With each step, the pointer gets gradually closer to the target. The figure also shows error circles indicating the area where the pointer will most likely end. These circles of error represent a probability density for the inaccuracies of the movement. The probability density could, but does not need to be, symmetrical. The derivation only demands that the expected value of the distance to the target after a step is proportional to the distance at the beginning of the step. This can be the case for unsymmetrical probability densities, for example, grabbing a cup, where overshooting is not allowed.

Let the distance to the target at each stage be A_i with the initial distance $A_0 = A$. After each step, the distance to the center of the target A_{i+1} is a constant fraction λ of the distance A_i at the beginning of the step.

[1] see also Alan Dix, https://www.hcibook.com/e3/plain/online/fitts-cybernetic/

$$A_{i+1} = \lambda \cdot A_i \tag{1.8}$$

and consequently:

$$A_i = \lambda^i \cdot A \tag{1.9}$$

The process stops after n steps when the distance to the target center is less than the radius R of the target:

$$A_n = \lambda^n \cdot A < R \tag{1.10}$$

As it is possible to choose a logarithm with any base, we choose the binary logarithm log_2 and get:

$$n = \frac{log_2(\frac{R}{A})}{log_2(\lambda)} \tag{1.11}$$

Each step takes a fixed time τ. The total time T to reach the target is:

$$T = \tau n = \tau \cdot \frac{log_2(\frac{R}{A})}{log_2(\lambda)} = b \cdot log_2(\frac{A}{R}) \tag{1.12}$$

where $b = -\tau / log_2(\lambda)$. As the pointer gets closer to the target with each step, λ is smaller than 1 and $log_2(\lambda)$ is negative, so b is positive.

The formula derived from the discrete step model is exactly Fitts' formula. Together with an initial time a for the brain to get started, i.e. reaction time, we get the popular form:

$$T = a + b \cdot log_2(\frac{A}{R}) \tag{1.13}$$

Again, we got Fitts' formula 1.7 (and not one of the alternative formulas), but this time without information theory and terms like ‘bits’ or ‘noise’.

1.6 The Continuous Approach Model

The problem with the discrete step model is its discreteness, which does not allow one to describe the movement on a continuous time scale (equation of motion). However, it is not very difficult to extend the discrete model to a continuous model by making the steps smaller and finally doing an infinitesimal transition (see also [5]).

Let $x(t)$ be the distance to the target center at time t and $x(0) = A$. In an time step Δt the pointer gets Δx closer to the target. Together with the assumption that Δx is proportional to the current distance $x(t)$, we get the equation:

$$\Delta x = c \cdot x(t)\Delta t \tag{1.14}$$

Doing a infinitesimal transformation and replacing Δ with d we get:

$$dx = c \cdot x(t)dt \tag{1.15}$$

with c as the proportionality factor. The factor c is negative as we move towards the target at the origin of the coordinate system. With a simple transformation:

$$\frac{dx}{dt} = c \cdot x(t) \tag{1.16}$$

we get a (very simple) differential equation, which is well-known from atomic decay or from discharging a capacitor. The solution for the differential equation, the equation of motion, is an exponential function (e is the Euler number):

$$x(t) = A \cdot e^{ct} \tag{1.17}$$

It is easy to see that $x(0) = A$ and that $x(t)$ tends towards zero when time goes towards infinity as c is negative.

Of course, it is possible to derive Fitts' Law from the equation of motion 1.17. When the pointer reaches the target edge, the pointer is radius R (or half the width $W/2$) away from the target center. With T as the time to reach the target, we get the following equation:

$$x(T) = A \cdot e^{cT} = R \tag{1.18}$$

Solving this equation for T by taking the logarithm we get:

$$T = \frac{1}{c} \cdot ln(\frac{R}{A}) \tag{1.19}$$

We can transform this equation to:

$$T = -\frac{1}{c} \cdot ln(\frac{A}{R}) \tag{1.20}$$

Together with $ln(x) = log_2(x)/log_2(e)$ we can write:

$$T = b \cdot log_2(\frac{A}{R}) \tag{1.21}$$

with $b = -1/(c \cdot log_2(e))$. This is again Formula 1.3 given by Fitts. However, the reason for introducing the continuous approach model was not to derive Fitts' formula a fourth time. We will need the equation of motion in the next section (Section 1.7) to calculate the pointer speed over time.

1.7 Reality versus Model

In general, it is desirable to have a model which predicts reality with high accuracy. However, accuracy is not the only criterion for a model. For any measured data, it is possible to find an empirical function, for example, a polynomial, which approximates the data quite well. In the eyes of an engineer, such an empiric function is valuable because it allows for the prediction of accurate values. In the eyes of a scientist, this function does not help much, as it does not explain underlying mechanisms. Even if the accuracy is lower, scientists prefer a formula derived from assumptions. If measured data fit the derived formula, this strongly supports the assumptions made. Even if an empirically derived function produces better results because of 'dirty effects' not considered by the model assumptions, the theoretically derived formula has more explanatory power.

Sometimes the benefit of a model lies in simplification. Typically, physicists derive formulas for a perfect sphere rolling down a perfect plane. The resulting formula does not predict the motion of a real rock rolling down a hill with high accuracy. However, the formula reflects concepts of translation, rotation, and potential energies and allows for general statements. A formula with the ambition to predict the motion of a real rock rolling down a hill, if such a formula exists, would need many parameters specifying the shape of the stone and the hill. For most practical purposes, this would not be worth the effort.

After this discussion on the value of models, we will now look at the continuous model introduced in Section 1.6. This model is a simplification and does not match reality perfectly. The main problem is the initial speed of the movement.

To get the speed $v(t)$ we take the derivative of the equation of motion (1.17) with respect to time:

$$v(t) = \frac{dx(t)}{dt} = c \cdot A \cdot e^{ct} \tag{1.22}$$

and the initial speed at $t = 0$ is $v(0) = c \cdot A$.

Figure 1.11 shows the speed over time as calculated with the continuous model. This initial speed contradicts the fact that the pointer rests at the beginning of the task; nothing can be accelerated within zero time. Figure 1.12 shows speed over time as expected by common sense, i.e. considering acceleration.

To show that common sense matches reality, Figure 1.13 shows true mouse speed data from a user study [6]. The study was not designed for measuring mouse speeds, and due to constraints on the temporal resolution of the data imposed by the 10 ms time slice length of the operating system (Windows XP), the data is a little bit noisy. The first 200 milliseconds are the reaction time. The rest of the curve looks similar to Figure 1.12.

The consequence of the discrepancy between theory and common sense is that the time to reach the target is a little bit longer than predicted by the theory because

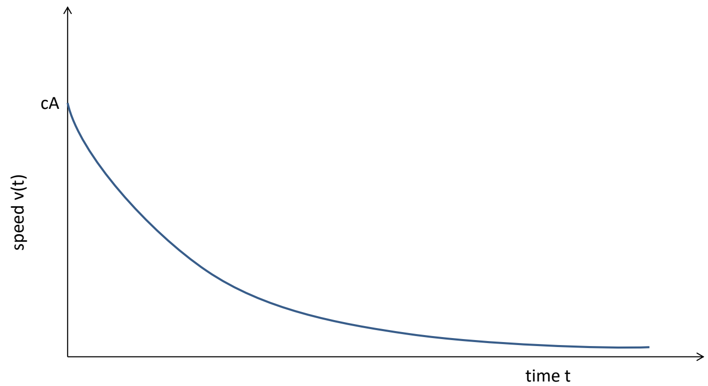

Fig. 1.11 Speed over time for the continuous approach model

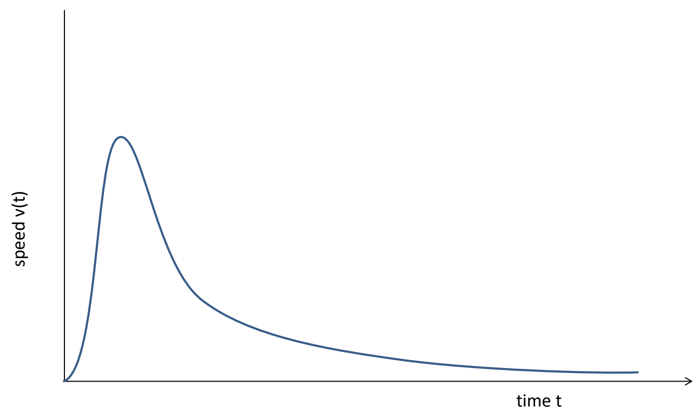

Fig. 1.12 Speed over time as expected by common sense

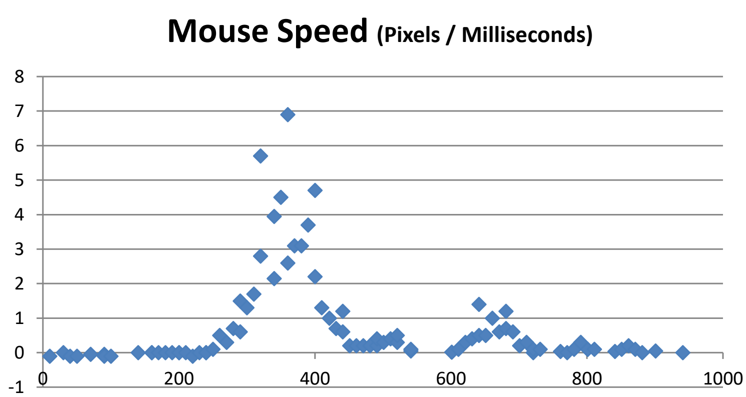

Fig. 1.13 Speed (in pixels/millisecond) over time (in milliseconds) from measured data

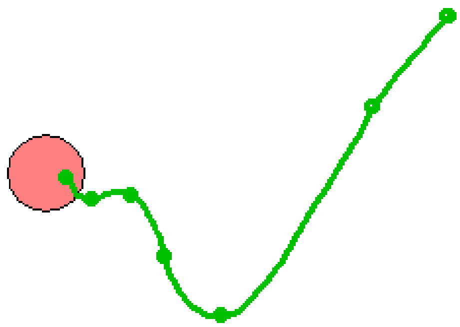

Fig. 1.14 Curved mouse path to target. The green dots indicate 100 ms intervals.

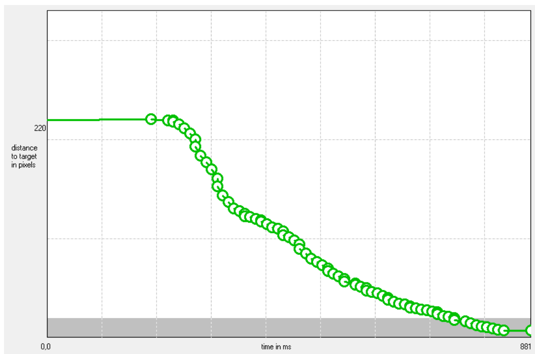

Fig. 1.15 Distance to target over time until mouse click for Figure 1.14 (curved mouse path)

of the additional time required to accelerate to the initial speed. For this reason, an alternative Fitts' Law formula may fit marginally better to actual data.

People who go for a more realistic model with better predictive power can enhance the continuous model by adding an acceleration phase. The initial speed is given above (Formula 1.22), and the time to reach this speed can be calculated with the assumption of gradually increasing muscle force [1]. During the acceleration phase, the pointer has already covered some distance toward the target. Therefore, the Fitts' Law phase must be calculated with a reduced distance. It is not very difficult to derive the formula for such an enhanced model, but this brings along two further parameters, the mass (of mouse and arm) and the achievable acceleration (individual muscle strength or precisely the constant of the constantly increasing force). The increase in model complexity introduced by the addition of the two extra parameters is not worth the expected slight improvement in the prediction of the movement time. The acceleration phase is short (see Figure 1.13) in comparison to the total execution time and does not add completely as the pointer covers some distance in the acceleration phase. This confirms the thoughts from the beginning of this section – a simple model can be more helpful than a complex model.

The continuous approach model was introduced to calculate an initial speed. As this model considers only radial speeds, we are free to choose tangential movements. This means we can model curved paths to the target. Figure 1.14 shows a mouse path recorded in a user study. The green dots indicate the mouse position at 100 ms intervals. Figure 1.15 shows the corresponding distance to the target over time. The gray area represents the target.

However, this model does not match all situations. In many cases, especially when asked to hit the target as quickly as possible, people tend to overshoot the target. Overshooting the target means that the pointer crosses the target and has to move backward. Therefore the velocity changes the sign. Figure 1.16 and Figure 1.17 shows an example recorded in a user study. The continuous approach model cannot explain this situation.

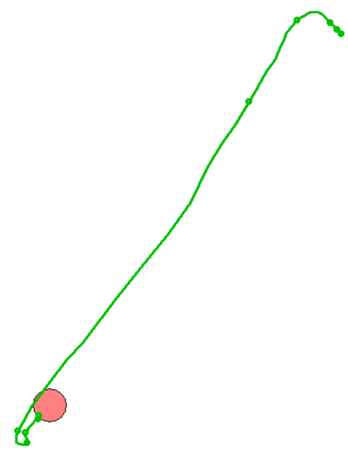

Fig. 1.16 Overshooting the target

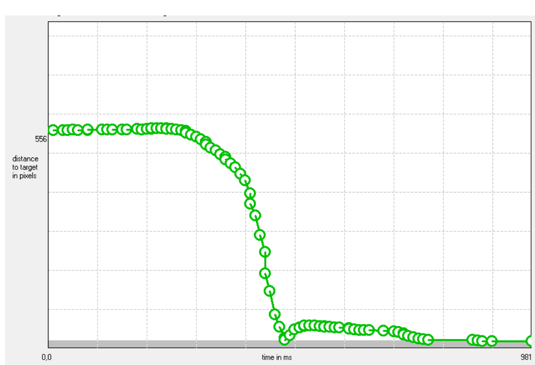

Fig. 1.17 Distance to target over time until mouse click for Figure 1.16 (overshooting the target)

1.8 Limits of Fitts' Law

Every theory is only valid within a certain range, and this is also the case for Fitts' Law. If the ID is very small or very large, Fitts' Law is not applicable. If a person stands next to a 1 m x 1 m table and has her or his hand 20 cm away from the edge of this table and is asked to knock on the table, it will take a certain time until the hand reaches the table. If we replace the table with a bigger 2 m x 2 m table and ask the person to do the same task again, the time to reach the table will be the same, although the ID is smaller because of a bigger target size. The reason for the same execution time is that for both tasks, the challenge is to bridge the gap of 20 cm. As the table is big enough in both cases, its size is irrelevant. The person fulfilling the task does not aim at the table center but to a point behind the table edge.

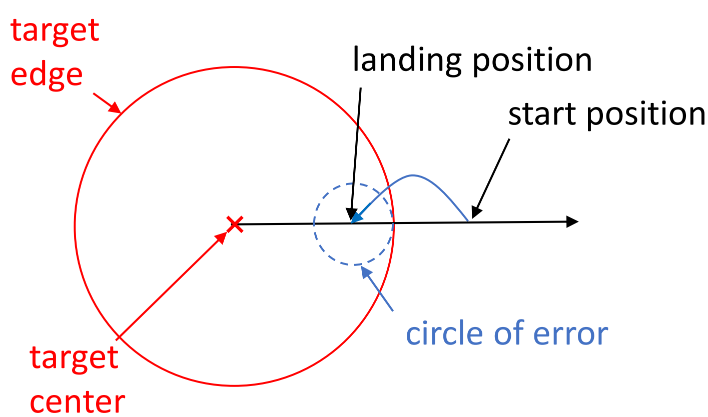

Fig. 1.18 If the pointer is close to a big target, the aim is not the target center, but a point across the target edge. How far this point is away from the edge depends on the size of the error circle.

Figure 1.18 shows the situation. The distance the pointer covers is the distance to the target edge plus a safety distance, which is about the accuracy of the move without a feedback-control loop. Figure 1.18 shows this accuracy as the circle of error. In this situation the time to reach the target does not depend on the target size but only on the distance of the pointer to the target edge, which is a ballistic movement. This means that if the pointer is close to the target, Fitts' Law does not apply.

This raises the question of how far the pointer has to be away from the target that Fitts' Law applies. Figure 1.19 tries to answer the question with common sense. Let us assume an error of about 25% for the ballistic movement. If the pointer is only one target radius away from the target edge, this means an ID of 1, the aim is not the target center. For a short move, the circle of error is small, and the distance to the target edge plus the radius of the error circle is shorter than the distance to the target center. If the pointer is three target radii away from the target edge, this means

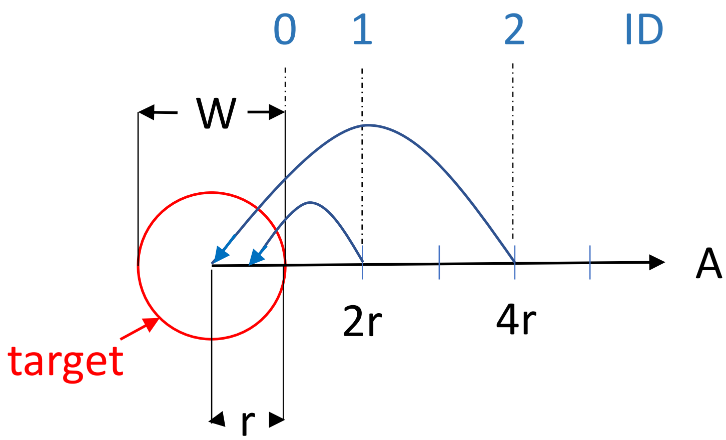

Fig. 1.19 When the pointer is only one target radius away from the target edge ($ID = 1$), the aim is not the target center. If the pointer is three target radii away from the target edge ($ID = 2$), the aim is the target center, if the error circle is bigger than the target.

an ID of 2, the radius of the error circle exceeds the target radius and the aim is the target center. With this rough estimation, we can say that Fitts' Law applies if the ID is bigger than two. Hoffmann's review [15] reports on values between 1.5 and 4 for the lower bound of the ID.

This lower bound for the ID has relevance for HCI. When operating a graphical user interface with a mouse, it happens quite frequently that the target is large and the pointer is not far away from the target. A typical example is to activate a window that covers half of the screen.

There is also an upper limit for the ID. The maximum distance to the target and the minimum target size determine the upper limit. Typically, the maximum distance in a pointing experiment is the arm length, and the minimum target size is the visual resolution – the target must be visible – and bigger than the amplitude of muscle tremor. For a pointing task on a computer screen, the maximum distance depends on the screen size, and the minimum target size is the pixel size. From our daily experience, we know that addressing a single pixel on a high-resolution display (2K or bigger) is already difficult. It depends on the individuals' vision and muscle tremor until which resolution she or he is still able to address a single pixel. If we take $A = 2048$ pixels and $D = 1$ pixel, the corresponding ID is 12.

For the application of Fitts' Law, the ID should be between 2 and 12 or, to be on the safe side, between 3 and 10.
For smaller IDs, the movement becomes ballistic. For higher IDs, the task becomes impossible.

There is a further limit for Fitts' Law by the pointer size. Ideally, the pointer's tip size is infinitely small. Practically, the pointer's tip has a finite size. If the pointer is the fingertip, the task to hit a target of 1 mm, 2 mm, or 4 mm in diameter does not make a relevant difference as these targets are all smaller than the fingertip. If the hit condition is that the target area and the pointer tip area overlap, the task is equivalent to a pointing task with an infinite small pointer tip and a target with a diameter, which is the sum of the diameters of the original target and pointer tip. In such a situation, the smallest possible effective target size is the pointer size (see Figure 1.20).

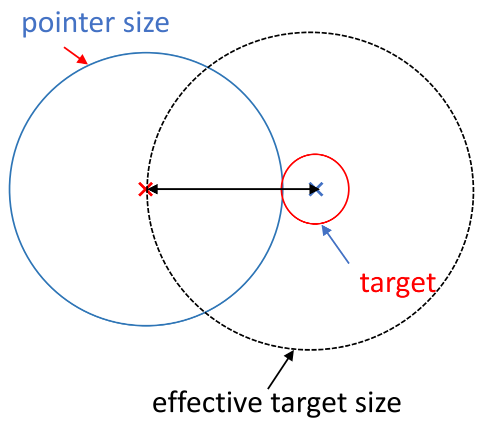

Fig. 1.20 If the pointer has a finite size the effective target width is the sum of target size and pointer size.

Besides pointing with a finger, the reason this is mentioned here is gaze pointing, where the size of the pointer is the size of the fovea, the small region where we can see with high resolution. However, Fitts' Law does not apply to eye pointing (see 3.4).

1.9 Evaluation of Fitts' Law Data

A typical Fitts' Law study lets the participants move a pointer from distance A to a target of size W (width or diameter) and measures the time T to reach the target. The evaluation of a Fitts' Law study depends on the design of the experiment, or the other way round, the study design depends on the planned evaluation.

One important question is whether reaction time is involved. If there is no reaction time involved, as it is the case in Fitts' reciprocal tapping task (see 1.2), the completion time is proportional to the ID, which means $T = bID$. The evaluation has to be done accordingly, which means a linear fitting model without an intercept. The derivation of the needed formula is given below.

The measured times deviate by error e_i from $T_i = bID_i$.

$$T_i = bID_i + e_i \tag{1.23}$$

The square error function $E(b)$ is:

$$E(b) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (T_i - bID_i)^2 \tag{1.24}$$

Differentiating 1.24 with reference to b and setting it to zero delivers the b for the minimum error.

$$E(b)' = \sum_{i=1}^{n} 2(T_i - bID_i)(-ID_i) = 0 \tag{1.25}$$

So the b with the minimum error is:

$$b = \frac{\sum_{i=1}^{n} T_i ID_i}{\sum_{i=1}^{n} ID_i^2} \tag{1.26}$$

The second derivative of $E(b)$ is twice the sum of ID^2 which is positive and confirms that $E(b)$ is minimized for this b.

If there is no reaction time involved, but the data is evaluated with linear regression, it is likely that the intercept is not exactly zero. In consequence it is necessary to discuss whether the intercept's deviation from zero is within an error interval determined by the noise in the data, or whether there is a hidden reaction time not considered in the study design.

In a setup that involves reaction times, the completion time T for the task obeys $T = a + bID$, where a is the reaction time. The intercept calculated with regression analysis delivers the reaction time. The derivation of the formula to estimate the a and the b is similar to the derivation of the linear fitting model without an intercept as given above but requires finding the minimum for two variables. However, most spreadsheet applications offer a linear regression just by ticking a checkbox.

The other important question is whether the independent variables, target size and distance, are chosen randomly from an interval or from discrete values. Fitts had a mechanical setup with a limited number of physical targets and could use only discrete values for the target size. Fitts constructed these values for target size and distance by doubling the previous value. The advantage of this approach is that several combinations of size and distance which result to the same ID.

The evaluation of a Fitts' Law study should have two goals:

1. Showing that Fitts' Law applies
2. Estimating the value of b

Most publications reporting on Fitts' Law studies do not prove the first goal rigorously but either assume that Fitts' Law is valid or that a value for R^2 close to 1 in the regression analysis proves the validity. The later is not true, especially for regression analysis on already averaged data (see 1.10).

Fitts proved the validity of his law by estimating the performance rate ($1/b$) for all conditions and presented them in a three-dimensional figure (see Figure 1.21). For a perfect fit, all values have to be on a horizontal plane, as the performance rate should be the same in all conditions. Figure 1.21 shows that Fitts' data does not fit perfectly but are constant within an error tolerance of 20%. Fitts did not report and indicate the performance rate's error interval, such as the standard deviation in his figure, and did not discuss whether the data is constant in consideration of the error range. Instead, Fitts argued that the data fit his law only within an optimum range.

'The results indicate that rate of performance in a given type of task is approximately constant over a considerable range of movement amplitudes and tolerance limits, but falls off outside this optimum range.' [10]

Another way to show the validity of Fitts' Law would be to show that execution times for the tasks depend only on the ratio of distance and target size. The discrete setup is best to do this as there are tasks with the same ratio of distance and target size but different target sizes, or distances, respectively. A t-test on the execution times of pointing tasks with the same ID but different target sizes, or distances, respectively, could show that the execution time is independent of the distance. However, the t-test may fail if the data set is large enough, which means it may show that the execution times for different distances are not from the same distribution, even if the IDs are equal. The reason is that the derivation of Fitts' Law is based on idealizations as discussed in the previous section (1.7). The (short) acceleration phase of the pointing movement is not considered and leads to a small dependency on the distance independent of the target size. Statistical tests are optimized to find even

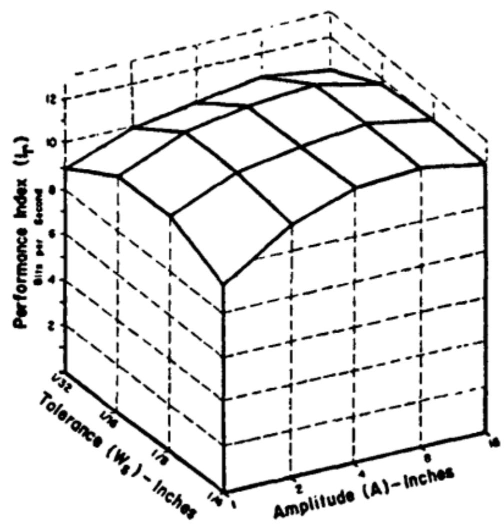

Figure 4. **Three-dimensional representation of performance rate (I_p) in bits per second for the pin-transfer task as a function of amplitude and tolerance requirements.**

Fig. 1.21 Fitts [10] estimated the performance rate $(1/b)$ for all conditions and presented them in a three-dimensional figure. For a perfect fit all values have to be on a horizontal plane.

small dependencies.

In the case of a setup with randomly chosen values, one way to show that the times do only depend on the ratio of distance and target size would be to show that the data points above and below the regression line are balanced according to the distance and target size.

The second step would be to show the linear dependency of execution times from the ID. From distance A and target size W it is possible to calculate an ID ($ID = log_2(2A/W)$). The execution times are plotted over ID in a scatter plot. For a linear dependency, the execution times should lie on a line. If no reaction times are involved, the line has to go through the origin. If the execution times include reaction times, the intersection of the line with the ordinate, also called intercept, specifies the reaction time. Figure 1.22 is an example of such a plot and analysis. A regression test calculates the line of best fit, also called the regression line. The slope of the regression line is the value of b.

As mentioned already, most spreadsheet applications offer a scatter plot function and the calculation of a regression line with a few simple mouse clicks. Figure 1.22 was made in this way. It additionally shows the equation of the regression line and the R^2-value, which is also called the coefficient of determination. R^2 is a measure

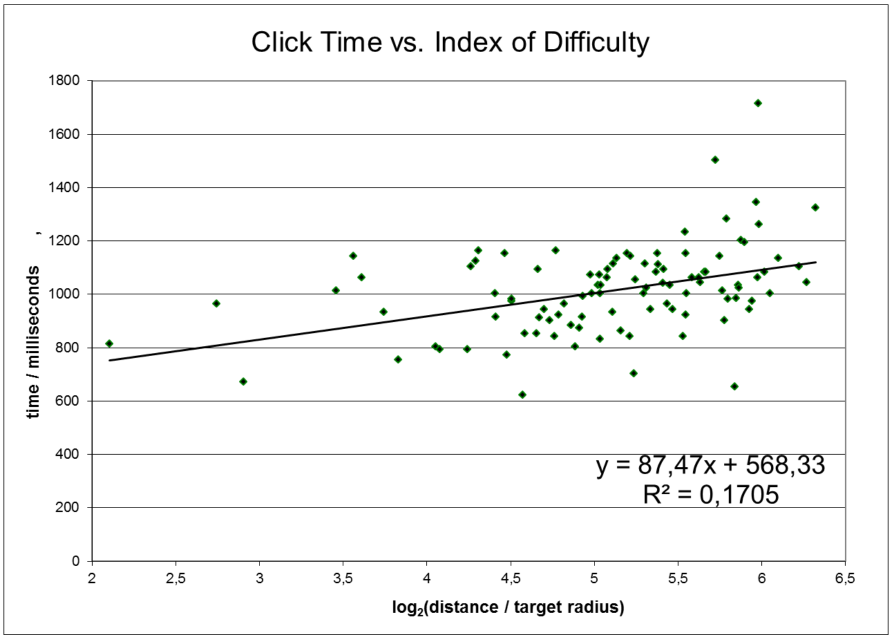

Fig. 1.22 Measured Fitts' Law data and regression line from a user study for 100 mouse clicks. Typically, the data are more a cloud than on a straight line.

of the goodness of fit. A value of R^2 equal to 1 indicates a perfect fit. In the case of simple linear regression, R^2 is the square of the correlation coefficient. Therefore many evaluations of Fitts' Law experiments report a correlation value. As the correlation is the square root of R^2, which is R, it is even closer to 1. The theory of linear regression is a topic of countless textbooks and standard knowledge in statistics, but beyond the scope of this lecture. It is beneficial to study these textbooks, as they help to avoid misconceptions.

Most studies on Fitts' Law report a correlation, but correlation does not tell much. It would be much better to know the accuracy of the *a*- and *b*-constant given as an interval. The textbooks mentioned above explain how to calculate these intervals. However, there are software packages that do the job. One such software package is `gnuplot`. The program `gnuplot` provides these intervals, together with the other values, using the following commands shown in Figure 1.23 (The file `fit100.txt` contains the measured data used in Figure 1.22):

`gnuplot` gives us a standard error and now we know that the a-constant is in the range from 468 to 668 milliseconds and the *b*-constant is in the range from 68 to 107 milliseconds/bit. This statement helps much more to compare different measurements of Fitts' Law data.

```
gnuplot> f(x) = a + b*x
gnuplot> fit f(x) 'fit100.txt' using 2:1 via a, b

...

After 4 iterations the fit converged.
final sum of squares of residuals : 2.24853e+006
rel. change during last iteration : -2.89397e-010

degrees of freedom    (FIT_NDF)                        : 98
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 151.473
variance of residuals (reduced chisquare) = WSSR/ndf   : 22944.1

Final set of parameters            Asymptotic Standard Error
=======================            ==========================

a               = 568.329          +/- 100.4        (17.66%)
b               = 87.4702          +/- 19.49        (22.28%)


correlation matrix of the fit parameters:

               a      b
a              1.000
b             -0.989  1.000
```

Fig. 1.23 gnuplot's output for the data in Figure 1.22

An evaluation is not done with stating the values, but needs a critical discussion. From the *b*-constant we know that it took 87.5 milliseconds to transmit one bit or that 11.4 bits were transmitted in a second. This matches with the values measured by Fitts:

'Performance on the pin-transfer task varied from 8.9 to 12.6 bits/sec, for the 20 conditions studied, ...' [10]

The *a*-constant, which is equivalent to reaction time, is around 568 milliseconds. As typical reaction times are 200 to 300 milliseconds this value seems to be rather high. However, the reason is that there are two reaction times involved in the experiment – the first reaction time is the time to start moving the mouse (see Figure 1.13) and the second reaction time is the time to click the mouse key when arriving in the target (see Figure 1.15). `gnuplot` can also plot the data (see Figure 1.24).

The evaluation given so far is still not perfect. Measured data, in this case distance and time, always have measurement errors. This error can be visualized by error bars in the plot. The time for the presented data was measured on an operating system with ten milliseconds time slices (Windows XP) and therefore the error in the time measurement is around one percent. With respect to the high standard error reported

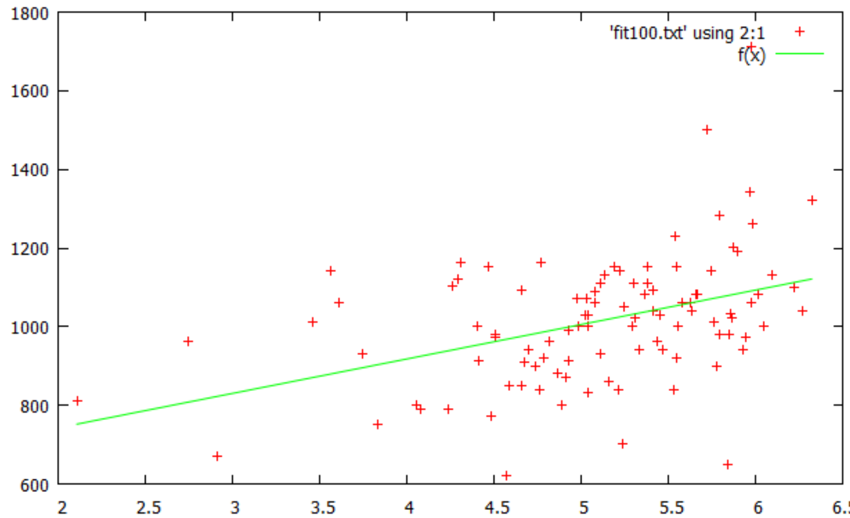

Fig. 1.24 gnuplot's plot for the data in Figure 1.22 (time in milliseconds over ID in bits)

by `gnuplot`, it is legitimate to neglect the error of the input data. However, it is a good scientific style to report error intervals for the raw data.

Finally, the standard error reported by `gnuplot` assumes certain properties for the data distribution. For a perfect evaluation, it is necessary to show that these assumptions are true. However, this is cumbersome and sometimes even difficult.

1.10 How Not to Evaluate Fitts' Law Data

Most HCI publications which report about Fitts' Law studies have severe deficits in their evaluation. It starts with associating the *b*-parameter with the device, going on with using the wrong formula, continuing with very questionable statistic evaluations, and finally forgetting to show that Fitts' Law applies at all. There is a check list with recommendations for correct Fitts' Law studies at the end of this section.

The logarithm can approximate most data on a curved line quite well. The reason is that the logarithm offers all curvature values. By scaling and shifting the logarithmic curve, it is possible to find parameters that approximate the data well within an interval. This makes it possible to claim the validity of Fitts' Law for most data, even if it is not the case. A regression line over a logarithmic scale does not prove a logarithmic relation, even if the correlation is good.

To achieve a good correlation, many HCI researchers use the discrete setup, averaging the values for the same ID, doing a regression analysis with the few data points left after averaging, and reporting a correlation close to 1. However, correlation strongly depends on the number of data points used for the calculation. Furthermore, correlation does not say anything about significance – it needs another statistical test to show this, and this test uses $n-2$ degrees of freedom for n data points. In other words, averaging the data first and calculating a correlation afterward reduces the significance of the correlation value.

If one is doing Fitts' Law studies with a mouse device and a computer screen, the restrictions of a mechanical setup do not exist, and it is possible to design the study on a nearly continuous scale (depending on screen resolution) instead of discrete values. The data in Figure 1.22 is from an experiment on a continuous scale. The value of R^2 is not close to 1, indicating a not perfect fit. To demonstrate that correlation depends on the number of data points, Figure 1.25 shows the same data of 100 mouse clicks as in Figure 1.22 but averaged first over groups of 10 data points. The slope and offset for now only ten data points did not change much, but the R^2 went up from 0.17 to 0.66 (0.81 for R). With two data points only, R^2 will have the perfect value of 1.0.

It is worth mentioning that Fitts did not argue that a high correlation proves his assumption. He did a critical analysis of his data with the result that his or her data was not perfect. Fitts' mentioning of correlation sounds more like an apology:

'The Pearsonian correlation between the 16 values for the two variations in the tapping task was large however (r = .97).' [10]

This sentence also shows that Fitts was aware that correlation depends on the number of values and therefore mentions the 16 values. As Fitts' experiment had four different target sizes and four distances, the 16 values also tell that Fitts did not average the values for the same ID first before doing the evaluation. However, Fitts calculated the correlation with averaged values for each condition, and this is the reason why the correlation was good.

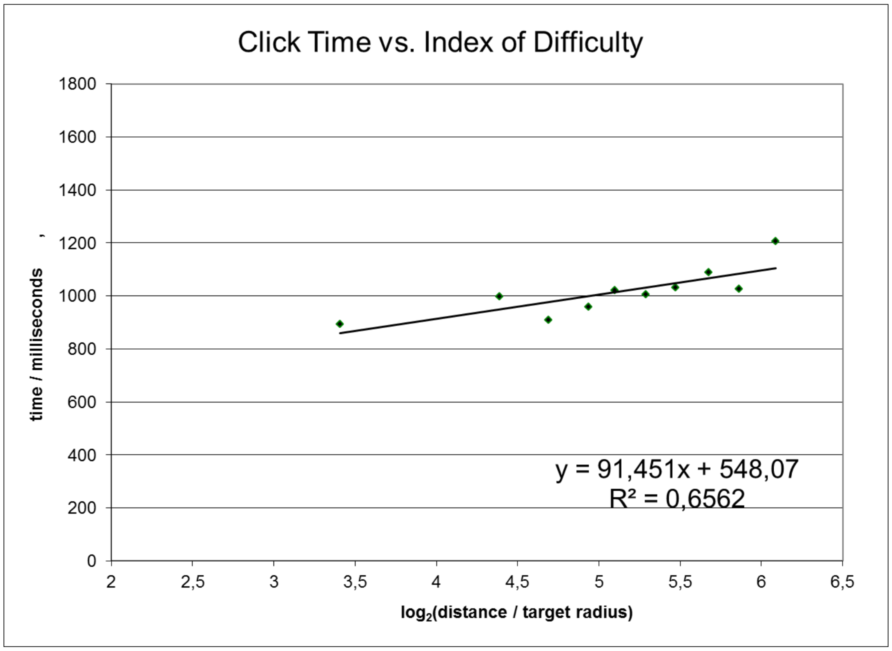

Fig. 1.25 Fitts' Law data for 100 mouse clicks, but averaged over sets of 10 data points first. The a- and the b-parameter do not differ much in value compared to the evaluation given in Figure 1.22. However, R^2 is much better.

Fitts had a mechanical setup and therefore had only a few different IDs. He used four different target sizes and four different distances. He chose sizes and distances in a ratio of 1, 2, 4, and 8. This is an advantageous choice as it includes different pointing tasks with the same ID. This allows for testing the validity of Fitts' Law: pointing tasks with different target sizes but the same ID should have the same completion time.

With a discrete setup, the corresponding scatter plot looks like the schematic diagram given in Figure 1.26. In the HCI community, it seems to be common practice to average the completion time for the same ID before evaluating the data with a regression test. This is not legitimate because it makes the correlation meaningless. Figure 1.26 shows two different data sets, which will produce the same value for the correlation if averaged first. However, it is obvious that the left data set fits much better. Figure 1.27 shows the data set from Figure 1.26 with the standard deviation as error bars. The dotted lines show the visual estimation of a minimal and a maximal slope of the regression line. If the data set is too big to visualize in the style of Figure 1.26, it should be visualized with error bars, as this allows us to estimate the range of the regression line's slope.

Averaging the data points over IDs from different target sizes, respectively distances, is not legitimate because it assumes the validity of Fitts' Law already before testing the validity. The consequence of such ill method is that the test confirms the

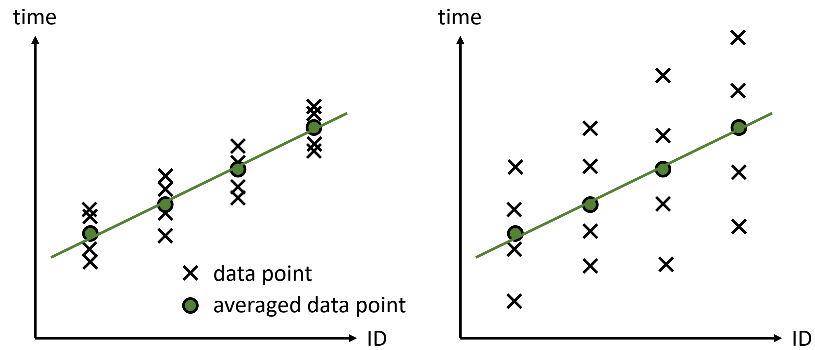

Fig. 1.26 Two different data sets with the same correlation if averaged first. The crosses represent the data, and the circles represent the means.

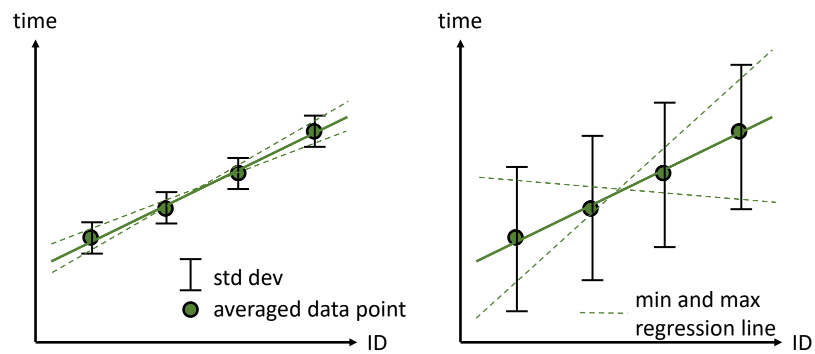

Fig. 1.27 The data sets from Figure 1.26 with the standard deviation as error bars. The dotted lines show regression lines with the minimal and the maximal slope.

assumption, which was put implicitly into the data, even if Fitts' Law does not apply. This is what has happened in many publications, for example [20], which state that Fitts' Law is valid for eye movements.

To illustrate this, let us assume that the formula of Carpenter (see Chapter 3.4, formula 3.10) applies to eye movements. The formula of Carpenter is an early and not very accurate approximation for eye movements, but it is a good example to demonstrate the effect of averaging over IDs before doing the regression analysis. With Carpenter's formula, the time to hit the target depends linearly on the distance but not at all from the target size.

Let us do the evaluation as given in *'Application of Fitts' Law to Eye Gaze Interaction'* [20], but let us assume a linear dependency from the distance only. This means the completion time is proportional to the distance. Figure 1.28 shows a schematic diagram for three target sizes and three distances fully crossed, which results in nine data points. For the middle ID there are three combinations of distance and size. These three combinations have different completion times as the completion time depends only on the distance by assumption. Under this assumption, the data are not on a straight line, but the averaged data are, and a regression test on the five averaged data produces a 'very good' correlation.

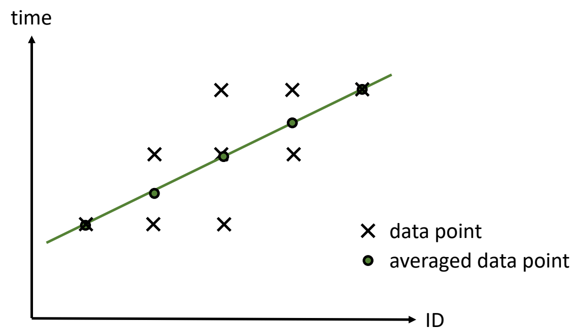

Fig. 1.28 Data set which assumes a completion time only linearly depending on the distance, but not on target size. Averaging over IDs first confirms Fitts' Law, even if it does not apply.

As the argumentation above is only on a qualitative level, let us have a closer look on a quantitative level. Let us again assume a linear dependency for the completion time T, which only depends on the distance A and not on the target size. This means $T \propto A$. For convenience , the target sizes and distances are 1, 2, 4, and 8 in arbitrary and perhaps different units. This is legitimate because the correlation is independent of scales. The four target sizes and distances with the lengths 1, 2, 4, and 8 result in sixteen values with seven different ratios, as shown in Table 1.1.

Table 1.1 Ratios for all combinations for four distances A and four target radii R

A $\quad$ R	1	2	4	8
1	1	2	4	8
2	1/2	1	2	4
4	1/4	1/2	1	2
8	1/8	1/4	1/2	1

Typically, the subjects in such a user study perform pointing tasks over all combinations of target sizes and distances, which results in data probes of uniform distribution over all combinations. With a linear relation it is possible to calculate the average execution time from the average distance. Table 1.2 shows all possible IDs and the resulting average distances ($\sum A/\#$), which by assumption is proportional to the completion time T. Again, for convenience, we assume $T = A$.

Figure 1.29 shows the completion time (as circles), which is equal to the average distance, over ID as given in Table 1.2 together with their trend line (solid) and all 16 data (as crosses) according to Table 1.1 also together with their trend line (dashed). For all 16 data R^2 is only 0.46 but for the 7 averaged data R^2 is 0.97 which

Table 1.2 All possible ratios from Table 1.1 and the corresponding IDs and average distance.

A/R	$\|log_2(A/R)$	#	$\sum A$	$\sum A/\#$
1/8	-3	1	1	1
1/4	-2	2	1+2	3/2
1/2	-1	3	1+2+4	7/3
1	0	4	1+2+4+8	15/4
2	1	3	2+4+8	14/3
4	2	2	4+8	6
8	3	1	8	8

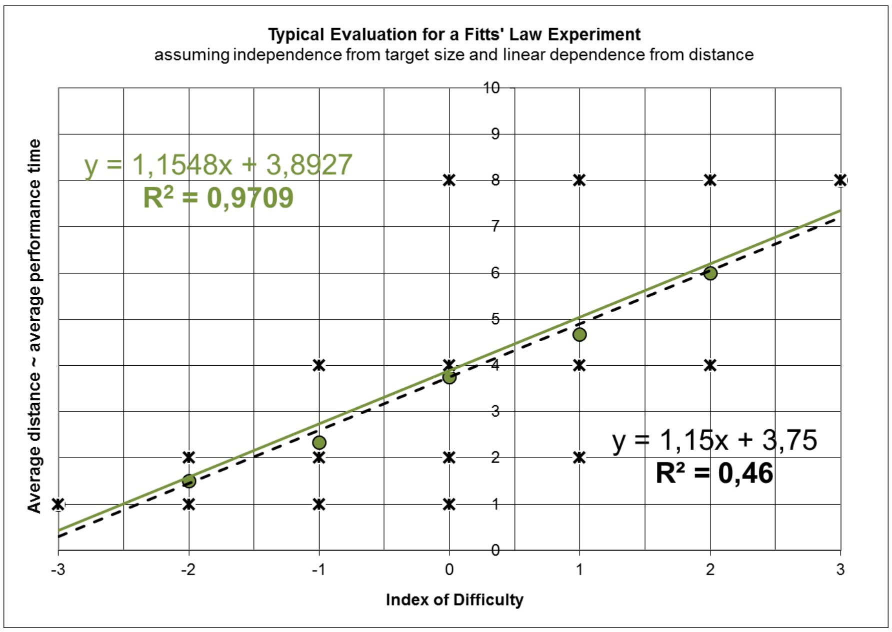

Fig. 1.29 Assuming a pointing task where the completion time depends only on the distance, but not the target size. There are 16 data points (crosses) in a 4x4-condition, which can be averaged for seven IDs. The regression analysis for the 16 data points is black, and for the seven data points is green. Averaging the data for same IDs forces the validity of Fitts' Law.

is very close to 1.0. However, the calculation here was done under the assumption that Fitts' Law does not apply. In consequence, a value for R^2 or for the correlation close to 1 is no proof of the validity of Fitts' Law if the data were averaged before.

The following list summarizes recommendations to avoid flawed evaluations.

- Make clear what the meaning of the *b*-parameter you want to measure is.
  The *b*-parameter characterizes the processing power of the human brain and is not a property of a device or interface (see section 3.3). Consider reporting times without a Fitts' Law evaluation.

- Do not forget to show that Fitts' Law applies at all.
  Use the recorded data to show this (see section 1.9).

- Use Fitts' formula and neither MacKenzie's formula nor the ISO 9241-9 standard.
  MacKenzie's formula is unfounded (see section 3.1) and the ISO 9241-9 standard uses MacKenzie's formula.

- Use the correct formula for calculating the regression line.
  Depending on the experiment's setup the measured times may or may not contain reaction times. If no reaction times are involved, the intercept has to be zero (see section 1.9).

- Do not average the data before calculating the regression line.
  Averaging the data before calculating the regression line delivers perfect R^2-values close to 1 but these R^2-values do not have any significance (see above).

- State a confidence interval for the *b*-parameter instead of a correlation.
  The correlation value depends strongly on the number of data points used for the calculation and does not tell anything about the accuracy of the reported *b*-value (see section 1.9).

- Report the *b*-parameter with units, i.e. s/bit or ms/bit.

Chapter 2
Fitts' Law Extended Topics

Abstract This chapter deals with the application of Fitts' Law on multi-dimensional movements and target shapes. The main topic, however, is the Steering Law, which is a valuable contribution with practical applications to interface design.

2.1 Two- and Three-dimensional Movements and Target Shapes

Many publications on two-dimensional movements and targets exist, for example [18], [25], [3], [36].

There is no reason to make the two- and three-dimensional case more difficult than it is. Looking at an x-y-plotter, it is clear that we have to transmit double the amount of information, e. g. bits, to a two-dimensional plotter compared to a one-dimensional plotter. However, the positioning of the plotter pen does not take twice as much time as typically both step motors work in parallel. The situation is the same for the muscles of the human body. Every antagonistic pair of muscles is controlled by the nervous system, which needs b seconds to process a bit. These control processes take place in parallel, and therefore the execution time does not change with dimension. Fitts measured comparable values for the b-constant in the one- and two-dimensional tasks. Also a look at the discrete step model makes clear that the dimensionality of the task does not change the situation. The derivation of the discrete step model (see Section 1.5) does not need any assumption on the dimensionality; the mathematics is the same for the one-, two-, and three-dimensional case.

One question of practical importance for the HCI community is the question of target shapes. Pointing to a word, for example on a menu item, means that the target size differs in direction. Again, the situation is quite clear using common sense.

If a plotter has the task positioning its pen inside a rectangle, the number of bits to transfer does not only depend on the dimensions of the rectangle but also the orientation. Figure 2.1 shows the same rectangle in different orientations. For the

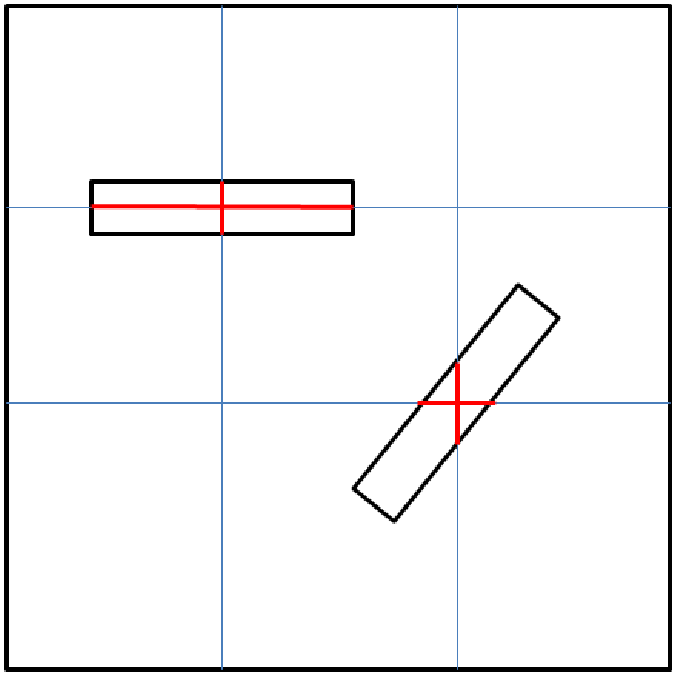

Fig. 2.1 Positioning accuracies depend on the orientation of the rectangle

horizontal rectangle, it takes the minimum number of bits to position the pen in the x-direction. However, in the y-direction, it needs the maximum number of bits. For the diagonally oriented rectangle, it needs a medium number of bits for both directions. With the assumption that the bits for both directions are transmitted in parallel, it means that the pen will arrive earlier in the diagonal-orientated rectangle. The length to hit the diagonally oriented rectangle is $\sqrt{2}$ bigger than the small length to hit in the horizontal-oriented rectangle. The target width is in the denominator when calculating the ID, and as $log_2(1/\sqrt{2}) = log_2(2^{-1/2}) = -1/2$, the ID is half a bit less. With a b-constant of 100 milliseconds/bit, the pointer will be 50 milliseconds earlier at the target.

Figure 1.22 shows that execution times in a Fitts' Law experiment have a standard deviation in the magnitude of 100 ms to 200 ms. A subject has to do quite a few pointing tasks to prove the 50 ms difference for the different target orientations.

The situation for steering a mouse with the hand into the target is the same as for the plotter. However, we have to look at the coordinate system of the motor space and not the device (screen) space. Typically, the motor space has a curved coordinate system (see Figure 2.2) because the angles of our joints define the motor space.

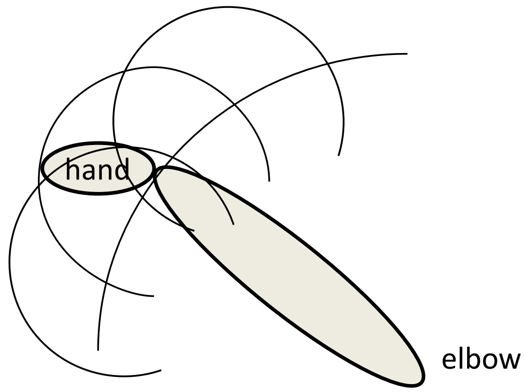

Fig. 2.2 Motor space of a hand controlling a mouse

In most situations, the motor space has a higher dimension than the device space. This means there are many possible arm postures with the fingertip in the same location. To calculate the consequences of target shape and orientation, the target has to be mapped into motor space first.

> For multidimensional targets, the minimum size rules the acquisition time.

2.2 The Steering Law

One valuable extension to Fitts' Law from the HCI community is the steering law of Accot and Zhai [2]. The idea behind the steering law is that steering a vehicle through a narrow tunnel needs more time than through a wide tunnel. The same is true for steering the mouse through cascading menus.

The question for the steering law is: How much time T does it need to steer the mouse through a tunnel of length A (A stands for the amplitude to be consistent with Fitts' Law) and width W?

Accot and Zhai give the following relation:

$$T = a + b \cdot \frac{A}{W} \tag{2.1}$$

where a and b are empirically determined constants.

This lecture discusses the derivation of this formula by Accot and Zhai later and presents another approach first. This approach[1] is a model with a speed-accuracy trade-off similar to the discrete step mode (see Section 1.5). A movement within a single time unit t can be short or long, but the movement accuracy is a fraction of the covered length. The maximum movement within one time unit is a movement where the error circle does not exceed the width of the tunnel. Figure 2.3 illustrates the situation.

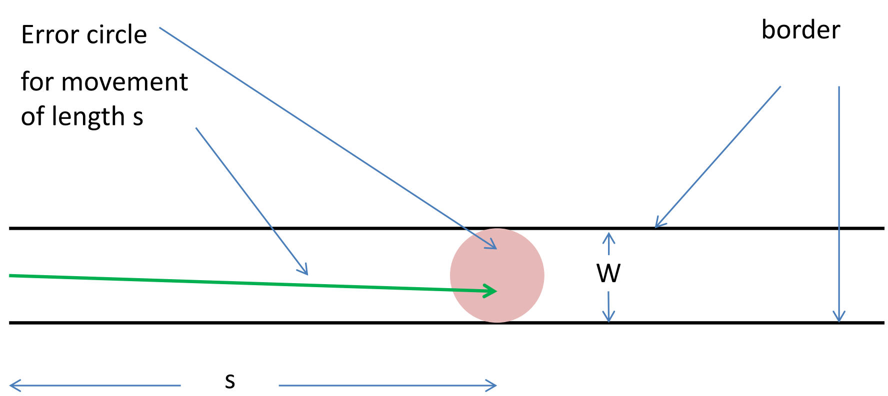

Fig. 2.3 Steering through a tunnel. The red area shows the error circle associated with the movement length s.

[1] This is the idea of the author; as it is simple and obvious it may be published by somebody already.

For simplicity, we assume a straight tunnel of constant width W. Let f be the factor to calculate the radius r of the error circle from the length s of the movement:

$$r = f \cdot s \tag{2.2}$$

For not bumping into the borders of the tunnel the error circle has to be smaller than the width of the tunnel W:

$$r < \frac{W}{2} \tag{2.3}$$

Together with the relation of distance and error (2.2) and an equal sign for the maximum distance we get

$$f \cdot s = \frac{W}{2} \tag{2.4}$$

and solved for s:

$$s = \frac{W}{2f} \tag{2.5}$$

Now it is easy to calculate the time T for steering through a tunnel of length A and width W. As we move distance s in one time unit t we have to divide the total length A by s and multiply by t:

$$T = t \cdot 2f \cdot \frac{A}{W} \tag{2.6}$$

Together with a reaction time, this formula is the same as given by Accot and Zhai (formula 2.1), however, derived from different assumptions and with much simpler mathematics.

The approach of Accot and Zhai is much more complex. They divide the steering task into n subtasks of Fitts' Law-type with tunnel length A/n, use MacKenzie's formula, do an infinitesimal transition, and do a Taylor series expansion.

$$ID_{tunnel} = n \cdot log_2(\frac{A}{n \cdot W} + 1) = log_2((\frac{A}{n \cdot W} + 1)^n) \quad n \rightarrow \infty \tag{2.7}$$

The argument of the logarithm is a well-known series (the limit definition of the exponential function $e^{A/W}$ by Euler). The problem is that with Fitts' original formula, which means without the 1, the infinitesimal transition converges to zero. Therefore, it is doubtful whether it is possible to compose a steering task from infinitely small Fitts' Law-type tasks. Nevertheless, Yamanaka replicated the infinitesimal approach in 2016 [31].

Formula 2.1 and 2.6 shows that the traveling time T is proportional to the length A of the tunnel, which is what we expect by common sense. If we know the time it needs to steer through a certain tunnel, we can connect the same tunnel at the end of the first tunnel, and the traveling time will be double. Concatenating n tunnels with traveling times t_i will result in a traveling time which is the sum of all t_i. If the tunnel has changing widths, it is possible to divide the tunnel into pieces with constant width and sum the times needed for the pieces. This gave Accot and Zhai the idea to calculate the steering time for a tunnel with changing widths from a path integral. They also proposed calculating complex paths, means curved paths, by integration over the curvilinear abscissa. This will hold for 'smooth' curves (see Figure 2.4). Smooth here means that the radius of the curvature is much bigger than the length s.

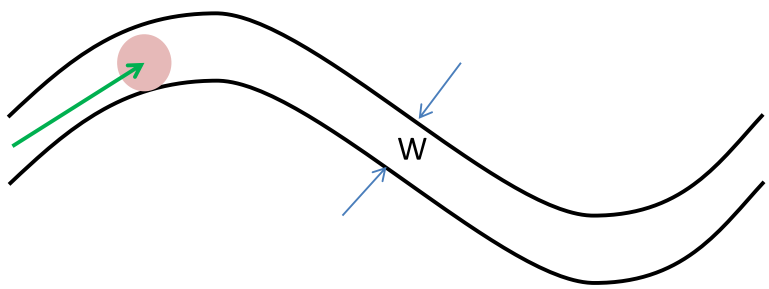

Fig. 2.4 The time to steer through a curved tunnel can be calculated by a composition of parts as long as the curves have only moderate changes in direction and width.

Accot and Zhai expressed it by:

$$ID_{tunnel} = \int_0^A \frac{dx}{W(x)} \tag{2.8}$$

This is a very elegant way to express it and works in 'nice' conditions. However, it suggests a universal validity, which does not exist, as it ignores that there is something like a characteristic length (the s in Figure 2.3). Strictly speaking it is a characteristic ratio (the f in formula 2.2) as the geometry is free of scale. Figure 2.5 shows two tunnels with the same ID according to formula 2.8. Even without conducting a user study it is clear that it takes more time to steer the mouse through the lower tunnel.

As soon as the radius of the curves is smaller than the distance s the situation becomes complex. Figure 2.6 shows a tunnel, where it is not even clear which is the path through it.

While the idea of the steering law is good and probably the best contribution from HCI to Fitts' Law, the mathematical derivation of the formula by Accot and Zhai seems to have some flaws. From a designer's point of view, however, the discussion presented above, and common sense, are good enough to design proper interfaces.

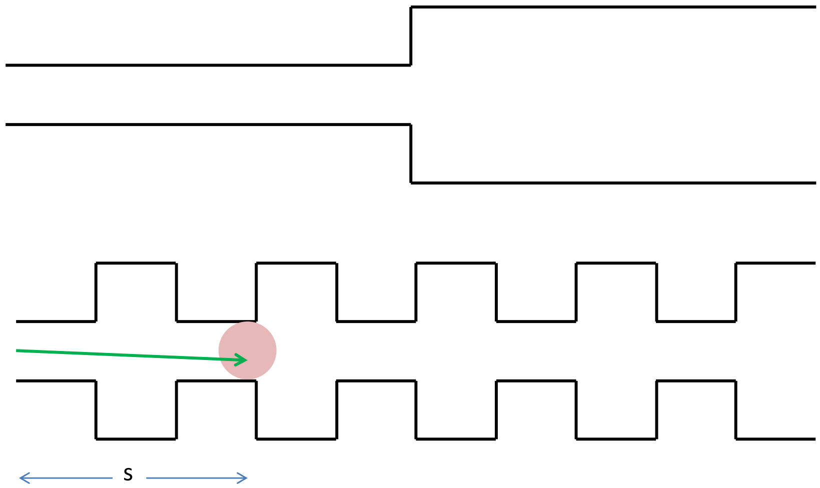

Fig. 2.5 Two tunnels with the same ID according to Formula 2.8

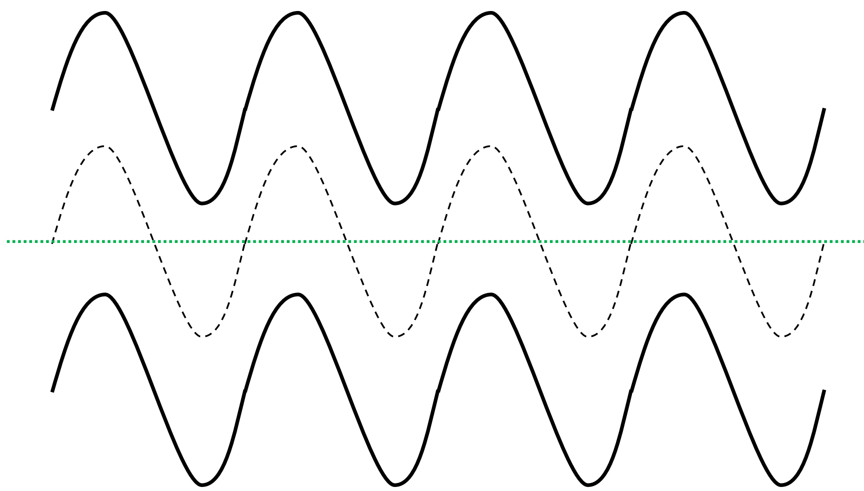

Fig. 2.6 Two possible paths - dashed and dotted - through a tunnel

Annotation: It is not possible to model the steering of a mouse through a cascading drop-down menu solely with the Steering Law. A drop-down menu creates a vertical tunnel. This tunnel is wide and the task is to hit the correct row, which is a Fitts' Law task. At the moment the mouse pointer is in the correct row, the task becomes a steering task as the mouse pointer has to stay in the row to keep the right cascading menu open. However, staying with the mouse pointer within the row is not mandatory. Finally, it is only necessary to exit the drop-down menu through the correct row. Again, this is a Fitts' Law task with the row height as the target size. Steering a mouse through a cascading drop-down menu is a task that is rather complicated to model (see Figure 2.7).

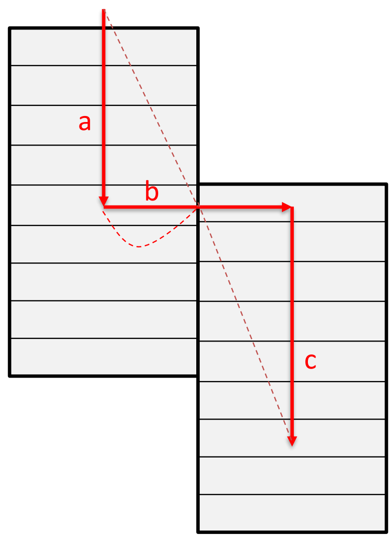

Fig. 2.7 Steering a mouse through a cascading menu is a complex task. Segments 'a' and 'c' are Fitts' Law tasks. Segment 'b' is either a Fitts' Law task or a steering task. The dotted segments are alternative paths through the menu.

Chapter 3
Fitts' Law in the HCI Community

Abstract This chapter discusses the problematic aspects of Fitts' Law in the HCI community. The first topic is MacKenzie's theory. A further topic is the trough-put defined in ISO 9241-9 and the question of whether the *b*-parameter is a property of the pointing device. Then a discussion follows whether Fitts' Law applies to eye movements. The chapter closes with the notion of a Fitts' Law filter bubble and echo chamber in the HCI community.

3.1 MacKenzie's Theory

Shannon published his Information Theory in 1948. A short time later, in 1954, Fitts applied information theory to the information processing capacity of the human nervous system [10]. In 1989 MacKenzie found Fitts' research and recognized that Fitts' Law applies to mouse devices. Up to that point, everything was fine. However, MacKenzie thought that Fitts' theory was incorrect and that he knew better.

MacKenzie writes in his *'Note on the Information Theoretic Basis for Fitts' Law'*:

> *'we demonstrate that Fitts' choice of an equation that deviates slightly from the underlying principle is perhaps unfounded'* [17]

and

> *'Fitts recognized that his analogy was imperfect.'* [17]

For a perfect formula, MacKenzie suggests a

> *'direct analogy with Shannon's Theorem 17'* [17]

and writes:

'It is the purpose of this note to suggest that Fitts' model contains an unnecessary deviation from Shannon's Theorem 17' [17].

These are strong statements, as they imply that psychology researchers were aware that the formula was imperfect but were unable to fix it within the last 35 years. Section 1.3 explains why Shannon's Theorem 17 does not apply to Fitts' Law, and this means that MacKenzie's theory is unfounded. In consequence, all research that uses MacKenzie's theory is unfounded too.

It seems that MacKenzie did not understand Fitts' theory and the definition of the Index of Difficulty (ID), especially his reason for the presence of a '2'. MacKenzie justifies the imperfectness of Fitts' formula with:

'The "2" was added to avoid a negative ID when $A = W$; however, $log_2(2A/W)$ is zero when $A = (W / 2)$ and negative when $A < (W / 2)$.' [17]

This statement is correct, but does not mean that Fitts' formula is incorrect. When $A < (W/2)$, the pointer is already inside the target (see Figure 3.1) so the pointer entered the target in the past, which means negative time.

MacKenzie suggests the following formula:

$$MT = a + b \cdot log_2((A + W)/W) \tag{3.1}$$

which can be also written as:

$$MT = a + b \cdot log_2(A/W + 1) \tag{3.2}$$

In MacKenzie's view, this solves the problem with Fitts' formula as the '+1' ensures positive IDs.

'It is noteworthy that, in the model based on Shannon's theorem, ID cannot be negative.' [17]

Figure 3.1 shows the Index of Difficulty according to Fitts in the upper row and according to MacKenzie in the lower row. The closer the pointer is to the target, the less difficult the task. When the pointer is at the edge of the target, the pointing task is fulfilled and Fitts' ID is zero. Without the '2' in the formula, the ID at the target edge would be '-1'. This is what Fitts mentioned in his paper (see also Section 1.2):

'The use of 2A rather than A is indicated by both logical and practical considerations. Its use insures that the index will be greater than zero for all practical situations.' [10]

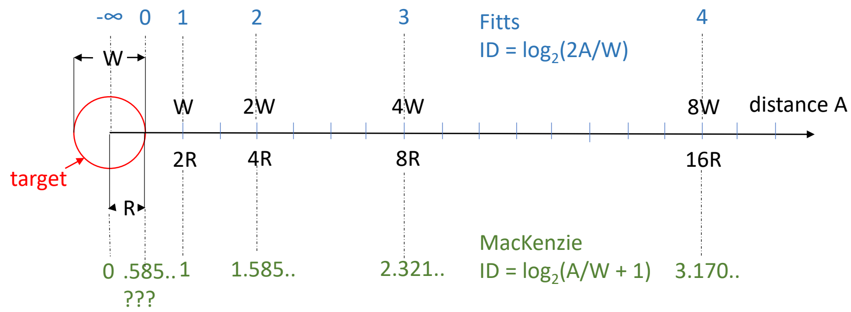

Fig. 3.1 Index of Difficulty over pointer distance to target according to Fitts (upper) and MacKenzie (lower). It is hard to understand why according to MacKenzie there is still some difficulty left when the pointer reached the target. And why is Fitts' formula imperfect?

Fitts' ID fulfills all expectations and is not 'imperfect'. In particular, the presence of the factor of '2' is correct. The ID is the logarithm of the ratio of the distances from the target center to the pointer and to the target edge. The distance to the target edge is half of the diameter and this is where the '2' comes from.

Fitts' definition of the ID has vivid interpretations. If the pointer moves with every step to the middle of the remaining distance to the target center, the ID is the number of steps needed to reach the target (see Section 1.4). Alternatively, the ID is the number of binary digits of the pointer distance measured in units of target radius.

With MacKenzie's definition, the ID for a pointer at the target edge is $log_2(3/2) \approx 0.585$. It is hard to understand why there is still some difficulty left when the pointer has reached the target and the task is done. Such ill-defined ID causes negative intercepts. If the pointer is at the target edge the task is fulfilled and the execution time is zero. With a positive ID it needs a negative a to get a time of zero. MacKenzie himself reports on negative intercepts in his publication from 1991 [19]

'however, a large, negative intercept appeared for the trackball-dragging combination (-349 ins). .... a negative prediction would only occur for ID< 0.5 bits.' [19]

There is nothing wrong with Fitts' definition of the ID, while MacKenzie's definition is not even plausible.

> If there is nothing wrong with Fitts' formula, there is no need for an alternative formula.

As MacKenzie's main argument is that Fitts' formula is imperfect, the arguments given above should be enough to ban MacKenzie's formula. However, as MacKenzie's theory is extremely popular, this lecture goes on with a discussion of the 'direct analogy'. MacKenzie complains:

> *'The reason Fitts did not use Shannon's original equation was not stated.'* [17]

However, MacKenzie did not give any reason why this theorem should apply to pointing movements. Theorem 17 implies that there are at least 16 further theorems which could be used for an analogy.

The reason why Fitts did not use Shannon's Theorem 17 is very simple. Theorem 17 is not helpful for defining an ID. Theorem 17 calculates information capacity or bandwidth. As information capacity is the limiting factor in Fitts research hypothesis, Fitts mentioned the formula of Theorem 17 in a footnote. Within the context of Fitts' Law, Theorem 17 allows one to estimate a value for the b-parameter, or $1/b$ to be precise, but has nothing to do with the ID. The ID is a pure geometric value depending only on lengths and has no time aspect. Therefore, Fitts used an analogy to amplitudes (explained in Section 1.2).

It is more than questionable whether MacKenzie's 'direct analogy' to Shannon's Theorem 17 is legitimate. Shannon's Theorems are valid for channels but the control-feedback loop for a Fitts' Law task is not a channel according to Shannon's definition (see Figure 1.7). This means that Shannon's Theorem 17 is not applicable.

Shannon's Theorem 17 has bandwidth as result, which is a frequency and is measured in bits/second. Why should bandwidth be analogous to completion time, which is measured in seconds? And why should power be analogous to diameter? Power is proportional to the square of the amplitude, and the amplitude relates to radius and not to diameter. Choosing a formula based on appearance and just swapping letters is naïve.

> Shannon's Theorem 17 has nothing to do with the Index of Difficulty (ID).

MacKenzie calls his theory the 'Shannon Model'. This is misleading as it sounds like Shannon himself made the model, giving MacKenzie's formula the scientific credibility of Shannon. Of course, MacKenzie could not name his formula after himself, but the HCI community could clearly refer to either Fitts' theory and Fitts' formula or MacKenzie's theory and MacKenzie's formula.

An unexplained aspect of MacKenzie's formula is the mysterious a-parameter. It appears out of nowhere without any explanation and is referred to later as the intercept. The a-parameter is not part of Shannon's Theorem 17 and also does not occur in Fitts' publication. The reason why it does not appear in Fitts' publication is that reaction time was involved in his experiment. If used together with Fitts' ID, the a-parameter is a reaction time. Together with MacKenzie's ID, the a-parameter loses this meaning.

Dropping the factor of 2 in Fitts' formula (1.7) does not affect the value of the b-parameter. However, it affects parameter a.

$$T = a + b \cdot log_2(\frac{2A}{W}) \tag{3.3}$$

$$= a + b \cdot (log_2(\frac{A}{W}) + log_2 2) = a + b + b \cdot log_2(\frac{A}{W}) \tag{3.4}$$

$$= a' + b \cdot log_2(\frac{A}{W}) \tag{3.5}$$

with

$$a' = a + b \tag{3.6}$$

This means that parameter a does not have the meaning of reaction time anymore. Therefore, some people call the a'-constant the 'non-informal parameter', which contributes to the confusion.

MacKenzie's theory damages the meaning of the a-parameter.

MacKenzie states that with his definition for the ID, experimental data yield a better correlation. This seems to be true. However, if a high value for correlation is the goal, the addition of 2 or 3 instead of 1 in MacKenzie's formula (3.2) results in even better correlations (at least with the author's data sets). MacKenzie did not examine his data in that direction. However, the question is, whether correlation, especially on pre-averaged data, is the right value for judging the quality of a formula. Section 1.9 discusses this in detail.

Section 1.7 explains that empirically fitted formulas can match experimental data better than a model. However, empirically fitted formulas do not explain the underlying principles.

In MacKenzie's next publication, together with A. Selen and W. Buxton, he refers to his own formula with:

'There is recent evidence that the following formulation is more theoretically sound' [19].

Scientists should not confirm their own results, and if they do, these results should not be taken as evidence. The statement that MacKenzie's formulation is more theoretically sound should therefore be doubted.

A further problem with this publication is that the value of the b-parameter is assigned to the pointing devices. According to Fitts, however, the b-parameter is a property of the human nervous system. This is the topic of the Section 3.3.

3.2 Other Theories and Formulas for Fitts' Law

There are further formulas for Fitts' Law. One is known as the Welford formulation:

$$T = a + b \cdot log_2(\frac{A}{W} + 0.5) \tag{3.7}$$

MacKenzie published his theory online[1] and added a link to a review[2] of his publication, which was done by Welford. In this review Welford approves of MacKenzie's formula. As Welfod agrees to MacKenzie's formula he votes against his own formula.

There are plenty of extensions and alternatives for Fitts' Law. These contributions are spread over different disciplines, such as psychology, human computer interaction, human movement science, and biological cybernetics. Some publications in the field build their research on MacKenzie's formula, others use Fitts' formula and do not even mention MacKenzie's theory. Some of them are solid, valuable science, while others are questionable. The problem lies in finding out which is which. The number of publication is enormous and it will probably take a lifetime to work through all of them. MacKenzie published a bibliography of research on Fitts' Law[3] with 310 entries, but the last update is from 25-Jun-02, which is nearly two decades ago. However, research on Fitts' Law did not stop.

The following formulas for the ID are only few examples selected by impressiveness. The following definition for the ID is from Zhang et al. [35] who used it for eye pointing target acquisition. However, Fitts' Law does not apply to eye movements (see Section 3.4).

$$ID_{eye} = \frac{e^{\lambda A}}{W - \mu} \tag{3.8}$$

In 2011, Soukoreff, Zhao, and Ren published a paper with the title *'The Entropy of a Rapid Aimed Movement: Fitts' Index of Difficulty versus Shannon's Entropy'* [27]. It is a further application of information theory and also an example for a very impressive definition of a further ID:

$$ID_{entropy}(U, W) = m + log_2(U) - \frac{1}{2} \cdot log_2(\pi e \cdot \frac{W^2}{8}) + 1) \tag{3.9}$$

The authors write that they examine the question: *'What is the precise relationship between the index of difficulty and entropy?'* [27]. It is up to the reader to find the answer in the paper.

[1] http://www.yorku.ca/mack/JMB89.html

[2] http://www.yorku.ca/mack/WelfordReview.html

[3] http://www.yorku.ca/mack/RN-Fitts_bib.htm

In 2018, Gori published a TOCHI paper [13] and a PhD thesis with the title '*Modeling the Speed-Accuracy Tradeoff Using the Tools of Information Theory*'. Gori writes:

'*Elias [..], an important figure of the information theory society to which we will return in Chapter 6, urged authors – using a very ironic, even aggressive tone – to stop writing approximative papers that abused information theoretic results and concepts. Fitts' work, based on a loose analogy with Shannon's Theorem 17, is a good example of abuse of information theory*

- *Why should D/W of Fitts' Law be analogous to P/N as defined in Shannon's Theorem 17?*
- *What is the bandwidth BW of Shannon's Theorem 17 analogous to in Fitts' Law? There seems to be no reason to identify BW to 1/MT beyond the fact that both are expressed in the same physical units (s^{-1}).*
- *Since D and W are amplitudes while P and N are powers, what happened to the squares?*' [12, p. 36]

Gori's accusation that Fitts abused information theory definitely does not have an ironic tone. Gori uses Drewes' [7] arguments, originally formulated against MacKenzie's theory, without reference and turns them into an allegation against Fitts. This perverts the facts. Fitts used Shannon's Theorem 17 only to argue that bandwidth limits the pointing speed and did not make an analogy to Shannon's Theorem 17. Fitts' analogy uses amplitudes, which is legitimate and has nothing to do with Shannon's Theorem 17. Actually, it was MacKenzie who introduced the analogy to Shannon's Theorem 17. He writes (already cited in Section 3.1):

'It is the purpose of this note to suggest that Fitts' model contains an unnecessary deviation from Shannon's Theorem 17' [17].

and

'The reason Fitts did not use Shannon's original equation was not stated.' [17]

This means that MacKenzie believed that an analogy to Shannon's Theorem 17 is necessary and complained that Fitts did not use it.

Gori's reference to Elias [9] points to an editorial from the year 1958, where Elias writes about papers using the vocabulary and conceptual framework of information theory:

'*I suggest that we stop writing them, and release a large supply of manpower to work on the exciting and important problems which need investigation*' [9].

This sounds very reasonable – and even polite – and Gori and the HCI community could have heeded Elias' advice.

Authors	Equation	Remarks
Crossman (1956)	$MT = a + b\log_2\left(\frac{A}{W}\right)$	Applying this expression to experimental data, Crossman found that the fit was better and that the constant a had a value of 0.05 sec, which was the time he found that the subject spent lingering on the target.
Welford (1968)	$MT = k\log_2\left(\frac{A}{W} + 0.5\right)$	k is an experimentally determined constant.
Welford et al. (1969)	$MT = a + b_A\log_2(A) + b_W\log_2\left(\frac{1}{W}\right)$	
Jagacinski et al. (1980b)	$MT = c + dA + e(V+1)\left(\frac{1}{W} - 1\right)$	V is the mean velocity of the target movement and c, d, e are fitting constants.*
Jagacinski et al. (1980b)	$MT = p + q\log_2\left\{2\left[A + \frac{V}{W}(MT + T)\right]\right\}$	T is a constant, corresponding to the length of time the cursor had to be held over the target to capture it, and p, q or x, y, z are fitting parameters. It should be noted that this equation is transcendent and, as such, does not provide an analytical solution for MT.*
Jagacinski et al. (1980b)	$MT = x + y\log_2\left(\frac{2A}{W}\right) + z\log_2\left[\frac{V}{W/T} + 1\right]$	T is a constant, corresponding to the length of time the cursor had to be held over the target to capture it, and p, q or x, y, z are fitting parameters.**
Hoffman (1991a)	$MT = \frac{1}{K}\ln\left[\frac{A + \frac{V}{K}}{\frac{W}{2} - \frac{V}{K}}\right]$ and $MT = a + b\log_2\left(A + \frac{V}{K}\right) - c\log_2\left(\frac{W}{2} - \frac{V}{K}\right)$	K, a, b, c are fitting parameters.*
Hoffman (1992)	$MT = -a + b(c + D)\log_2\left(\frac{2A}{W}\right)$	D is the delay; a, b, and c are regression coefficients.***
MacKenzie (1989; 1992)	$MT = a + b\log_2\left(\frac{A}{W} + 1\right)$	
Gan & Hoffmann (1988)	$MT = a + b\sqrt{A}$	
Johnsgard (1994)	$MT = a + b\log_2\left(\frac{A/W}{G} + 1\right)$	
Kvalseth (1980)	$MT = a\left(\frac{A}{W}\right)^b$	

*If $V = 0$, we obtain an expression of MT for the case of static targets, which is different from Fitts' law.
**If $V = 0$, we obtain an expression of MT for two cases of static targets, similar to Fitts' law.
***If $D = 0$, we obtain an expression of MT for the no-delay case, similar to Fitts' formulation.

Fig. 3.2 An overview on variants of Fitts' Law as given by Plamondon et al. [21]

To finish this section, Figure 3.2 shows an overview of variants of Fitts' Law as given by Plamondon et al. [21]. This lecture refrains from discussing all these theories or formulas. However, it raises the question of the benefit of these theories. Is there any practical impact? Do these theories contribute to knowledge, clarification, and understanding or do they increase confusion? How much Fitts' Law research do we need until the topic is understood?

3.3 The *b*-parameter and the Throughput in ISO 9241-9

Beside the fact that the performance, or bit transfer, in Fitts' experiments did not depend on the weight of the stylus, the bit transfer was within 8 to 12 bits/second for all tasks. This means that the human nervous system has a general limit for control tasks independent from the details of the task.

Therefore it is surprising that the International Standardization Organization defined a throughput TP based on Fitts' Law to characterize the performance of input devices in ISO 9241-9. There is a critical voice against the definition of TP [32] by Zhai. His critique is that ISO 9241-9 does not use the a- and b-constant but defines the throughput TP by the ratio of the Index of Difficulty and trial completion time.

However, it is questionable whether the ISO standard of throughput makes sense at all. Seeing the performance as a property of a device is the opposite of Fitts' idea, who sees performance as a property of the nervous system. Comparing a small laptop mouse against a big desktop mouse in the context of a student exercise did not show differences in performance. Comparing a trackball against a standard mouse device typically shows that people perform better with the standard mouse device. However, people who use a trackball for their daily work show the same performance as with a standard mouse device. This means the performance of a pointing device depends on the subject and especially on her or his training on the device.

Fitts showed that the pointing performance does not depend on the mass and the size of the pointing device (see 1.2). In consequence, the throughput of a mouse device does not depend on the size or weight of the mouse. This raises the question of which properties of the mouse influence the throughput. Or to ask the other way round: how to build a mouse with a 'good throughput'?

It seems that nobody uses this ISO standard, and manufacturers of mouse devices do not state a throughput value for their products. Instead, they refer to a dpi-value (dots per inch), which is the resolution of the mouse device's optical sensor. This dpi-value determines the maximum control-gain ratio. The control-gain ratio is the ratio of the distance the mouse pointer covered on the screen and the distance the mouse covered on the table. A high dpi-value allows for moving the mouse at a high speed and reduces the necessary space for the mouse on the table. However, this does not increase the accuracy. Mouse devices for gamers advertised with high dpi-values often come with a sniper mode button, which reduces the dpi-value.

Modern graphical operating systems offer adjustments to the control-gain ratio and mouse acceleration. Again, the mouse acceleration decreases the distance the mouse and the hand have to cover, and therefore the physical workload, but does not increase the number of bits a human can transfer to the mouse. A high control-gain ratio speeds up the work on a graphical user interface as many pointing operations aim for large targets, such as an application window, which are not ruled by Fitts' Law (see Section 1.8).

Exercise: Discuss the throughput TP for a mouse operated with the feet.

3.4 Fitts' Law does not Apply to Eye Movements

This chapter would not be necessary, if there were not a handful of publications, which did a Fitts' Law evaluation for eye movements, for example [30], [20], [34], [29], [35], [33]. Psychology textbooks state that the eyes move ballistically which is the opposite of Fitts' Law. Also, Sibert and Jacob expressed themselves as skeptical of the validity of Fitts' Law for the eyes [26].

Ballistic movements do not depend on target size. Psychology textbooks also give a formula for the time the eyes need to position on a target. This formula was given by Carpenter[4] [22] and is independent of the target size:

$$T = 21ms + 2.2ms/^{\circ}{\cdot}A \tag{3.10}$$

A is the amplitude of the eye movement measured in degrees, as the eye movement is a rotational movement. Formula 3.10 assumes a linear relation and was approximated by fitting experimental data. See Abrams, Meyer, and Kornblum [1] for a better eye speed model assuming a gradually increasing eye muscle force. Assuming a constantly increasing muscle force results in a cubic root relation. In both approaches, the eye movement time does not depend on target size. It is hard to understand why some publications in the field of HCI ignore the results of psychology completely, and also do not even listen to warning voices [26] from their own community. Additionally, the publications which experimentally 'proved' Fitts' Law for the eyes made severe mistakes in the evaluation by averaging over IDs first. This means to implicitly assume that Fitts' Law is valid before the proof, which then confirms it. There is a detailed discussion on this mistake in Section 1.9.

Saccades are abrupt eye movements with speeds up to 700°/sec. This means that the visual information on the retina changes quicker than the receptors can process it, and therefore the eye is virtually blind during a saccade. This means that the movement cannot be controlled by a feedback loop, and therefore the movement is ballistic. The situation is comparable to throwing a stone; when the stone leaves the hand, there is no further control over the movement, and the arrival at the target does not depend on the target size.

Assuming that Fitts' Law applies to eye movements not only contrasts the results of psychology but also leaves open questions. The first thing to be discussed is the question of whether target acquisition by the eye is a single- or multi-saccade process. It seems that the eye can position itself at a target with a single saccade with an accuracy, which is enough to bring the target into the small field of the high-resolution vision (fovea).

When aiming for a small target, it is sufficient if the target is finally within the field of high-resolution vision given by the fovea. The target doesn't need to be in the center. Looking at something is like pointing with a pointer of the size of the fovea. The target size is not relevant, as long as it is smaller than the fovea (see also

[4] http://wexler.free.fr/library/files/carpenter (1988) movements of the eyes.pdf, Figure 4.2

Section 1.8). The situation is similar to spotting an insect with a flashlight, depicted in Figure 3.3. Of course, it is possible, especially if demanded, to position the light cone that the insect is in the center, however, normally this is not necessary. For a Fitts' Law experiment with the gaze, it is crucial how the hit condition is defined. If the hit condition is that the reported x-y coordinates, which means the center of the fovea, is on the small target, the hit condition may force a corrective saccade. For seeing the target, the first saccade has sufficient accuracy in most cases. The corrective saccade is necessary to bring the target into the center of the fovea. A gaze-pointing experiment with such a hit condition creates an artificial situation. Additionally, eye trackers have an inaccuracy which should also be considered for the hit condition.

Fig. 3.3 Spotting an insect at the wall with a flashlight does not require that the insect is in the center of the light cone. Also the size of the insect does not matter.

The next thing to discuss is the target and its size. What are targets for the eye when watching a video, and what are the target sizes? What is the target size when looking at a face - the eye, the nose, the mouth, or the whole face? Gaze pointing experiments with big plain circle targets (see Figure 3.4) reveal that the gaze typically does not jump to the target center but rather crosses the target border, as discussed in Section 1.8. It seems that the eye does not have a concept of targets with size but only knows spots of interest. In consequence, the only relevant size for gaze pointing is the size of the fovea. Without a target size, it is impossible to calculate a time with Fitts' formula.

There is no Fitts' Law without a concept of target size.

It is also possible to think about the consequences it would have if Fitts' Law applies to eye movements. One consequence would be that completion times for eye

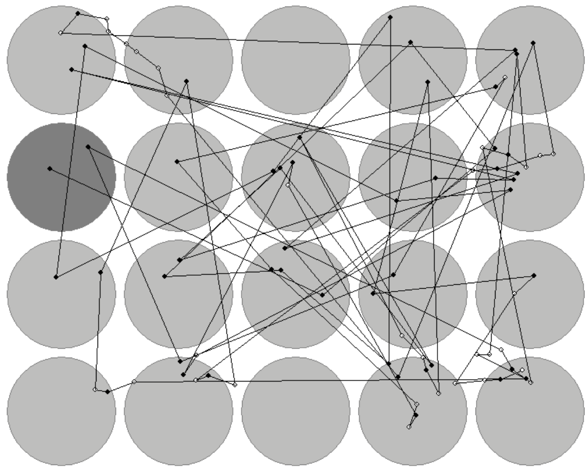

Fig. 3.4 Gaze trail from a gaze pointing experiment where the next target on a grid was highlighted in a random order immediately after the current target was hit by gaze. The gaze typically does not jump to the target centers. The targets had a size of 2°. Taken from [6].

pointing would be independent of the distance of the eyes to the screen. If the eyes change the distance to the screen, both target size and target distance are scaled with the same factor, and the ID does not change. This is valid as long the angles are in a moderate range, meaning $sin(x) \approx x$, or in other words, the effect of a flat display against a curved one is small. This independence from the distance to the screen would be a nice property for gaze interfaces, and it would be easy to show in a user study.

In 2019 Schuetz et al. published a paper with the title *'An Explanation of Fitts' Law-like Performance in Gaze-Based Selection Tasks Using a Psychophysics Approach'* [23]. They write:

'Therefore, saccades are pre-programmed, ballistic movements [...]. Given these reasons, Fitts' Law should not apply to saccadic eye movements'. [23]

and

'As we will show below, movement times of individual saccades are indeed independent of target size and do not follow Fitts' Law.' [23]

If this would be the central statement of the publication, it would be a good clean-up of the mistakes from the past. However, the authors argue that the eyes move Fitts' Law-like. They refer to Errol Hoffmann who published *'Fitts' Law With*

An Average of Two or Less Submoves?' [16]. In simple words, this means if we have a series of submoves, even with ballistic characteristics, the situation approaches the discrete-step model as explained in Section 1.5.

Schuetz et al. argue:

' However, humans frequently generate secondary (''corrective") saccades after a main target directed saccade, especially when aiming for small targets [...]. We show that these additional saccades can explain the Fitts-like relationship between movement time and target size.' [23]

Restricting the statement of Fitts' Law-likeness to eye movements with a secondary saccade, which may happen frequently but not the normally, is 'ingenious'. It avoids pointing out mistakes done by other authors and even legitimizes erroneous, previous research. This can be seen as a diplomatic concession to the HCI community. However, science should have a stricter concept of truth than diplomacy. Additionally, the secondary saccade may be caused by the hit condition as explained above.

The positive aspect of Schuetz's publication is that reviewers can not reject papers on gaze-pointing techniques anymore because of a missing Fitts' Law evaluation as done in the past. However, the publication does not prevent more Fitts' Law-for-the-eyes papers. Zhang et al. published such a paper in 2021 [33].

And as a final remark and just for completeness, a Fitts' Law study of pupil dilations got the result that

'Fitts' index of difficulty had no significant effect on pupil dilation' [4].

3.5 Fitts' Law and the HCI's Scientific Claim

HCI claims to be a science. There are some rules for science. One of these rules is to be free of contradictions. In the moment a contradiction arises a scientific community has to solve it to stay scientific. To have different formulas for the same thing is definitely a contradiction. It is not possible to have a good and a better formula.

There are people who see HCI as a 'soft science' and think that these rules should not be applied too strictly. However, the contradicting formulas of Fitts and MacKenzie are built on information theory, which is a hard science, and as such demands strict rules. People who want to do 'soft science' should stay away from information theory and other hard sciences.

When MacKenzie states that *'Fitts' choice of an equation that deviates slightly from the underlying principle is perhaps unfounded'* and *'Fitts recognized that his analogy was imperfect'* [17], he is definitely issuing a critique – not to the HCI community, but to psychology.

With the same right with which MacKenzie claims that Fitts' formula is imperfect, and in contrast well-founded, it is possible to state:

1. Fitts' formula is perfect.
2. MacKenzie's theory is unfounded and his direct analogy is naïve.
3. The *b*-parameter belongs to the human and is not a property of a device.
4. Fitts' Law does not apply to eye movements.

These claims have yet to be confirmed or disproved by the HCI community, Statement 1 and 2 at least since the publication of *'Only one Fitts' Law formula please!'* [7] in the year 2010.

Despite the misleading title *'An Explanation of Fitts' Law-like Performance in Gaze-Based Selection Tasks Using a Psychophysics Approach'*, Schuetz et al. [23] confirmed Statement 4 in 2019. The fact that manufacturers of mouse devices do not state the *'through-put TP'* in the data sheets supports Statement 3. Another critique of MacKenzie's theory, which agrees with Statement 2, was written by Errol Hoffmann [14] in 2013 . Hoffmann's critiques were published in the same journal (JMB) where MacKenzie published his note.

A general confirmation of the four statements by the HCI community, however, is still pending and seems unlikely to happen. The consequence of a confirmation would be the invalidation of several hundred peer-reviewed scientific HCI papers. Many authors of these papers still publish and are in leading positions within the HCI community. The problem is not MacKenzie's theory, but the 21286 citations[5] of his work.

[5] https://scholar.google.ca/citations?user=G9MSEncAAAAJ&hl=en looked up at 28.3.2023

In *'Seven HCI Grand Challenges'* the authors write:

'Besides "filter bubbles", other technological perils to democracy include fake news, echo chambers (i.e., shared social media bubble with like-minded friends, resulting in restricted access to a diversity of views), and agenda shaping by increased visibility of the most popular stories in media.' [28]

Fitts' Law research in HCI seems to fulfill the criteria of a filter bubble and an echo chamber. The filter is the review process. Critical paper submissions are typically reviewed by the criticized researchers, who reject critiques of their own research. In consequence, only research which confirms questionable theories gets published and this forms an echo chamber. The big number of already existing questionable publications is used as the argument to reject reasonable paper submissions. However, science is not a democracy.

The narrative told by the members of the the Fitts' Law filter bubble is that psychology applied information theory to pointing movements, but was not able to do it correctly. As mentioned above, MacKenzie [17] states that Fitts' theory is unfounded and Fitts' formula is imperfect and Gori [12, p. 36] says that Fitts abused information theory. The members of the bubble, however, have greater abilities and know how to do it right. They are so self-confident that they do not need consultation from external experts and can ignore critical voices. It is out of the question that it could be the other way round and they are wrong. External expertise could come from the information theory society[6] or the physics department of the local university.

The Fitts' Law filter bubble, or the MacKenzie bubble, puts into peril HCI's scientific claim and damages the credibility of the whole community. Members of the HCI community who are not involved in Fitts' Law research should worry and take action to save their reputation. With the current tendencies in society towards 'alternative facts', we are in danger of losing the truth. It is more important than ever for the HCI community to establish proper scientific standards and defend scientific truth. The author's recent publication, 'The Fitts' Law Filter Bubble' [8] is another attempt to create an awareness of the problem in the HCI community.

The Fitts' Law bubble in HCI has existed since at least 1989 with MacKenzie's theory and is older than any social media bubble. Social media bubbles and echo chambers are very difficult to shatter. However, ignoring the problem would be ignorant and the problem will get even worse. It seems that the HCI community has a challenge.

[6] https://www.itsoc.org/

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